Midterm Examination Sensing and Actuation AT74.03 March 1, 2022

Time: 10:00-11:30 h. Open Book
Marks: 100

Attempt all questions.

Q.1 The signal for a DC servo motor is a PWM signal at 50 Hz. The duty cycle of the signal indicates the desired position of the motor. When the on (5V) period of the PWM signal varies from 1 ms to 2 ms the motor moves from -90° to 90° linearly. Design a signal conditioning circuit that convert the 50-Hz PWM signal (1-2 ms on period) to analog signal (0-5 V) whose noise at 50 Hz is attenuated for 3.01 dB. (25)

Solution

1. LPF circuit is used to convert PWM signal to DC signal.

A low pass filter with attenuation rate of 3.01 db at 50 Hz is used to convert PWM signal to dc signal.

$$\frac{E_0}{E_i} = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$0.5 = \frac{1}{\sqrt{1 + (50 \times 2\pi RC)^2}}$$

$$RC = 0.0055$$

Select $R = 1k\Omega, C = C = 5.5\mu F$.

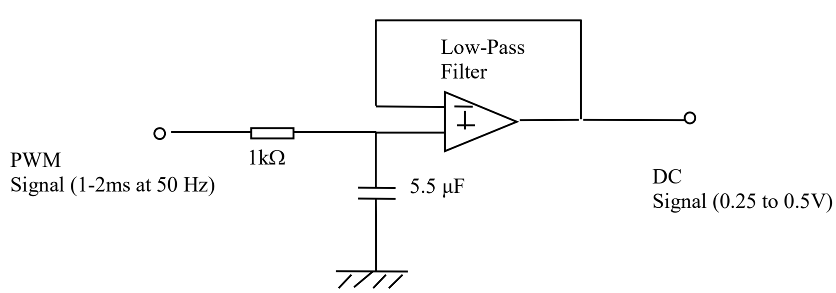
Determine the average of the PWM signal when the on period varies from 1 ms to 2 ms.

The average when the on period is 1 ms is determined as

$$V_{1ms} = \frac{on\ period}{whole\ period} V = \frac{0.001s}{1/(50Hz)} \times 5 = 0.25\ V$$

The average when the on period is 2 ms is determined as

$$V_{2ms} = \frac{on\ period}{whole\ period} V = \frac{0.002s}{1/(50Hz)} \times 5 = 0.5\ V$$



2. Differential amplifier circuit or summing circuit is used to convert dc (0.25-0.5 V) to (0-5 V)

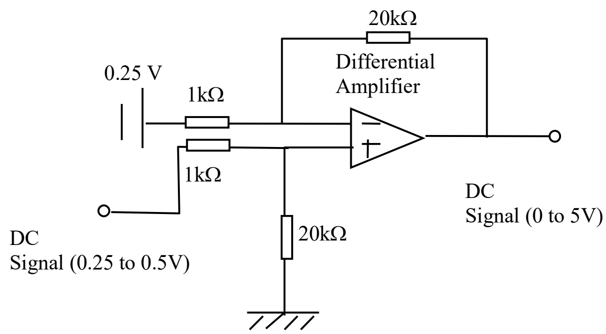
The required gain is determined.

