OPTIMIZING AN INTEGRATED VENDOR MANAGED INVENTORY SYSTEM FOR A SINGLE-SUPPLIER AND TWO-RETAILER SUPPLY CHAIN WITH CONTINUOUS REVIEW POLICY AND STOCHASTIC DEMAND

by

Bandara Pathiranage Supun Viraj Pathiranage

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Examination Committee:	Dr. Huynh Trung Luong (Chairperson)
	Dr. Pisut Koomsap
	Dr. Mongkol Ekpanyapong

Nationality:	Sri Lankan
Previous Degree:	Bachelor of Science in Engineering in Mechatronics Engineering Asian Institute of Technology, Thailand
Scholarship Donor:	AIT Partial Scholarship

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School of Engineering and Technology
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ABSTRACT

Due to increasing competitiveness, the demand for movement of goods around the world has been increased and every manufacturer prospect is to deliver the products rapidly to the customer. Therefore, Vendor Managed Inventory is becoming an essential service in the global supply chain. Vendor Managed Inventory is a collaborative commerce initiative in which vendors can manage buyer inventory. Vendor managed inventory method is a widely discussed partnership initiatives which is a frequently used method to improve supply chain efficiency. Reducing inventory cost for both supplier and buyer and improving customer service level is the main objective of implementing a VMI system. In this research, VMI models with one vendor and two retailers under continuous review policy was tackled. A mathematical model was developed to deal with the case considering the lateral transshipment between two retailers while the demand is stochastic and lost sales are allowed. This model was developed to determine the optimal delivery quantities and the allocated safety stock of each retailer so as to minimize the total cost of the whole system. Sensitivity analysis and numerical experiments are conducted in order to analyze the effect of various input parameters on the optimal solutions.

Keywords: Vendor managed inventory, Lateral transshipments, Stochastic demand, continuous review policy

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LIST OF ABBREVIATIONS

SCM	= Supply Chain Management
VMI	= Vendor Managed Inventory
SV	= Single Vendor
SR	= Single Retailer
MV	= Multiple Vendor
MB	= Multiple Buyers

CHAPTER 1
INTRODUCTION

1.1 Background of the Study

Vendor managed inventory is becoming an essential service in the global supply chain. Vendor Managed Inventory is a cooperative business initiative in which vendors can manage buyer inventory. This is the main difference between the traditional supply chain and a VMI supply chain. Therefore, vendor managed inventory is a frequently used method for widely discussed partnership initiatives to improve supply chain efficiency.

Reducing inventory cost for both vendor and retailer and improving customer service level are the main objectives of implementing a VMI system. The parties involved in VMI generally have sales, inventory and order processing systems that must be integrated to share information. The vital information is the customer demand and the point in time of replenishment.

Mutual understanding of each other's processes in a supply network guides to a stronger supply chain and can also gain a competitive advantage over other supply chains. Information transparency can guide to lowering the total cost of a VMI supply chain by providing better information for decision making and minimizing the risk of optimizing inventory. Therefore, this results in a flexible supply chain with smaller stocks.

Reducing transportation costs, increasing understanding and overview of the market for better forecasting, and increasing capital turnover are the benefits that a supplier can derive from a VMI supply chain. And for customers, VMI can help reduce costs by outsourcing the materials warehousing process, to have a shorter delivery time that can lead to lower inventory cost and also lower materials planning and processing costs. Implementing VMI can prevent financial risks associated with traditional inventory management. Features of Vendor managed Inventory system as follows;

- Inventory level transparency
- Transparency of tock ordering
- Items hare safely stored in a warehouse
- Market surveillance

- Shipping and handing over (delivery)
- Replenishment needs are covered
- Cost management

Advantages of a VMI system when applied correctly:

- Holding costs reduction
- Safety stock reduction
- Efficient supply chain management
- Better communication
- Streamlined execution
- Structured across the supply chain
- Productivity / cost savings

Inventory system can be classified in to two main types. They are continuous review systems and periodic review systems. In the continuous review system, normally the same quantity of items in each order is kept and the orders are placed when inventory reaches the reorder level. In the periodic review system, the quantity is ordered periodically at the same time and the order quantity is based on the inventory level at the reorder point.

The continuous review system requires an understanding about the physical information at all the time, therefore, the cost of applying this method is more expensive than periodic review system. But the required safety stock level is low since the demand quantity is the only uncertain parameter within the delivery period. In addition, there are many advantages such as real-time updates of inventory. When compared with the periodic reviews system the continuous review system is more suitable for products with high volume of sale and also this review system has more control on the inventory movements.

There are two types of inventory models. They are deterministic demand model and stochastic demand model. The model output is completely obtained by the parameter values and the initial status in deterministic models while stochastic models deal with some inherent randomness. According to the literatures most of the inventory models considered are deterministic because the demand is assumed to be known. But for many logistic systems such assumptions are not appropriate. Normally demand is a random variable whose distribution may be known. But stochastic models are considerably more complicated.

1.2 Statement of the Problem

Normally every organization's objective is to increase the profit by providing a valuable product to the customer while reducing the total supply chain cost. There are many ways to reduce the cost of supply chain. Some of them are Automating processes, Inventory Management, improve space utilization, review customer demand, streamline ordering process, outsourcing the production, improve packaging, sales and operations planning, analyze the cost of operation, design a flexible supply chain, performance measurement, etc. Normally transportation cost is only 2-5 % of the total cost while the inventory can cost more than 55% of the total cost. Therefore, currently most of the companies are trying to have a huge saving by implementing a Vendor Managed Inventory system to their supply chain.

Most of the related past authors have developed models to compare the traditional supply chain (RMI) and the VMI supply chain and the results of the models presented that implementing VMI system can reduce the total cost of the whole system. And most of them have considered periodic review policy because they have conducted the researches for slow moving items which have low demands. And also, according to the past researches most of them are conducted under deterministic demand. Therefore, there still exists a gap to develop models for fast moving items which have high demand. Hence, this study will focus on the development of a model to help minimize the total cost of the supply chain by using continuous review policy for fast moving items with stochastic demand under VMI system.

1.3 Objectives of the Study

This research aim is to identify economic order quantities and the allocated safety stocks to retailers in a VMI inventory system so that the total cost (including order or production setup cost at supplier and retailers, inventory holding cost, transportation cost and lost sale cost at retailers) is minimized under continuous review policy and stochastic demand for a one-supplier and two-retailer supply chain system.

1.4 Scope and Limitation

In this research, the aim is to identify the economic order quantities and the allocated safety stocks of the whole inventory system for a one-supplier and two-retailer supply chain system. The research will be conducted under following assumptions.

1. The supplier will use continuous review policy and his demand will be aggregated of the two retailers (which are random variables).
2. Production speed of the supplier is higher than the total demand rate, and the whole lot of size $(Q_i + Q_j)$ will be completely produced before sending Q_i to retailer i and Q_j to retailer j.
3. Shortage at the two retailers will be completely considered as lost sales.
4. The access inventory of the other retailer will be transshipped so as to prevent lost sales when shortage occurs at a retailer.
5. Lead times to delivered products to the two retailers are constants.
6. Safety stock (SS) of the vendor will be allocated to the retailer in advance.

CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

Nowadays most of the companies are facing challenges to reduce operational cost, increase profit margins and remain competitive. Therefore, various types of strategies are applied by the companies and Vender Managed Inventory is one of the best strategies identified by the professionals. Vendor managed Inventory is not a new strategy for the industry.

When discussing about the responsible for the inventory management, J.F. Magee was the one who developed a conceptual framework in 1958. Wal-Mart, is the first company who implemented the VMI and also VMI has been implemented in the several industries such as Gampbell Soup, Johnson &Johnson and Barilla (Yu et al., 2012). Significant benefits for the supply chain and also the each of the members, are provided by the VMI system and it helps to improve the productivity and efficiency of the supply chain (Ryu et al., 2013). Replenishment frequencies will be increased with smaller quantities and reduces inventory level in the supply chain. (Dong et al., 2007, Chen and Chang,2010).

