Midterm Examination Sensing and Actuation AT74.03 February 29, 2024

Time: 10:00-11:30 h. Open Book
Marks: 100

Attempt all questions.

Q.1 The frequency of the output voltage, V (in V), from a pressure sensor varies with the input pressure, P (in Bar), as expressed by

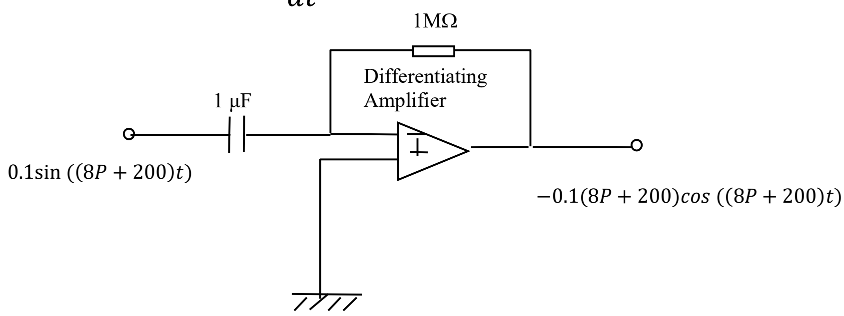
$$V = 0.1\sin((8P + 200)t)$$

When the the input pressure varies between 0-100 Bar. Design a signal conditioning that convert the pressure value (0-100 Bar) to the DC voltage, V_{dc} (0-5 V). The low-pass filter, if needed, has the cut-off frequency (3 dB attennuation) at 50 rad/s. (25)

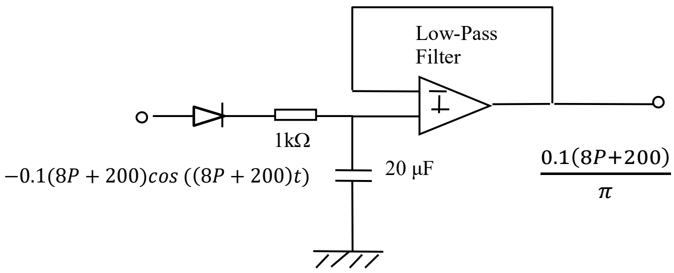
<u>Solution</u>

1. A differentiating circuit is used to differentiate the signal so that the amplitude of the signal is a function of the input pressure.

$$\frac{dV}{dt} = 0.1(8P + 200)cos((8P + 200)t)$$



2. A half-wave rectifier and a low pass filter is used to convert ac signal to dc signal.



Select a low-pass first-order filter with cut-off frequency (3db attennuation) at 50 rad/s.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + (RC\omega)^2}}$$

Select $R = 1$ kΩ,

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + (1000 \times 50C)^2}}$$

$$C = 20\ \mu\text{F}$$

The magnitude of the dc signal from half-wave rectifier and low pass filter is considered from

$$V_{av} = \frac{\int_{\pi/2}^{3\pi/2} -0.1(8P + 200)\cos(\theta)d\theta}{2\pi} = \frac{-0.1(8P + 200)\sin(\theta)[_{\pi/2}^{3\pi/2}}{2\pi} = \frac{0.1(8P + 200)}{\pi}$$

At 0 Bar,

$$V_{av} = \frac{20}{\pi} = 6.37V$$

At 100 Bar,

$$V_{av} = \frac{100}{\pi} = 31.83V$$

3. Differential amplifier circuit or summing circuit is used to convert dc (6.37 - 31.83 V) to (0-5 V)

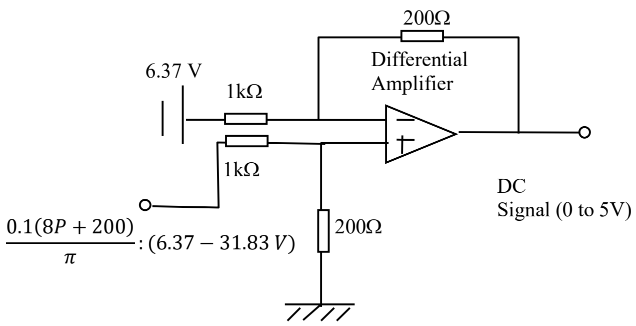
The required gain is determined.