$$G = \frac{Change\ of\ output}{Change\ of\ input} = \frac{5}{0.25} = 20$$

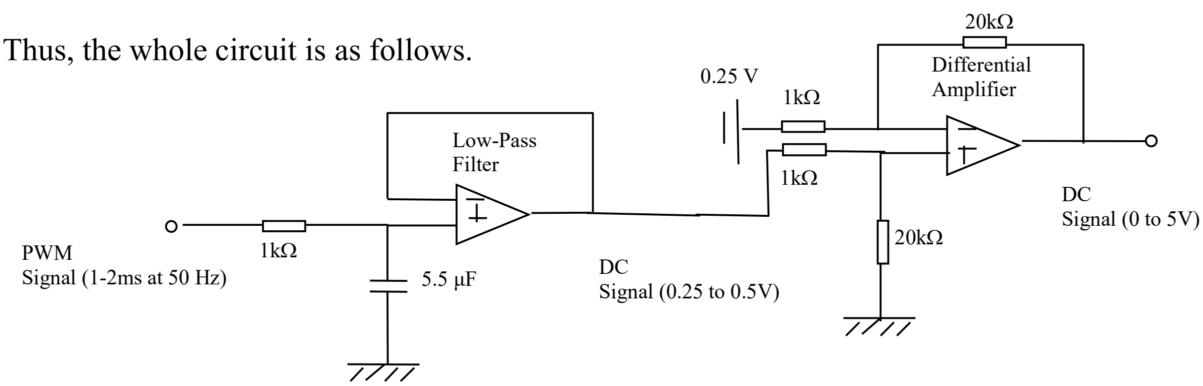
$$\frac{R_f}{R_1} = \frac{R_3}{R_2} = 20$$

Select $R_1 = R_2 = 1k\Omega, R_f = R_3 = 20k\Omega.$

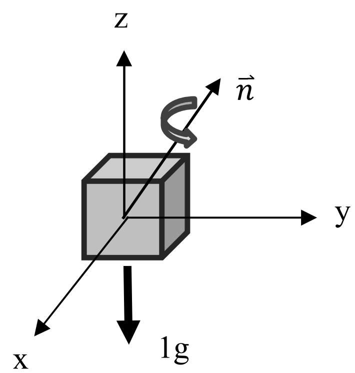
A constant voltage of 0.25 V is provided to the inverting input pin.



Thus, the whole circuit is as follows.



Q.2 A 3-axis accelerometer is used to determine the attitude of an object rotating freely at the sea level. Assume the accelerometers are identical in all axis, the seismic mass deflects 1 mm from the acceleration of 1g (9.8 ms^{-2}). Determine the deflections of seismic mass in all axes of the accelerometer when the object rotates around a normal vector $\vec{n} = \frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j + 0k$ for 90° as shown in the figure below. (25)



Solution

Determine new coordinate of $(\acute{x}, \acute{y}, \acute{z})$ from the old coordinate of $(x, y, z) = (i, j, k)$.

The new coordinate can be determined by any methods; for examples, using vector consideration, using Rodrigues formula, using Quarternion formula, etc.

By Rodrigues formula,

$$\acute{x} = \big(1 - cos(\theta)\big)(x \cdot \vec{n})\vec{n} + cos(\theta)x + sin(\theta)(\vec{n} \times x)$$

$$\acute{x} = \big(1 - cos(90°)\big)\left(i \cdot \left(\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j + 0k\right)\right)\left(\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j + 0k\right) + cos(90°)i$$
$$+ sin(90°)\left(\left(\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j + 0k\right) \times i\right)$$

$$\acute{x} = (1)\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j + 0k\right) + (0)i + (1)\left(0j - \frac{1}{\sqrt{2}}k\right)$$

$$\acute{x} = (0.5i + 0.5j - 0.707k)$$

Thus, the deflection of seismic mass along x axis from gravity acceleration is 0.707 mm.

$$\acute{y} = \big(1 - cos(\theta)\big)(y \cdot \vec{n})\vec{n} + cos(\theta)y + sin(\theta)(\vec{n} \times y)$$

$$\acute{y} = \left(1 - cos(90°)\right)\left(j \cdot \left(\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j + 0k\right)\right)\left(\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j + 0k\right) + cos(90°)j$$

$$+ sin(90°)\left(\left(\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j + 0k\right) \times j\right)$$

$$\acute{y} = (1)\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j + 0k\right) + (0)j + (1)\left(0i + \frac{1}{\sqrt{2}}k\right)$$

$$\acute{y} = (0.5i + 0.5j + 0.707k)$$

Thus, the deflection of seismic mass along y axis from gravity acceleration is -0.707 mm.