VMI may lead to improve the customer service levels and significant cost reduction of the system. (Kang and Kim, 2012). In addition, supplier will be able to obtain greater flexibility in production planning and managing on quantity and distribution (Claassen et al., 2008, Wong et al., 2009, Xu and Leung, 2009). Furthermore, supplier can gain higher forecasting accuracy so that the supplier is able to have a better response in uncertainty demand of the customer and random market changes. By having a better visibility may lead to reduce the bullwhip effect and demand uncertainty (Disney and Towill, 2003a, Zhu and Peng, 2008).

2.2 Continuous Review Policy (r, Q)

Most of the previous researches related to VMI programs focused on a policy such as newsboy policy for single period while the policy which is similar to periodic review policy (s, S) was considered for multi period problems. Few past researched have been focused on continuous review policy. A continuous review policy model was developed by Li and Liu (2006) to find a methodical solution for optimal order quantity by using a fixed reorder point and also implemented a quantity discount policy for a two-level supply chain. After that this model was extended by Chaharsooghi et al., (2011) to make it easy to choose the order size and connect the replenishment point.

A credit option was implemented to coordinate the two-level supply chain system by using as a mechanism under continuous review policy by Chaharsooghi and Heydari (2010). Yugang Yu et al. (2012) developed a VMI model to help the vendor to optimally select his retailer under a continuous review policy by using a Stackelberg game model. A mathematical model was developed by Reza Hosseini Rad et al. (2013) to optimize a single-vendor and two-retailer VMI supply chain model and also a weighting factor was obtained for the Supplier's ordering cost.

2.3 Vendor-Buyer Inventory System

According to the past research Goyal (1976) has developed an integrated inventory model for a single-vendor and single-buyer supply chain who has been one of the pioneers to illustrate integrated vendor buyer inventory problems. Then Banarjee generalized the Goyal's model in 1986 assuming the vendor as manufacturer with finite production rate. An optimal solution was obtained by Goyal and Nebebe (2000) and their objective is to minimizing total joint annual cost under different shipment strategies. Centinkaya and Lee (2000) developed an analytical model to coordinate inventory and transportation decisions with VMI system and their main aim is to find the optimal replenishment quantity and the shipment-release policy.

Chang et al. (2006) have investigated the single-vendor and single-buyer problem under continuous review policy. They have considered under two different situations like when there is a relationship between order cost reduction and the lead time and when there is not. For this problem Hoque and Goyal (2006) have proposed a heuristic solution to minimize the total cost. A VMI system was proposed by the Guan and Zhao (2010) under continuous review policy by designing revenue sharing contract. Reze Hosseini Rad et al. (2010) have developed a mathematical model to optimize an integrated vendor managed inventory system for single-vendor and two-buyer supply chain and it was compared with the traditional supply chain (RMI) and the results illustrate that greater reduction of total cost can be achieved by using VMI system.

Furthermore, Arqum Mateen et al. (2014) have developed an analytical model for a single-vendor and multiple-retailer supply chain and explored number of ways of coordinating partners. From the analysis they have illustrated that the difference in total cost in a supply chain for different replenishment approaches.

2.4 VMI System Stochastic Demand

Under stochastic demand problems Kiesmuller and Broekmeulen (2010) have studied the benefit of VMI in stochastic multi-product system dealing with slow moving items. A routing problem was formulated by Hemmelmayr et al. (2010) under VMI for blood banks. Optimal replenishment policy was determined by the Lee and Ren (2011) for a single-vendor and single-supplier VMI with stochastic demand. Another research which was conducted by Chen and Wei (2012) compared retailer managed inventory policy with VMI in single seller – single buyer system. Glock (2009) has investigated how stochastic demand affects the optimal order quantity and the reorder point.

Some recent studies under stochastic demand obtained an optimal replenishment polices for a vendor managed inventory single-vendor and single-buyer supply chain by considering variable lead time (Govindan, 2015) and also Sajadieh and Larsen, 2015 have studied on stochastic production rate for a single-vendor and single-buyer inventory model. Furthermore, some researchers have been carried out to identify the effects of delivery cost and stochastic transportation time on the expected total cost for an integrated vendor buyer supply chain model. Below table illustrates the summary of the vendor managed inventory policy research papers under stochastic demand.

2.5 Transshipment Models

Recently most of the past researches are focused on the transshipment models which has begun with Krishan and Rao in 1965.A general number of retailers were studied under centralized control and independent demands. Then this model was extended by Dong and Rudi (2004). A supply chain was developed which can transship inventories within retailers after demand materialization and one manufacturer and n retailers' model was studied to find out how transshipment affects both manufacturers and retailers under normal demand distribution.

The optimal transshipment policy has been described by Wee and Dada (2005) by developing a formal model focused on the role of transshipment for n-retailers. Herer et al. (2006) concluded that the replenishment strategies and transshipments must be used to coordinate a supply chain and order-up-to S policy must be used as the optimal replenishment policy for each retailer by developing a model with one supplier and several retailers. Xu Chen et al. (2012) developed a model to study the affect the demand variability and transshipment under VMI system. In this study they have proved that a unique optimal distribution policy can be obtained by employing transshipment and gain maximum expected profit within the supply chain.

According to the literature, finding the optimal order quantity and the reorder point are valued problem under vendor inventory managed system for single vendor and two buyer model. There are few research papers studied this kind of problem with several assumptions and controlled factors. Most of them aimed to minimize the total cost by reducing order or production setup cost, inventory holding cost at only for the supplier perspective and also most of the papers are not allowed backlogging cost. Therefore, there exists some gap to identify the optimal order quantity and the reorder point for whole inventory system so that the total cost is minimized by considering the order or production setup cost, inventory holding cost for supplier and both retailers including the backlogging cost.

CHAPTER 3
MATHEMATICAL MODEL DEVELOPMENT

This research focuses on deriving a VMI system with one vendor and two retailers under continuous policy. The mathematical model will be derived in this chapter to deal with the case when demand is stochastic and lost sales are considered

3.1 Development of the Mathematical Model

This study focusses on a VMI system consisting of two retailers with equal replenishment cycle times (which is decided by the vendor) and the demand rate is stochastic. The order is placed by the vendor for the whole system and the total order quantity will be split and deliver to the two retailers downstream. In order to develop the total cost function for the vendor and the retailers, the following notations are used**.**

i , j	=	Index for retailers (i = 1,2)
L	=	Lead Time of vendor (unit time)
T	=	Expected Cycle Length
X_L	=	Demand during lead time of the vendor where the probability function is $f(x_L)$ with the mean $\mu_L = L\mu$ and the standard deviation $\sigma_L = \sqrt{L}\sigma$
π_i	=	The unit cost of lost sales at retailer i
π_j	=	The unit cost of lost sales at retailer j
SS_i	=	Safety stock allocated to retailer i (decision variable)
SS_j	=	Safety stock allocated to retailer j (decision variable)
D	=	Total demand of the whole system
R	=	Reorder point of the whole system which is under the control of the vendor
D_i	=	The demand at retailer i which is normally distributed random variable with mean (μ_i) and standard deviation (σ_i); (i= 1,2)
Q	=	Total order quantity of the system
Q_i	=	The allocated order quantity to the i^{th} retailer (decision variable)
Q_j	=	The allocated order quantity to the j^{th} retailer (decision variable)
h_{ri}	=	Holding cost of retailer i per unit per unit time

h_{rj}	=	Holding cost of retailer j per unit per unit time
T_{ij}	=	The amount of transshipment from retailer i to retailer j
T_{ji}	=	The amount of transshipment from retailer j to retailer i
N_i	=	Lost sale amount at retailer i
N_j	=	Lost sale amount at retailer j
f_{ri}	=	Replenishment cost per shipment to retailer i
f_{rj}	=	Replenishment cost per shipment to retailer j
f_v	=	Fixed replenishment cost per shipment to vendor
s_r	=	Transportation cost per unit to retailer
TC_r	=	Total cost at both retailers
TC_v	=	Total cost at vendor
TC_{VMI}	=	Total cost of the whole VMI system

In VMI system, the vendor is in charge of supplying to several retailers in order to meet their customer's demand. Therefore, the demand of the vendor is equal to the total demand of retailers.