$$G = \frac{Change\ of\ output}{Change\ of\ input} = \frac{5}{(31.83 - 6.37)} = 0.2$$

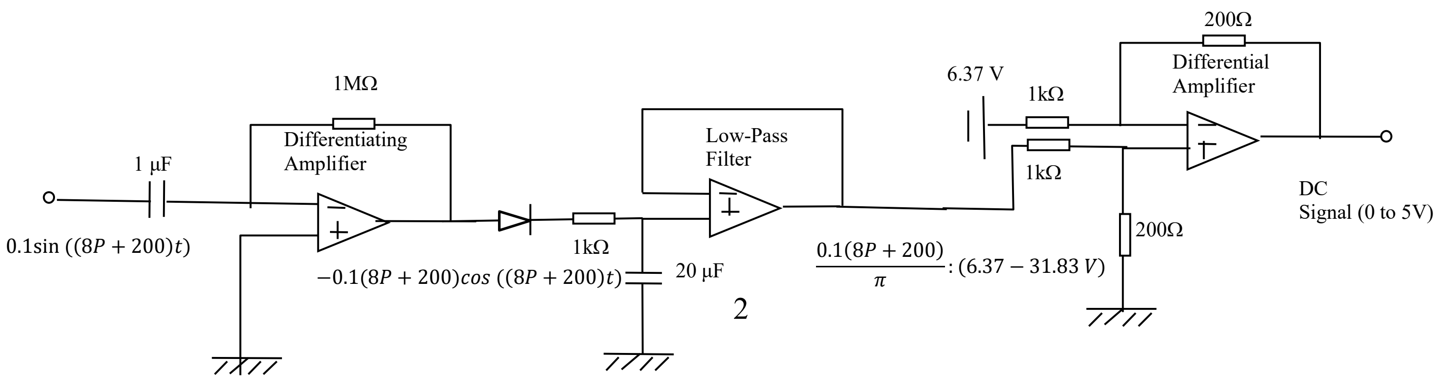
$$\frac{R_f}{R_1} = \frac{R_3}{R_2} = 0.2$$

Select $R_1 = R_2 = 1k\Omega, R_f = R_3 = 200\Omega$.

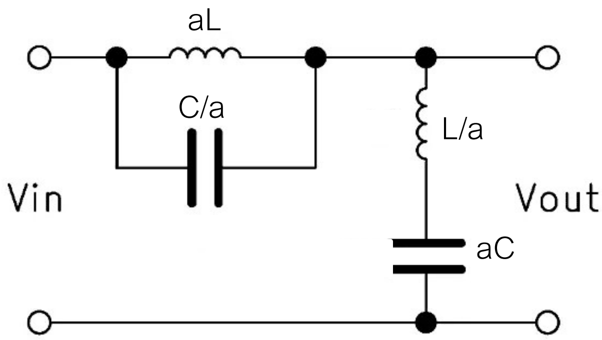
A constant voltage of 6.37 V is provided to the inverting input pin.



Thus, the whole circuit is as follows.



Q.2 Consider the band-stop filter circuit below. Determine the center of the rejected frequency and the 3 db rejection bandwidth when L = 4 H, C = 100 μF, a = 10. (25)



<u>Solution</u>

$$\frac{V_{out}}{V_{in}} = \frac{\frac{LCs^2+1}{saC}}{\frac{LCs^2+1}{saC} + \frac{aLs}{LC\ {}^{2}+1}} = \frac{(LCs^2+1)^2}{(LCs^2+1)^2 + LCa^2s^2}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{(1-LC\omega^2)^2}{(1-LC\omega^2)^2 - LCa^2\omega^2}\right|$$

The center of the rejected frequency is obtained when the magnitude ratio becomes 0.

$$1 - LC\omega^2 = 0$$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4\times100\times10^{-6}}} = \frac{1}{\sqrt{0.0004}} = 50\ rad/s \quad (1)$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{(1-0.0004\omega^2)^2}{(1-0.0004\omega^2)^2 - 0.04\omega^2}\right| = \frac{1}{\sqrt{2}}$$

Consider

$$\frac{(1-0.0004\omega^2)^2}{(1-0.0004\omega^2)^2 - 0.04\omega^2} = \frac{1}{\sqrt{2}}$$

$$\omega^2 = -0.0001\times10^5, -5.9854\times10^5$$

Which is not possible.