$$\acute{z} = \left(1 - cos(\theta)\right)(z \cdot \vec{n})\vec{n} + cos(\theta)z + sin(\theta)(\vec{n} \times z)$$

$$\acute{z} = \left(1 - cos(90°)\right)\left(k \cdot \left(\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j + 0k\right)\right)\left(\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j + 0k\right) + cos(90°)k$$

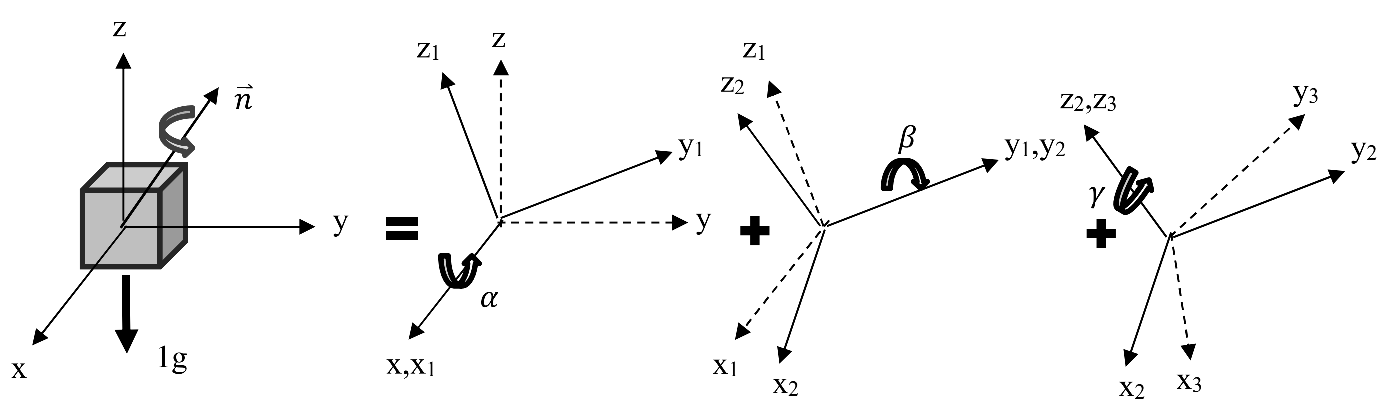
$$+ sin(90°)\left(\left(\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j + 0k\right) \times k\right)$$

$$\acute{z} = (1)(0)\left(\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j + 0k\right) + (0)k + (1)\left(\frac{1}{\sqrt{2}}i - \frac{1}{\sqrt{2}}j\right)$$

$$\acute{z} = (0.707i - 0.707j + 0k)$$

Thus, the deflection of seismic mass along z axis from gravity acceleration is 0 mm.

Q.3 If 3 single-axis rate-integrating gyros are used to measure rotations around the Euler coordinate of (x, y, z) or (roll (α), pitch (β), and yaw (γ)) of the object in **Q.2**. Assume all the gyros are identical, the gyro deflects 1° from the angular displacement of 10°. Determine the deflections of the gyros in successive rotations of roll, pitch, and yaw when the object rotates around a normal vector $\vec{n} = \frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j + 0k$ for 90° as shown in the figure below. (25)



Solution

Rotation matrices are determined.

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\alpha) & -sin(\alpha) \\ 0 & sin(\alpha) & cos(\alpha) \end{bmatrix}$$

$$R_y = \begin{bmatrix} cos(\beta) & 0 & sin(\beta) \\ 0 & 1 & 0 \\ -sin(\beta) & 0 & cos(\beta) \end{bmatrix}$$

$$R_z = \begin{bmatrix} cos(\gamma) & -sin(\gamma) & 0 \\ sin(\gamma) & cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotations in three axis.