$$Retailer\ _i \sim N(\mu_i, \sigma_i{}^2) \qquad - \qquad X_i$$

$$Retailer\ _j \sim N\left(\mu_j, \sigma_j{}^2\right) \qquad - \qquad X_j$$

Therefore, Total demand:

$$X = X_i + X_j$$

$$X \sim N(\mu, \sigma^2) \qquad \mu = \mu_i + \mu_j\ , \ \ \sigma^2 = \sigma_i{}^2 + \sigma_j{}^2$$

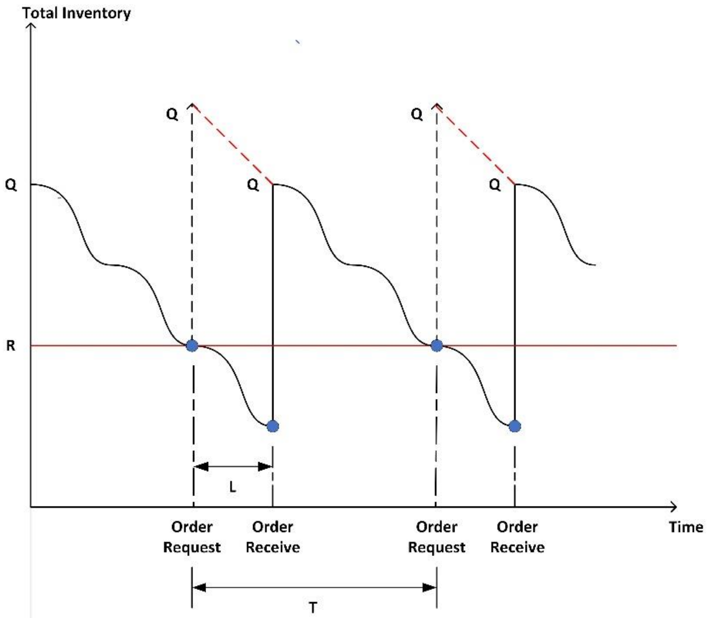
Demand during lead time:

$$X_L \sim N(\mu_L, \sigma_L{}^2) \qquad \mu_L = L\mu\ , \ \ \sigma_L = \sqrt{L}\sigma$$

The inventory distribution of the whole system when using the (Q, R) policy can be figured as below.

Figure 3.1

Inventory Distribution of the Whole System when using (Q,R) Policy



The amount $Q + R$ will cover the demand during $L + T$ period and the amount R will cover the demand during lead time.

Reorder level consists with mean demand during lead time ($\mu_L = \mu L$) and the safety stock (SS).

Mean demand during lead time can be expresses as follow:

$$\mu_L = \mu L = \mu_i L + \mu_j L$$

According to the VMI policy, the mathematical model consists of two parts, one of the retailers and the other for vendor.

3.2 Total Cost at the Vendor

The vendor's total inventory cost includes only the ordering cost.

Expected cycle length:

$$T = Q/D \tag{3.10}$$

Ordering cost per unit time of the vendor:

$$OC_v = \frac{f_v}{T} = f_v \frac{D}{Q} \tag{3.11}$$

Therefore, total cost at vendor per unit time: $TC_V = f_v \frac{D}{Q}$

$$TC_V = f_v \frac{D}{Q} \tag{3.12}$$

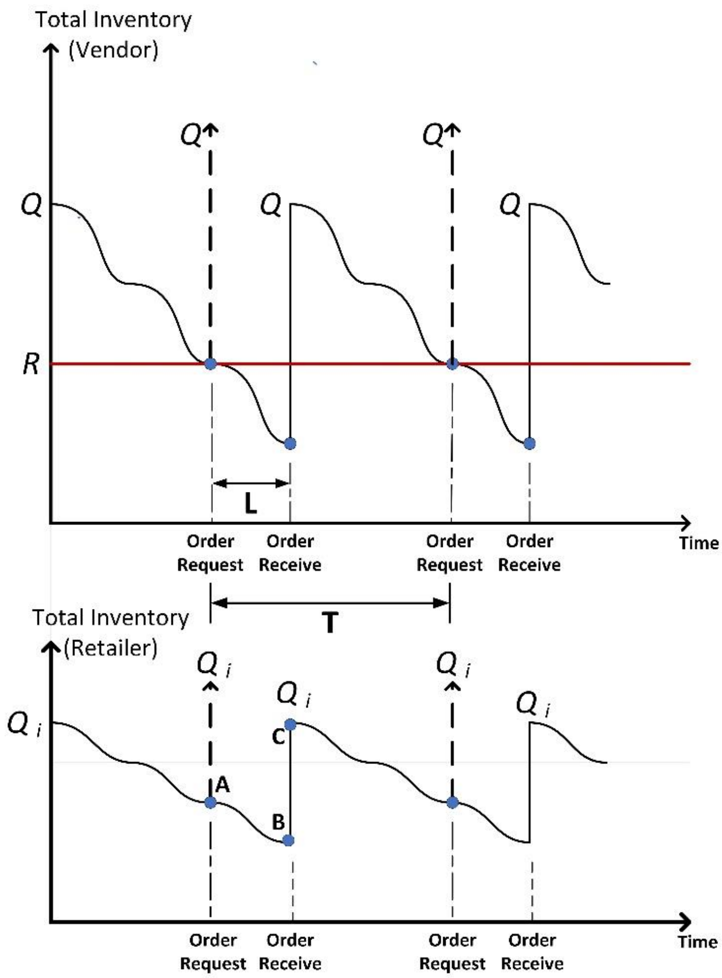
3.3 Total Cost at the Retailers

The cost incurred at a retailer will consist of the replenishment cost (a constant), the transportation cost to deliver to the two retailers which is proportional to the quantity, the holding cost, and the lost sales cost. The cost to allocate the safety stock can be ignored because this is a one-time cost.

The relationship between the inventory of the vendor and the inventory of the retailer is showed below.

Figure 3.2

Inventory Distribution at Vendor and Retailer



The above graph shows the inventory distribution of the retailer when compared to the vendor. According to the graph the average inventory level at retailer can be expressed as below.

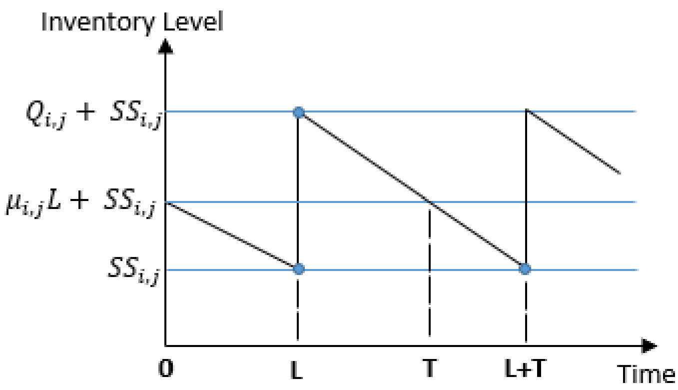
At point A (the inventory level at vendor reaches to reorder point), the average inventory level at retailer is $\mu_{i,j}L + SS_{i,j}$.

At point B (order is received at vendor), the average inventory level at retailer is $SS_{i,j}$.

At point C (order is delivered from the vendor to the retailer), the average inventory level at retailer is $Q_{i,j} + SS_{i,j}$.

Figure 3.3

Inventory Distribution at Retailer



At retailers, transshipment will be considered to help reduce shortage. In order to analyze the transshipment, the following four scenarios must be considered.

1. **Scenario 1**: Total demand of retailer i is lower than the order quantity and the safety stock while the demand of retailer j is lower than the order quantity and the safety stock.

- Transshipment will not occur
- lost sales will not occur

2. **Scenario 2**: Total demand of retailer i is lower than the order quantity and the safety stock while the demand of retailer j is greater than the order quantity and the safety stock.

- Transshipment from retailer i to retailer j
- Lost sales may occur a retailer j

3. **Scenario 3**: Total demand of retailer i is greater than the order quantity and the safety stock while the demand of retailer j is lower than the order quantity and the safety stock.

