Midterm Examination   Sensing and Actuation AT74.03   March 2, 2023

Time: 10:00-11:30 h.   Open Book
Marks: 100

Attempt all questions.

Q.1 The output from a capacitive force sensor follows the relation expressed by

$$C = 0.07F + 10$$

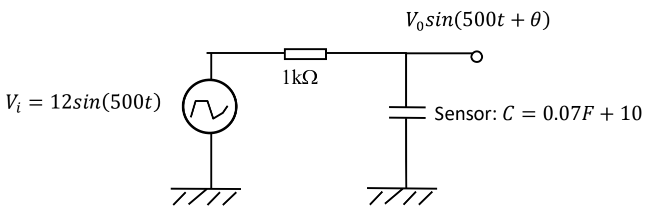
When the the input force, F, varies between 0-1000 N and the unit of the output capacitance, C, is in μF. Design a signal conditioning that convert the force value to the DC voltage, V (in V unit), expressed by

$$V = 0.005F$$

Assume that an AC voltage source of $V_i = 12sin(500t)$ is available and to be used in the circuit. The low-pass filter, if needed, has the cut-off frequency (3 dB attennuation) at 500 rad/s. (25)

<u>Solution</u>

1. RC ac circuit is used to convert the C value to ac voltage signal.



The voltage output across the C is determined.

$$V_0 = \frac{V_i}{\left(R + \frac{1}{sC}\right)} \times \frac{1}{sC} = \frac{V_i}{RCs + 1}$$

$$|V_0| = \frac{|V_i|}{\sqrt{1 + (\omega RC)^2}}$$

Select $R = 1$ kΩ,

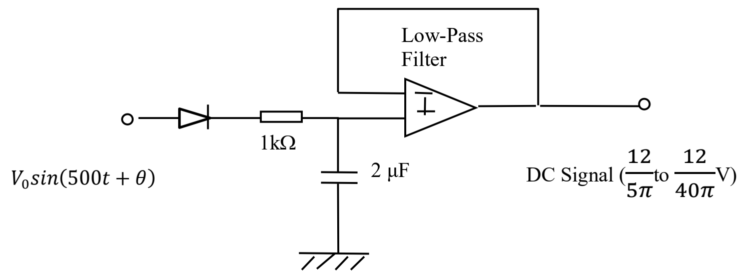
At 0 N,

$$|V_0| = \frac{12}{\sqrt{1 + \left(500 \times 1000 \times (10 \times 10^{-6})\right)^2}} \approx \frac{12}{5} V$$

At 1000 N,

$$|V_0| = \frac{12}{\sqrt{1 + \left(500 \times 1000 \times (80 \times 10^{-6})\right)^2}} \approx \frac{12}{40} V$$

2. A half-wave rectifier and a low pass filter is used to convert ac signal to dc signal.



Select a low-pass first-order filter with cut-off frequency (3db attennuation) at 500 rad/s.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + (RC\omega)^2}}$$

Select $R = 1$ kΩ,

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + (1000 \times 500C)^2}}$$

$$C = 2 \text{ μF}$$

The magnitude of the dc signal from half-wave rectifier and low pass filter is considered from

$$V_{av} = \frac{\int_0^{\pi} V_0 sin(\theta) d\theta}{2\pi} = \frac{-V_0 cos(\theta)[_0^{\pi}}{2\pi} = \frac{V_0}{\pi}$$

At 0 N,

$$V_{av} = \frac{12}{5\pi} V$$

At 1000 N,

$$V_{av} = \frac{12}{40\pi} V$$

3. Differential amplifier circuit or summing circuit is used to convert dc ($\frac{12}{5\pi}$ - $\frac{12}{40\pi}$ V) to (0-5 V)

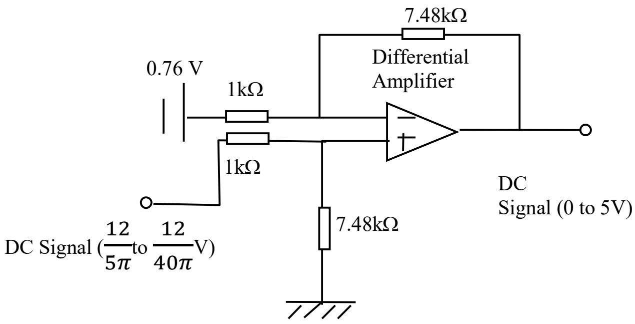
The required gain is determined.