$$R = R_x R_y R_z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\alpha) & -sin(\alpha) \\ 0 & sin(\alpha) & cos(\alpha) \end{bmatrix} \begin{bmatrix} cos(\beta) & 0 & sin(\beta) \\ 0 & 1 & 0 \\ -sin(\beta) & 0 & cos(\beta) \end{bmatrix} \begin{bmatrix} cos(\gamma) & -sin(\gamma) & 0 \\ sin(\gamma) & cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} cos(\beta)cos(\gamma) & -cos(\beta)sin(\gamma) & sin(\beta) \\ -sin(\alpha)sin(\beta)cos(\gamma) + cos(\alpha)sin(\gamma) & sin(\alpha)sin(\beta)sin(\gamma) + cos(\alpha)cos(\gamma) & -sin(\alpha)cos(\beta) \\ -cos(\alpha)sin(\beta)cos(\gamma) + sin(\alpha)sin(\gamma) & cos(\alpha)sin(\beta)sin(\gamma) + sin(\alpha)cos(\gamma) & cos(\alpha)cos(\beta) \end{bmatrix}$$

$$[\acute{x} \quad \acute{y} \quad \acute{z}] = R$$

$$\begin{bmatrix} 0.5 & 0.5 & 0.707 \\ 0.5 & 0.5 & -0.707 \\ -0.707 & 0.707 & 0 \end{bmatrix} = \begin{bmatrix} cos(\beta)cos(\gamma) & -cos(\beta)sin(\gamma) & sin(\beta) \\ -sin(\alpha)sin(\beta)cos(\gamma) + cos(\alpha)sin(\gamma) & sin(\alpha)sin(\beta)sin(\gamma) + cos(\alpha)cos(\gamma) & -sin(\alpha)cos(\beta) \\ -cos(\alpha)sin(\beta)cos(\gamma) + sin(\alpha)sin(\gamma) & cos(\alpha)sin(\beta)sin(\gamma) + sin(\alpha)cos(\gamma) & cos(\alpha)cos(\beta) \end{bmatrix}$$

Consider element (1,3),

$$0.707 = sin(\beta)$$

$$\beta = 45°$$

Thus the gyro used to detect pitch angle deflects 4.5°.

Divide element (2,3) by element (3,3),

$$\frac{-0.707}{0} = -tan(\alpha)$$

$$\alpha = 90°$$

Thus the gyro used to detect roll angle deflects 9°.

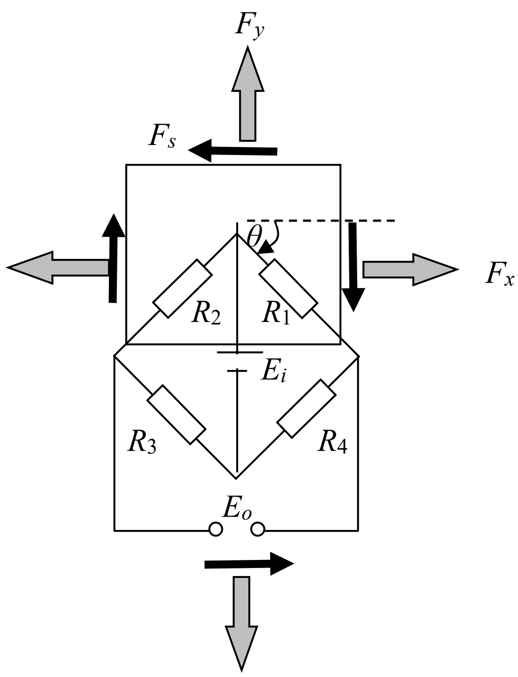
Divide element (1,2) by element (1,1),

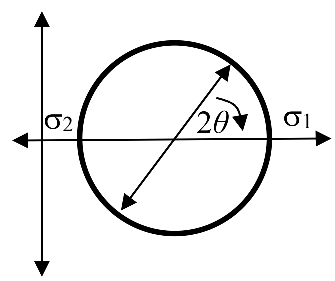
$$\frac{0.5}{0.5} = -tan(\gamma)$$

$$\gamma = -45°$$

Thus the gyro used to detect yaw angle deflects -4.5°.