- Transshipment from retailer j to retailer j
- Lost sales may occur a retailer i

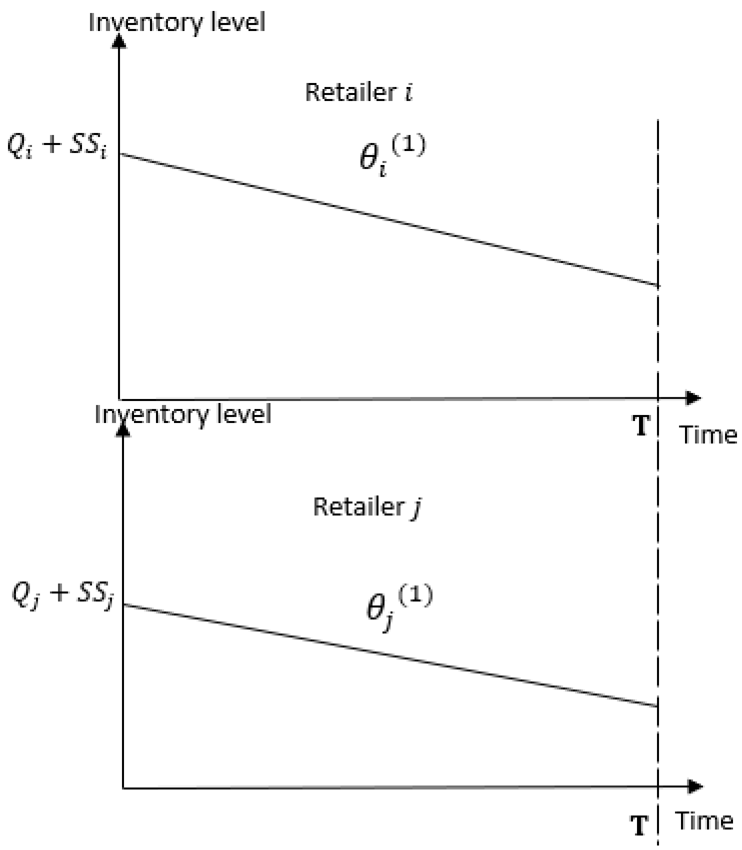
4. **Scenario 4**: Total demand of retailer i and the demand of retailer j is greater than the order quantity and the safety stock of each retailer.

- Transshipment will not occur
- Lost sales may occur at both retailer

3.3.1 Scenario 01: ($D_i(T) \leq Q_i + SS_i$ and $D_j(T) \leq Q_j + SS_j$)

Figure 3.4

Inventory Level at Retailer i and Retailer j at Scenario 1



For scenario 1, the demand of both retailers is lower than the order quantity and the safety stock. Therefore, transshipment between the retailers and the lost sales will not occur. So that the probability for scenario 1 to occur is given by,

$$P_1 = P(D_i(T) \le Q_i + SS_{\boldsymbol{i}})P(D_j(T) \le Q_j + SS_j) \tag{3.13}$$

Let $\theta_i{}^{(1)}$ be the conditional average demand rate at retailer i in scenario 1 then,

$$\theta_i{}^{(1)} = \frac{1}{T} E\left[D_i(T)|_{D_i(T)\le Q_i+SS_{\boldsymbol{i}},D_j(T)\le Q_j+SS_j}\right] \tag{3.14}$$

We have,

$$f_{D_i(T)|D_i(T)\le Q_i+SS_{\boldsymbol{i}}}(x) = \frac{dF_{D_i(T)|D_i(T)\le Q_i+SS_{\boldsymbol{i}}}}{dx} \tag{3.16}$$

$$F_{D_i(T)|D_i(T)\le Q_i+SS_{\boldsymbol{i}}}(x) = P(D_i(T)|D_i(T) \le Q_i + SS_{\boldsymbol{i}}) = \frac{P(D_i(T)\le x, D_i(T)\le Q_i+SS_{\boldsymbol{i}})}{P(D_i(T)\le Q_i+SS_{\boldsymbol{i}})} = \frac{P(D_i(T)\le x)}{P(D_i(T)\le Q_i+SS_{\boldsymbol{i}})}$$

Therefore,

$$F_{D_i(T)|D_i(T)\le Q_i+SS_{\boldsymbol{i}}}(x) = \frac{F_{D_i(T)}(x)}{F_{D_i(T)}(Q_i + SS_i)} \tag{3.17}$$

From equation (3.16) and (3.17),

$$f_{D_i(T)|D_i(T)\le Q_i+SS_{\boldsymbol{i}}}(x) = \frac{f_{D_i(T)}(x)}{F_{D_i(T)}(Q_i + SS_i)} \tag{3.18}$$

Therefore,

$$E\left[D_i(T)|_{D_i(T)\le Q_i+SS_{\boldsymbol{i}},D_j(T)\le Q_j+SS_j}\right] = \frac{\int_0^{Q_i+SS_{\boldsymbol{i}}} x f_{D_i(T)}(x)dx}{F_{D_i(T)}(Q_i + SS_i)} \tag{3.19}$$

Let $\theta_j{}^{(1)}$ be the conditional average demand rate at retailer j in scenario 1 then,

$$\theta_j{}^{(1)} = \frac{1}{T} E\left[D_j(T)|_{D_i(T)\le Q_i+SS_{\boldsymbol{i}},D_j(T)\le Q_j+SS_j}\right] \tag{3.20}$$

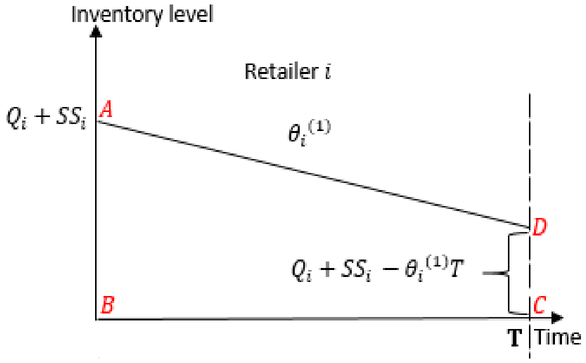
Similarly, we have,

$$E\left[D_j(T)|_{D_i(T)\le Q_i+SS_{\boldsymbol{i}},D_j(T)\le Q_j+SS_j}\right] = \frac{\int_0^{Q_j+SS_j} y f_{D_j(T)}(y)dy}{F_{D_j(T)}(Q_j + SS_j)} \tag{3.21}$$

Expected holding cost at retailer i can be expressed as,

Figure 3.5

Inventory Level at Retailer i at Scenario 1

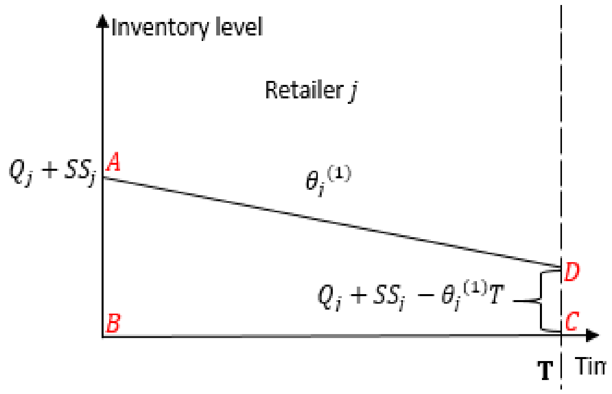


$$E\left[H_{ri}^{(1)}\right] = \frac{h_{ri}}{T}\frac{(2Q_i - \theta_i^{(1)}T + 2SS_i)}{2}T = h_{ri}\frac{(2Q_i - \theta_i^{(1)}T + 2SS_i)}{2} \tag{3.22}$$

Expected holding cost at retailer j can be expressed as,

Figure 3.6

Inventory Level at Retailer j at Scenario 1



$$E\left[H_{rj}^{(1)}\right] = \frac{h_{rj}}{T}\frac{(2Q_j - \theta_j^{(1)}T + 2SS_j)}{2}T = h_{rj}\frac{(2Q_j - \theta_j^{(1)}T + 2SS_j)}{2} \tag{3.23}$$

Therefore,

$$E\left[H_r^{(1)}\right] = h_{ri}\frac{(2Q_i - \theta_i^{(1)}T + 2SS_i)}{2} + h_{rj}\frac{(2Q_j - \theta_j^{(1)}T + 2SS_j)}{2} \tag{3.24}$$