$$G = \frac{Change\ of\ output}{Change\ of\ input} = \frac{5}{\left(\frac{12 \times 35}{200\pi}\right)} = 7.48$$

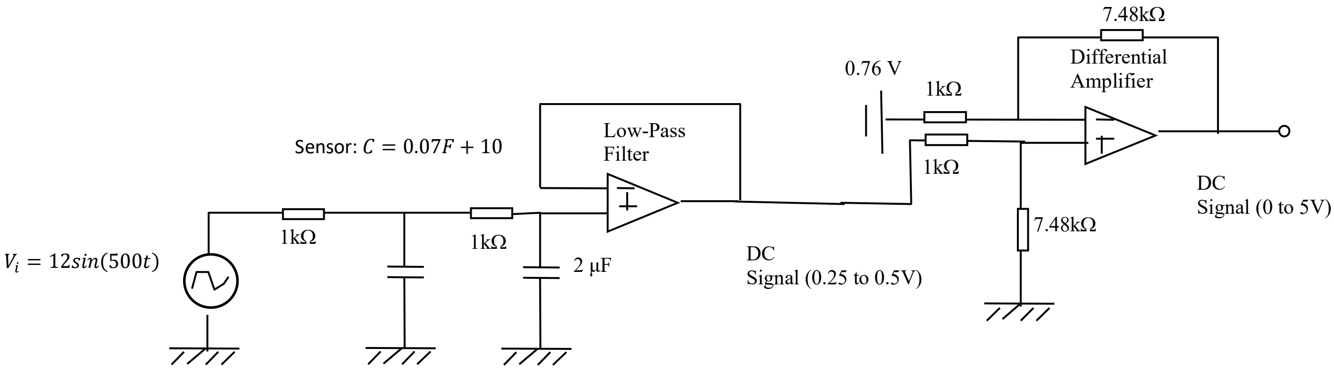
$$\frac{R_f}{R_1} = \frac{R_3}{R_2} = 7.48$$

Select $R_1 = R_2 = 1k\Omega, R_f = R_3 = 7.48k\Omega.$

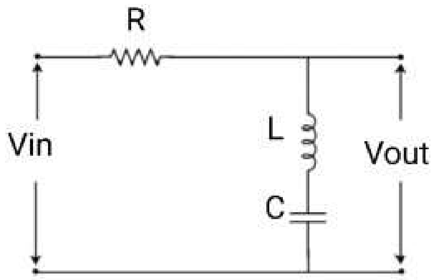
A constant voltage of $\frac{12}{5\pi} = 0.76$ V is provided to the inverting input pin.



Thus, the whole circuit is as follows.



Q.2 Consider the band-stop filter circuit below. Determine the center of the rejected frequency and the 3-db stop bandwidth when R = 1 kΩ, L = 2 H, C = 50 µF. (25)



<u>**Solution**</u>

$$\frac{V_{out}}{V_{in}} = \frac{sL + \frac{1}{sC}}{R + sL + \frac{1}{sC}} = \frac{LCs^2 + 1}{LCs^2 + RCs + 1}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{1 - LC\omega^2}{1 - LC\omega^2 + RC\omega}\right| = \left|\frac{1 - LC\omega^2}{\sqrt{(1 - LC\omega^2)^2 + (RC\omega)^2}}\right|$$

The center of the rejected frequency is obtained when the magnitude ratio becomes 0.

$$1 - LC\omega^2 = 0$$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2 \times 50 \times 10^{-6}}} = 100\ rad/s$$

The 3db-stop bandwidth is determined when the magnitude ratio becomes $\frac{1}{\sqrt{2}}$.

$$\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{1 - LC\omega^2}{\sqrt{(1 - LC\omega^2)^2 + (RC\omega)^2}}\right| = \frac{1}{\sqrt{2}}$$

$$L^2C^2\omega^4 - (R^2C^2 + 2LC)\omega^2 + 1 = 0$$

$$\omega^2 = \frac{(R^2C^2 + 2LC) \pm \sqrt{(R^2C^2 + 2LC)^2 - 4L^2C^2}}{2L^2C^2}$$

$$\omega = \sqrt{\frac{(R^2C^2 + 2LC) \pm \sqrt{R^4C^4 + 4R^2C^3L}}{2L^2C^2}}$$

$$\omega = \sqrt{\frac{((1 \times 10^3)^2(50 \times 10^{-6})^2 + 2 \times 2 \times 50 \times 10^{-6}) \pm \sqrt{(1 \times 10^3)^4(50 \times 10^{-6})^4 + 4(1 \times 10^3)^2(50 \times 10^{-6})^3 2}}{2 \times 4 \times (50 \times 10^{-6})^2}}$$