Q.4 4 identical strain gages; 2 gages, R_1 and R_2, attached to a cubic block of a material and 2 gages, R_3 and R_4, floated as shown below, are used to determine the principal normal stresses when a two axial forces, F_x, F_y and shearing force, F_s are applied to the block as shown in the figure below. It's Mohr's circle is also shown in the same figure. Assume each side the cubic block has length of 10 cm and made of the material with Young's modulus, E, 200 GPa and Poisson's ratio, ν, 0.25. The 4 identical strain gages, each has the nominal resistance, R, 120 Ω with gage factor, S_g, of 2. E_i is 5 V. If axial forces (F_x, F_y) and shearing force (F_s) have the magnitude of 800,000, 400,000, and 200,000 N respectively are applied to the block, determine the alignment angle, θ, of strain gage R_1 that can detect the principal normal stress, σ_1. Strain gage R_2 is always aligned 90˚ relative to strain gage R_1 and used to detect σ_2. What is the voltage output reading? (25)





Solution

Axial stress along x-axis,

$$\sigma_x = \frac{F_x}{A_x} = \frac{800{,}000}{0.01} = 80{,}000{,}000\ Pa = 80\ MPa$$

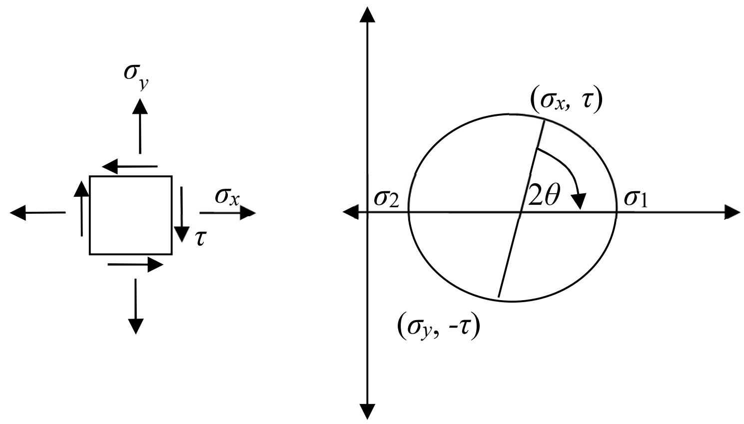
Axial stress along y-axis,

$$\sigma_y = \frac{F_y}{A_y} = \frac{400{,}000}{0.01} = 40{,}000{,}000\ Pa = 40\ MPa$$

Shearing stress,

$$\tau = \frac{F_s}{A_s} = \frac{200{,}000}{0.01} = 20{,}000{,}000\ Pa = 20\ MPa$$

Cross section and Mohr's circle,



Center of the Mohr's circle,

$$\frac{\sigma_x + \sigma_y}{2} = 60\ MPa$$

Radius of the Mohr's circle,

$$\sqrt{20^2 + 20^2} = 28.28\ MPa$$

$$\sigma_1 = 60 + 28.28 = 88.28\ MPa$$

$$\sigma_2 = 60 - 28.28 = 31.72\ MPa$$

$$2\theta = atan\left(\frac{20}{20}\right) = 45°$$

The alignment of strain gage R_1,

$$\theta = 22.5°$$

$$\frac{\Delta R_1}{R_1} = \frac{\sigma_1 - \nu\sigma_2}{E} S_g$$

$$\frac{\Delta R_2}{R_2} = \frac{\sigma_2 - \nu\sigma_1}{E} S_g$$

$$E_0 = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2}\right] E_i$$

$$E_0 = \frac{1}{4}[88.28 - 0.25 \times 31.72 - 31.72 + 0.25 \times 88.28]\frac{2 \times 5}{200{,}000} = 0.88\ mV$$