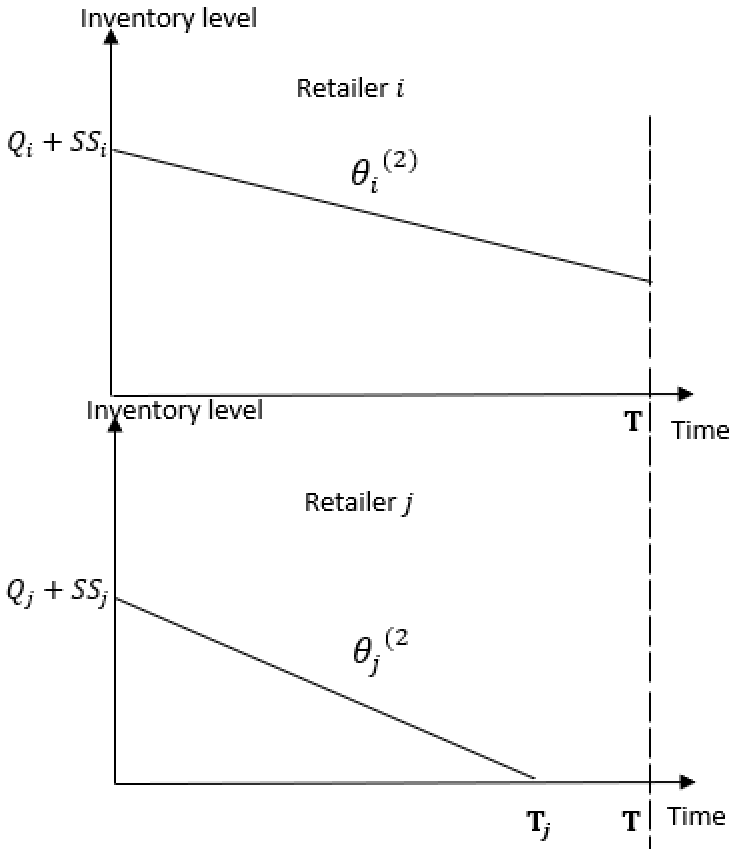
Expected total cost (excluding order cost and transportation cost) per unit time at retailer is (Scenario 1):

$$E\left[TC_r{}^{(1)}\right] = h_{ri}\frac{(2Q_i - \theta_i{}^{(1)}T + 2SS_i)}{2} + h_{rj}\frac{(2Q_j - \theta_j{}^{(1)}T + 2SS_j)}{2} \tag{3.25}$$

3.3.2 Scenario 02: ($D_i(T) \leq Q_i + SS_i$ and $D_j(T) > Q_j + SS_j$)

Figure 3.7

Inventory Level at Retailer i and Retailer j at Scenario 2



For scenario 2, the demand of retailer j is greater than the order quantity and the safety stock while the demand of retailer i is lower than the order quantity and the safety stock. Therefore, excess amount of retailer i will be transshipped to retailer j to fulfil the demand. The probability for scenario 2 to occur is given by,

$$P_2 = P(D_i(T) \leq Q_i + SS_i)P(D_j(T) > Q_j + SS_j) \tag{3.26}$$

Let $\theta_i{}^{(2)}$ be the conditional average demand rate at retailer i in scenario 2 then,

$$\theta_i{}^{(2)} = \frac{1}{T}E\left[D_i(T)|_{D_i(T)\leq Q_i+SS_i, D_j(T)>Q_j+SS_j}\right] \tag{3.27}$$

We have,

$$E\left[D_i(T)|_{D_i(T)\le Q_i+SS_i, D_j(T)>Q_j+SS_j}\right] = \int_0^{Q_i+SS_i} x f_{D_i(T)|D_i(T)\le Q_i+SS_i}(x)dx \tag{3.28}$$

$$f_{D_i(T)|D_i(T)\le Q_i+SS_i}(x) = \frac{f_{D_i(T)}(x)}{F_{D_i(T)}(Q_i+SS_i)} \tag{3.29}$$

Therefore,

$$E\left[D_i(T)|_{D_i(T)\le Q_i+SS_i, D_j(T)>Q_j+SS_j}\right] = \frac{\int_0^{Q_i+SS_i} x f_{D_i(T)}(x)dx}{F_{D_i(T)}(Q_i+SS_i)} \tag{3.30}$$

Let $\theta_j{}^{(2)}$ be the conditional average demand rate at retailer j in scenario 2 then,

$$\theta_j{}^{(2)} = \frac{1}{T} E\left[D_j(T)|_{D_i(T)\le Q_i+SS_i, D_j(T)>Q_j+SS_j}\right] \tag{3.31}$$

We have,

$$E\left[D_j(T)|_{D_i(T)\le Q_i+SS_i, D_j(T)>Q_j+SS_j}\right] = \int_{Q_j+SS_j}^{\infty} y f_{D_j(T)|D_j(T)>Q_j+SS_j}(y)dy \tag{3.32}$$

$$f_{D_j(T)|D_j(T)>Q_j+SS_j}(y) = \frac{f_{D_j(T)}(y)}{P(D_j(T)>Q_j+SS_j)} \tag{3.33}$$

$$= \frac{f_{D_j(T)}(y)}{1-P(D_j(T)>Q_j+SS_j)} \quad = \frac{f_{D_j(T)}(y)}{1-F_{D_j(T)}(Q_j+SS_j)}$$

$$E\left[D_j(T)|_{D_i(T)\le Q_i+SS_i, D_j(T)>Q_j+SS_j}\right] = \frac{\int_{Q_j+SS_j}^{\infty} y f_{D_j(T)}(y)\,dy}{1-F_{D_j(T)}(Q_j+SS_j)} \tag{3.34}$$

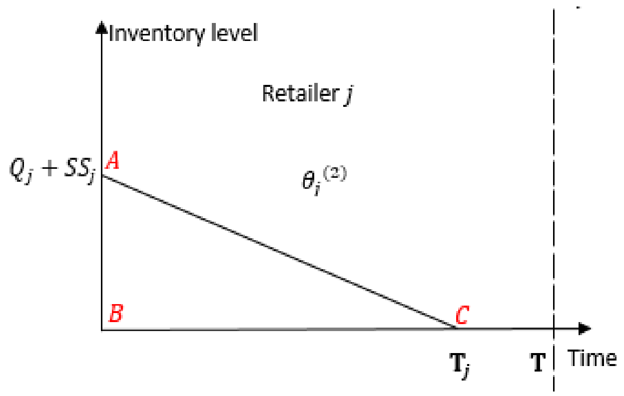
Expected holding cost at retailer i can be expressed as same as the scenario 1.

$$E\left[H_{ri}{}^{(2)}\right] = h_{ri}\frac{(2Q_i - \theta_i{}^{(2)}T + 2SS_i)}{2} \tag{3.35}$$

Expected holding cost at retailer j can be expressed as below.

Figure 3.8

Inventory Level at Retailer j at Scenario 2



$$E\left[{H_{rj}}^{(2)}\right] = \ h_{rj}\frac{(Aera\ of\ the\ ABC)}{T} = \frac{h_{rj}}{T}\left(\frac{1}{2}(Q_j + SS_j)T_j)\right) \tag{3.36}$$

In scenario 2, the whole inventory of retailer j is fully consumed before the end of the cycle. The expected time to consume the whole inventory at retailer j, i.e., $E[Tj]$ is given by,

$$E\left[T_j\right] = \frac{Q_j + SS_j}{{\theta_j}^{(2)}} \tag{3.37}$$

Therefore, the expected holding cost per unit time at the retailer j is given below.

$$E\left[{H_{rj}}^{(2)}\right] = \frac{h_{rj}}{T}\left(\frac{1}{2}(Q_j + SS_j)\frac{Q_j}{{\theta_j}^{(2)}})\right) = \frac{h_{rj}}{T}\left(\frac{(Q_j + SS_j)^2}{2{\theta_j}^{(2)}}\right) \tag{3.38}$$

Therefore, total expected holding cost for scenario 2 is given below.