$$\omega = 19.26, 519.26 \text{ rad/s}$$

Thus the stop bandwidth

$$BW = 519.26 - 19.26 = 500 \text{ rad/s}$$

Q.3 A stroboscope lamp is used to determine the speed (in rps (revolution per second)) of a rotating disk. When the flashing rate is 10 Hz, a marker is seen motionless. When the flashing rate is 9 Hz, the marker is seen moving clockwise and is back to the original position in 3 flashes. When the flashing rate is 8 Hz, the marker is seen moving counter clockwise and is back to the original position in 4 flashes. Determine the three slowest possible solutions of the speed and direction of the rotating disk. (25)

<u>Solution</u>

Assume the disk is rotating at the speed n rps and k_1, k_2, k_3 are integer number.

At 10 Hz, the disk is seen motionless,

$$\frac{n}{10} = k_1 \quad (1)$$

At 9 Hz, the disk is seen moving clockwise and is back in 3 flshes,

$$\frac{n}{9} = k_2 + \frac{1}{3} \quad (2)$$

At 8 Hz, the disk is seen moving counter clockwise and is back in 4 flshes,

$$\frac{n}{8} = k_3 + \frac{3}{4} \quad (3)$$

(2)-(1),

$$\frac{n}{90} = (k_2 - k_1) + \frac{1}{3} \tag{4}$$

Thus,

$$n = 30 + 90(k_2 - k_1) \tag{5}$$

(3)-(1),

$$\frac{n}{40} = (k_3 - k_1) + \frac{3}{4} \tag{5}$$

Thus,

$$n = 30 + 40(k_3 - k_1) \tag{6}$$

Least common multiple of 40 and 90 is 360, thus

$$n = 30 + 360k \tag{7}$$

If $k = 0$,

$$n = 30 \text{ rps clockwise}$$

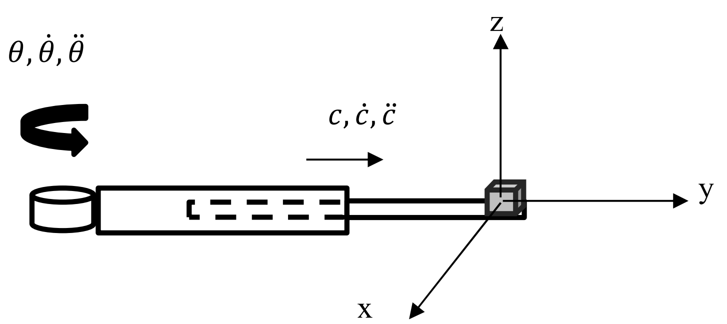
If $k = 1$,

$$n = 390 \text{ rps clockwise}$$

If $k = -1$,

$$n = 330 \text{ rps counter clockwise}$$

Q.4 A 3-axis accelerometer is attached at the end of a 2-DOF robotics arm moving in a horizontal plane as shown in the figure below.



If the seismic mass deflects 1 mm from the acceleration of 10 m/s^2 in each axis, determine the deflections of seismic mass in all axes of the accelerometer at the instance when the first joint of the robotics arm is rotating at the angular position, θ, of 0 rad, the angular velocity, $\dot{\theta}$, of 3 rad/s and at the angular acceleration, $\ddot{\theta}$, of 1 rad/s^2 and the second joint is extending and the accelerometer is at the distance, c, of 5 m from the center of rotation, at the velocity, $\dot{c}$, of 0.2 m/s and at the acceleration, $\ddot{c}$, of 0.1 m/s^2. (25)

Solution

The acceleration along x axis is determined.

$$a_x = -\ddot{\theta}c - 2\dot{\theta}\dot{c} = -(1 \times 5) - (2 \times 3 \times 0.2) = -6.2\, m/s^2$$

Thus, the deflection along x axis is -0.62 mm.

The acceleration along y axis is determined.

$$a_y = -\dot{\theta}^2 c + \ddot{c} = -(3^2 \times 5) + (0.1) = -44.9\, m/s^2$$

Thus, the deflection along y axis is -4.49 mm.

The acceleration along z axis is determined.

$$a_z = -9.8\, m/s^2$$

Thus, the deflection along z axis is -0.98 mm.