Consider

$$\frac{(1-0.0004\omega^2)^2}{(1-0.0004\omega^2)^2 - 0.04\omega^2} = -\frac{1}{\sqrt{2}}$$

$$\omega^2 = 0.0006\times10^5, 1.0850\times10^5$$

$$\omega = 7.5899, 329.3870$$

Thus, 3 db rejection bandwidth

$$BW = 329.3870 - 7.5899 = 321.7971\ rad/s \quad (2)$$

Q.3 An accelerometer and a rate gyro are used to measure the acceleration and angular velocity respectively. When the acceleration is 1g, the output from the accelerometer shows 2 V. When the angular velocity is π/8000 rad/s, the output from the rate gyro shows 0.5 V. Determine the outputs from the accelerometer and the rate gyro when they are located on an orbiting satellite at the height of 100 km from the earth surface to measure the gravitational acceleration and angular velocity of the satellite. Determine a period of an orbit of the satellite. Use the earth radius of 6,371 km and 1g of 9.8 m/s^2. (25)

Solution

Gravitational acceleration is determined from

$$g = \frac{GM}{r^2}$$

At the earth surface

$$g_0 = \frac{GM}{6{,}371{,}000^2}$$

Af the height of 100 km above the earth surface

$$g_{100} = \frac{GM}{6{,}471{,}000^2} = 0.9693 g_0$$

If, the satellite is stationary, the output voltage from the accelerometer

$$V_{accel\_100} = 0.9693 V_{accel\_0} = 0.9693(2) = 1.9387\, V$$

However, the outut voltage from the accelerometer on an orbiting satellite

$$V_{accel\_100} = 0\, V \quad (1)$$

The relation of the centrifugal force,

$$mg = m\omega^2 r$$

Af the height of 100 km above the earth surface

$$g_{100} = 0.9693(9.8) = \omega^2(6{,}471{,}000)$$

Thus

$$\omega = 0.0012\, rad/s$$

Thus, the output voltage from the rate gyro

$$V_{rategyro\_100} = 0.0012(0.5)/(\tfrac{\pi}{8000}) = 1.5279\, V \quad (2)$$

The satellite period,

$$T_{100} = \frac{2\pi}{0.0012} = 5{,}236.0\, seconds = 87.2665\, minutes \quad (3)$$

Q.4 An axial force is applied on a steel rectangle block with the cross section area of 1 cm^2 and the length of 10 cm. Young's modulus of steel is 200 GPa. Determine the axial elastic stiffness of this block. If the force of 10,000 N is applied, determine the change of the length of the block. Then determine the voltage output from the DC Wheatstone bridge circuit, if a strain gage of 350 Ω nominal resistance and gage factor of 2 is used to measure the axial strain, the remaining three resistors in the bridge circuit have a constant resistance of 350 Ω each, and the supplied voltage is 5 V. (25)

Solution

$$E = \frac{\sigma}{\varepsilon} = \frac{FL}{A\Delta L} = \frac{kL}{A}$$

$$k = \frac{EA}{L} = \frac{(200\times10^{9})(1\times10^{-4})}{(10\times10^{-2})} = 2\times10^{8} N/m \quad (1)$$

$$\Delta L = \frac{F}{k} = \frac{(1\times10^{4})}{(2\times10^{8})} = 5\times10^{-5} m = 50\mu m \quad (2)$$

$$V = \frac{r}{(1+r)^2}\frac{\Delta R}{R}E = \frac{r}{(1+r)^2}\frac{\Delta L}{L}SE == \frac{1}{(1+1)^2}\frac{(5\times10^{-5})}{(10\times10^{-2})}(2)(5) = 1.25\times10^{-3}V = 1.25mV \quad (3)$$