$$E\left[H^{(2)}\right] = h_{ri}\frac{(2Q_i - {\theta_i}^{(2)}T + 2SS_i)}{2} + \frac{h_{rj}}{T}\left(\frac{(Q_j + SS_j)^2}{2{\theta_j}^{(2)}}\right) \tag{3.39}$$

The expected requested amount of retailer j during T is given by

$$r_j{}^{(2)} = {\theta_j}^{(2)}T - \left(Q_j + SS_j\right) \tag{3.40}$$

The expected excess amount of retailer i during T is given by

$$I_i{}^{(2)} = (Q_i + SS_i) - {\theta_i}^{(2)}T \tag{3.41}$$

Then, the expected transshipped amount from i to j is given by,

$$T_{ij} = min\left\{(Q_i + SS_i) - {\theta_i}^{(2)}T, {\theta_j}^{(2)}T - \left(Q_j + SS_j\right)\right\} \tag{3.42}$$

Then the lost sale amount at retailer j can be determined as,

$$N_j = {\theta_j}^{(2)}T - \left(Q_j + SS_j\right) - min\left\{(Q_i + SS_i) - {\theta_i}^{(2)}T, {\theta_j}^{(2)}T - \left(Q_j + SS_j\right)\right\} \tag{3.43}$$

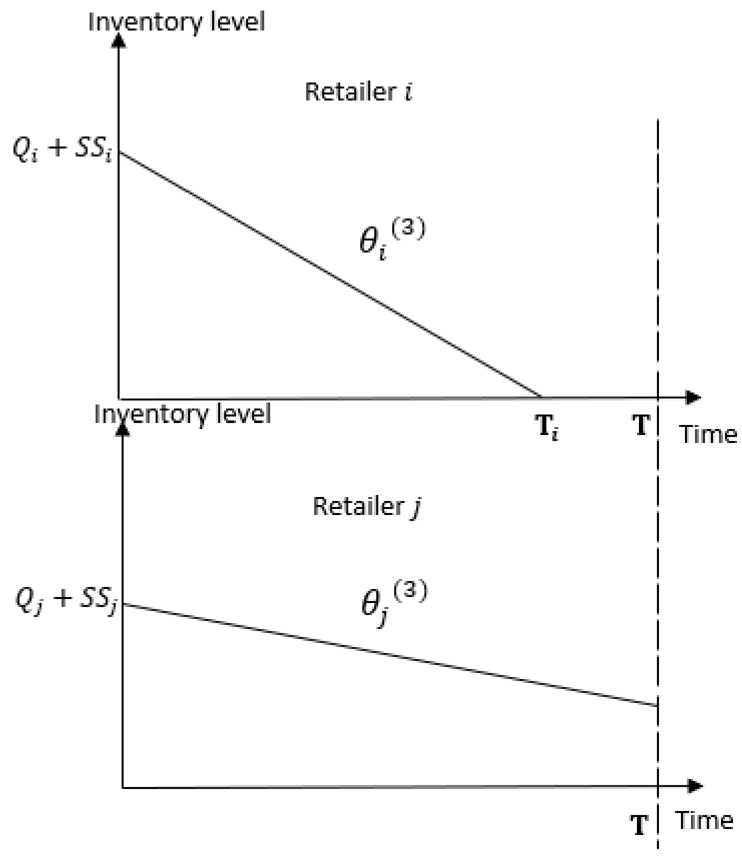
The expected total cost (excluding order cost and transportation cost) per time unit at scenario 2 can be determined as,

$$E\left[TC_r{}^{(2)}\right] = h_{ri}\frac{(2Q_i - \theta_i{}^{(2)}T + 2SS_i)}{2} + \frac{h_{rj}}{T}\left(\frac{(Q_j + SS_j)^2}{2\theta_j{}^{(2)}}\right)$$
$$+\frac{\pi_j}{T}\Big[\theta_j{}^{(2)}T - \left(Q_j + SS_j\right)$$
$$- min\left\{(Q_i + SS_i) - \theta_i{}^{(2)}T, \theta_j{}^{(2)}T - \left(Q_j + SS_j\right)\right\}\Big] \quad (3.44)$$

3.3.3 Scenario 03: ($D_i(T) > Q_i + SS_i$ and $D_j(T) \leq Q_j + SS_j$)

Figure 3.9

Inventory Level at Retailer i and Retailer j in Scenario 3



For scenario 3, the demand of retailer i is greater than the order quantity and the safety stock while the demand of retailer j is lower than the order quantity and the safety stock. Therefore, excess amount of retailer j will be transshipped to retailer i to fulfil the demand. The probability if scenario 3 to occur is given by,

$$P_3 = P(D_i(T) > Q_i + SS_i)P(D_j(T) \le Q_j + SS_j) \tag{3.45}$$

Let $\theta_i{}^{(3)}$ be the conditional average demand rate at retailer i in scenario 3 then,

$$\theta_i{}^{(3)} = \frac{1}{T}E\left[D_i(T)|_{D_i(T)>Q_i+SS_i, D_j(T)\le Q_j+SS_j}\right] \tag{3.46}$$

We have,

$$E\left[D_i(T)|_{D_i(T)>Q_i+SS_i, D_j(T)\le Q_j+SS_j}\right] = \int_{Q_i+SS_i}^{\infty} x f_{D_i(T)|D_i(T)>Q_i+SS_i}(x)dx \tag{3.47}$$

$$f_{D_i(T)|D_i(T)>Q_i+SS_i}(x) = \frac{f_{D_i(T)}(x)}{P(D_i(T) > Q_i + SS_i)} = \frac{f_{D_i(T)}(x)}{1 - P(D_i(T) > Q_i + SS_i)} \tag{3.48}$$

$$= \frac{f_{D_i(T)}(x)}{1 - F_{D_i(T)}(Q_i + SS_i)}$$

Therefore,

$$E\left[D_i(T)|_{D_i(T)>Q_i+SS_i, D_j(T)\le Q_j+SS_j}\right] = \frac{\int_{Q_i+SS_i}^{\infty} x f_{D_i(T)}(x)dx}{1 - F_{D_i(T)}(Q_i + SS_i)} \tag{3.49}$$

Let $\theta_j{}^{(3)}$ be the conditional average demand rate at retailer j in scenario 3 then,

$$\theta_j{}^{(3)} = \frac{1}{T}E\left[D_i(T)|_{D_i(T)>Q_i+SS_i, D_j(T)\le Q_j+SS_j}\right] \tag{3.50}$$

We have,

$$E\left[D_j(T)|_{D_i(T)>Q_i+SS_i, D_j(T)\le Q_j+SS_j}\right] = \int_0^{Q_j+SS_j} y f_{D_j(T)|D_j(T)\le Q_j+SS_j}(y)dy \tag{3.51}$$

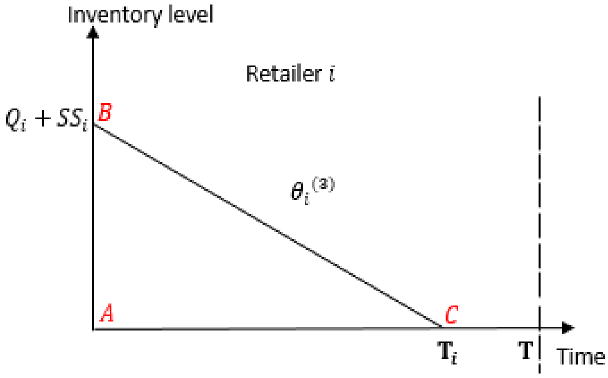
$$f_{D_j(T)|D_j(T)\le Q_j+SS_j}(y) = \frac{f_{D_j(T)}(y)}{F_{D_j(T)}(Q_j + SS_j)} \tag{3.52}$$

$$E\left[D_j(T)|_{D_i(T)>Q_i+SS_i, D_j(T)\le Q_j+SS_j}\right] = \frac{\int_0^{Q_j+SS_j} y f_{D_j(T)}(y)\,dy}{F_{D_j(T)}(Q_j + SS_j)} \tag{3.53}$$

Expected holding cost at retailer i can be expressed as same as the scenario 3

Figure 3.10

Inventory Level at Retailer i at Scenario 3



In scenario 3, the whole inventory of retailer i is fully consumed before the end of the cycle. The expected time to consume the whole inventory at retailer i, i.e., $E\ [T_i]$ is given by,

$$E[T_i] = \frac{Q_i + SS_i}{\theta_i^{(3)}} \tag{3.54}$$

Therefore, the expected holding cost per unit time at the retailer i is given below.

$$E\left[H_{ri}^{(3)}\right] = h_{ri} \frac{(Aera\ of\ the\ ABC)}{T} \tag{3.55}$$

$$= \frac{h_{ri}}{T}\left(\frac{1}{2}(Q_i + SS_i)T_i)\right)$$

$$= \frac{h_{ri}}{T}\left(\frac{1}{2}(Q_i + SS_i)\frac{Q_i + SS_i}{\theta_i^{(3)}})\right)$$

$$= \frac{h_{ri}}{T}\left(\frac{(Q_i + SS_i)^2}{2\theta_i^{(3)}}\right)$$

The expected holding cost per unit at the retailer j can be computed as scenario 1.

$$E\left[H_{rj}^{(3)}\right] = \frac{h_{rj}}{T}\frac{(2Q_j - \theta_j^{(3)}T + 2SS_j)}{2}T = h_{rj}\frac{(2Q_j - \theta_j^{(3)}T + 2SS_j)}{2} \tag{3.56}$$

Therefore, total expected holding cost for scenario 3 is given below.

$$E\left[H^{(3)}\right] = \frac{h_{ri}}{T}\left(\frac{(Q_i + SS_i)^2}{2\theta_i^{(3)}}\right) + h_{rj}\frac{(2Q_j - \theta_j^{(3)}T + 2SS_j)}{2} \tag{3.57}$$

The expected requested amount of retailer i during T is given by

$$r_i^{(3)} = \theta_i^{(3)}T - (Q_i + SS_i) \tag{3.58}$$

The expected excess amount of retailer j during T is given by

$$I_j^{(3)} = (Q_j + SS_j) - \theta_j^{(3)}T \tag{3.59}$$

Then, the expected transshipped amount from j to i is given by,

$$T_{ji} = min\{\theta_i^{(3)}T - (Q_i + SS_i), (Q_j + SS_j) - \theta_j^{(3)}T\} \tag{3.60}$$

Then the lost sale amount at retailer i can be determined as,

$$N_i = \theta_i^{(3)}T - (Q_i + SS_i) - min\{\theta_i^{(3)}T - (Q_i + SS_i), (Q_j + SS_j) - \theta_j^{(3)}T\} \tag{3.61}$$

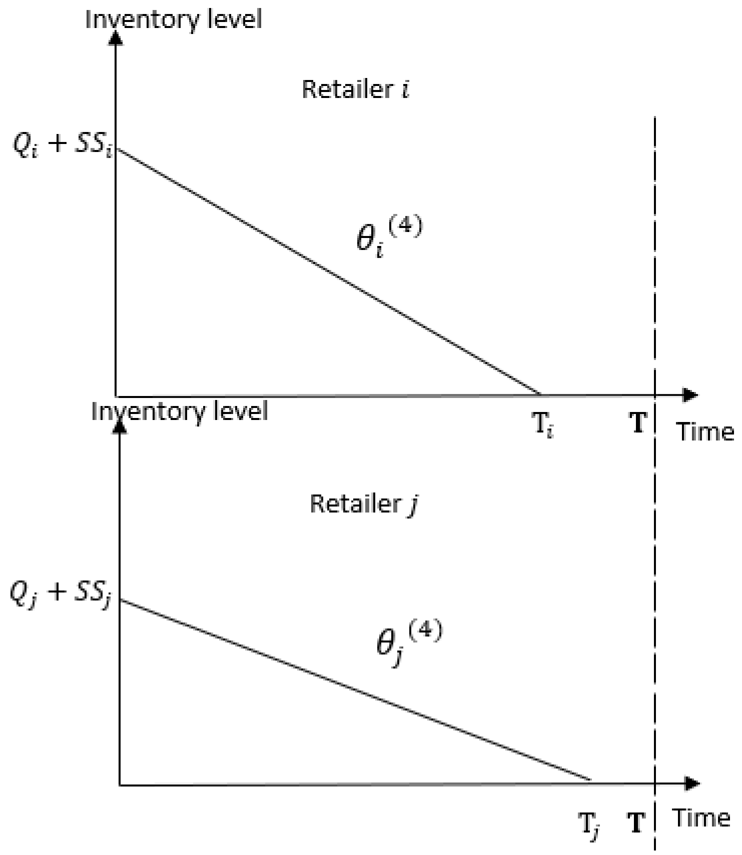
The expected total cost (excluding order cost and transportation cost) per time unit at scenario 3 can be determined as,

$$E[TC_r^{(3)}] = \frac{h_{ri}}{T}\left(\frac{(Q_i + SS_i)^2}{2\theta_i^{(3)}}\right) + h_{rj}\frac{(2Q_j - \theta_j^{(3)}T + 2SS_j)}{2} + \frac{\pi_i}{T}\left[\theta_i^{(3)}T - (Q_i + SS_i) - min\{\theta_i^{(3)}T - (Q_i + SS_i), (Q_j + SS_j) - \theta_j^{(3)}T\}\right] \tag{3.62}$$

3.3.4 Scenario 04: ($D_i(T) > Q_i + SS_i$ and $D_j(T) > Q_j + SS_j$)

Figure 3.11

Inventory Level at Retailer i and Retailer j in Scenario 4



For scenario 4, the demand of retailer i and the demand of retailer j are greater than the whole inventory of each retailer. The probability for scenario 4 to occur is given by,

$$P_4 = P(D_i(T) > Q_i + SS_i)P(D_j(T) > Q_j + SS_j) \tag{3.63}$$

Let ${\theta_i}^{(4)}$ be the conditional average demand rate at retailer i in scenario 4 then,

$${\theta_i}^{(4)} = \frac{1}{T} E\left[D_i(T)|_{D_i(T)>Q_i+SS_i, D_j(T)>Q_j+SS_j}\right] \tag{3.64}$$

We have,

$$E\left[D_i(T)|_{D_i(T)>Q_i+SS_i, D_j(T)>Q_j+SS_j}\right] = \int_{Q_i+SS_i}^{\infty} x f_{D_i(T)|D_i(T)>Q_i+SS_i}(x)dx \tag{3.65}$$

$$f_{D_i(T)|D_i(T)>Q_i+SS_i}(x) = \frac{f_{D_i(T)}(x)}{P(D_i(T) > Q_i + SS_i)} = \frac{f_{D_i(T)}(x)}{1 - P(D_i(T) > Q_i + SS_i)} \tag{3.67}$$
$$= \frac{f_{D_i(T)}(x)}{1 - F_{D_i(T)}(Q_i + SS_i)}$$

Therefore,

$$E\left[D_i(T)|_{D_i(T)>Q_i+SS_i, D_j(T)>Q_j+SS_j}\right] = \frac{\int_{Q_i+SS_i}^{\infty} x f_{D_i(T)}(x)dx}{1 - F_{D_i(T)}(Q_i + SS_i)} \tag{3.68}$$

Let ${\theta_j}^{(4)}$ be the conditional average demand rate at retailer j in scenario 4 then,

$${\theta_j}^{(4)} = \frac{1}{T} E\left[D_j(T)|_{D_i(T)>Q_i+SS_i, D_j(T)>Q_j+SS_j}\right] \tag{3.69}$$

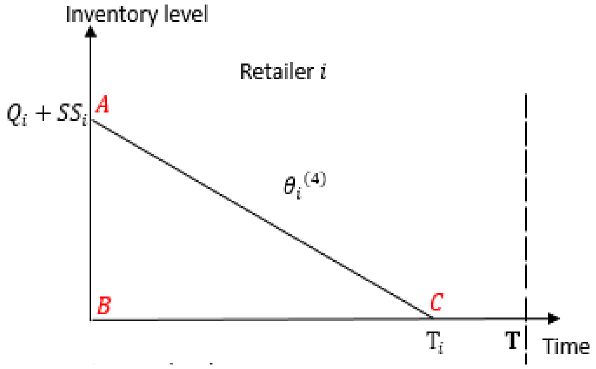
Similarly, we have,

$$E\left[D_j(T)|_{D_i(T)>Q_i+SS_i, D_j(T)>Q_j+SS_j}\right] = \frac{\int_{Q_j+SS_j}^{\infty} y f_{D_j(T)}(y)dy}{1 - F_{D_j(T)}(Q_j + SS_j)} \tag{3.70}$$

Expected holding cost per unit at retailer i can be expressed as below.

Figure 3.12

Inventory Level at Retailer i at Scenario 4



In scenario 4, the whole inventory of retailer i is fully consumed before the end of the cycle. The expected time to consume the whole inventory at retailer i, i.e., $E\ [T_i]$ is given by,

$$E[T_i] = \frac{Q_i + SS_i}{\theta_i{}^{(4)}} \tag{3.71}$$

Therefore, the expected holding cost per unit time at the retailer i is given below.

$$E\left[H_{ri}{}^{(4)}\right] = \ h_{ri} \frac{(Aera\ of\ the\ ABC)}{T} \tag{3.72}$$

$$= \frac{h_{ri}}{T}\left(\frac{1}{2}(Q_i + SS_i)T_i)\right)$$

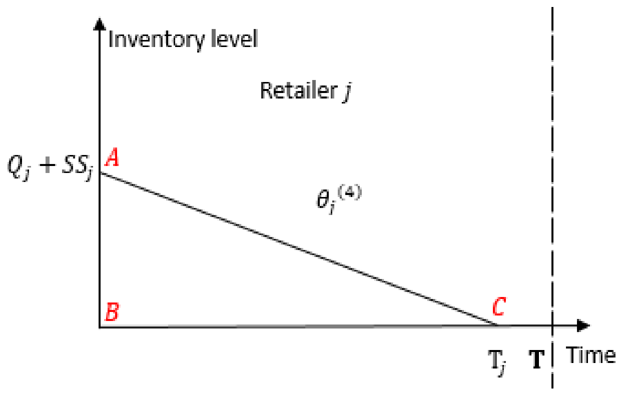
$$= \frac{h_{ri}}{T}\left(\frac{1}{2}(Q_i + SS_i)\frac{Q_i + SS_i}{\theta_i{}^{(4)}})\right)$$

$$= \frac{h_{ri}}{T}\left(\frac{(Q_i + SS_i)^2}{2\theta_i{}^{(4)}}\right)$$

Expected holding cost per unit at retailer j can be expressed as below.

Figure 3.13

Inventory Level at Retailer j at Scenario 4



In scenario 4, the whole inventory of retailer j is fully consumed before the end of the cycle. The expected time to consume the whole inventory at retailer j, i.e., $E\ [T_j]$ is given by,

$$E[T_j] = \frac{Q_j + SS_j}{\theta_j{}^{(4)}} \tag{3.73}$$

Therefore, the expected holding cost per unit time at the retailer j is given below.

$$E\left[H_{rj}{}^{(4)}\right] = \ h_{rj} \frac{(Aera\ of\ the\ ABC)}{T} \tag{3.74}$$

$$= \frac{h_{rj}}{T}\left(\frac{1}{2}(Q_j + SS_j)T_j)\right)$$
$$= \frac{h_{rj}}{T}\left(\frac{1}{2}(Q_j + SS_j)\frac{Q_j + SS_j}{\theta_j{}^{(4)}})\right)$$
$$= \frac{h_{rj}}{T}\left(\frac{(Q_j + SS_j)^2}{2\theta_j{}^{(4)}}\right)$$

Therefor total expected holding cost for scenario 4 is given below.

$$E[H^{(4)}] = \frac{h_{ri}}{T}\left(\frac{(Q_i + SS_i)^2}{2\theta_i{}^{(4)}}\right) + \frac{h_{rj}}{T}\left(\frac{(Q_j + SS_j)^2}{2\theta_j{}^{(4)}}\right) \tag{3.75}$$

The lost sale amount at retailer i can be determined as,

$$N_i = \theta_i{}^{(4)}T - (Q_i + SS_i) \tag{3.76}$$

The lost sale amount at retailer j can be determined as,

$$N_j = \theta_j{}^{(4)}T - (Q_j + SS_j) \tag{3.77}$$

The expected total cost (excluding order cost and transportation cost) per time unit at scenario 4 can be determined as,

$$E[TC_r{}^{(4)}] = \frac{h_{ri}}{T}\left(\frac{(Q_i + SS_i)^2}{2\theta_i{}^{(4)}}\right) + \frac{h_{rj}}{T}\left(\frac{(Q_j + SS_j)^2}{2\theta_j{}^{(4)}}\right) \tag{3.78}$$
$$+ \frac{\pi_i}{T}[\theta_i{}^{(4)}T - (Q_i + SS_i)] + \frac{\pi_j}{T}[\theta_j{}^{(4)}T - (Q_j + SS_j)]$$

For all four scenarios the ordering cost and the transportations cost will be fixed because these costs do not depend on the scenarios.

Therefore, ordering cost at retailer per unit time:

$$OC_r = \frac{f_{ri}}{T} + \frac{f_{rj}}{T} = \frac{f_{ri} + f_{rj}}{T} = (f_{ri} + f_{rj})\frac{D}{Q} \tag{3.79}$$

Transportation cost at retailers per unit time:

$$S_r = \frac{s_r(Q_i + Q_j)}{T} = \frac{s_r(Q_i + Q_j)}{Q}D \tag{3.80}$$

Therefore, the total cost at retailer can be derived as below.

$$E[TC_r] = OC_r + S_r + E[TC_r{}^{(1)}]P_1 + [TC_r{}^{(2)}]P_2 + [TC_r{}^{(3)}]P_3 + [TC_r{}^{(4)}]P_4 \tag{3.81}$$

Therefore, expected total per unit time cost of the system is given by,

$$E[TC_{VMI}] = E[TC_V] + E[TC_r] \tag{3.82}$$

$$E[TC_{VMI}] = f_v\frac{D}{Q} + (f_{ri} + f_{rj})\frac{D}{Q} + \frac{s_r(Q_i + Q_j)}{Q}D + E[TC_r{}^{(1)}]P_1 + [TC_r{}^{(2)}]P_2$$
$$+ [TC_r{}^{(3)}]P_3 + [TC_r{}^{(4)}]P_4$$

CHAPTER 4
NUMERICAL EXPERIMENTS

4.1 Numerical Example

In this chapter, numerical experiments were conducted by using Python to validate the mathematical model. The optimal values for the four decision variables, i.e. retailer i delivery quantity (Q_i), retailer j delivery quantity (Q_j), safety stock at retailer i (SS_i) and the safety stock at retailer j (SS_j), were determined so as to minimize the total cost of the whole VMI system.

The following constraints was added to the model to get more realistic values.

$$SS_i + SS_j = Zalpha * sqrt(L) * sqrt(\sigma_i{}^2 + \sigma_j{}^2) \tag{4.1}$$

$$Q_i + Q_j = T * (\mu_i + \mu_j) \tag{4.2}$$

The following options were applied for the optimization model.

- Bounds: Lower – [1 1 1 1], Upper – [250 250 250 250]

Input parameters as follows,

Table 4.1

Input Parameters

Input Parameter	Vendor	Retailer i	Retailer j
Mean Demand	-	12	20
Standard Deviation	-	24	23
Ordering Cost per unit	148	200	250
Lost sale Cost per unit time	-	7	8
Holding Cost per unit per time	-	5	6
Transportation cost per unit (Fixed)	-	9	9
Safety factor	-	2.236	2.236
Cycle Time	7	7	7
Lead Time	-	2	2

The optimal values obtained for the above input parameters are as below.

Table 4.2

Optimal Values

Decision Variables	Optimal Values
Q_i	125
Q_j	99
SS_i	66
SS_j	39

4.2 Sensitivity Analysis

In this section the effects of input parameters will be examined. The parameters such as the mean demand of two retailers, standard deviation of demand of two retailers, cycle time, lead time, holding cost, ordering cost, transportation cost, lost sale cost and the safety factor with stock-out probability α are investigated.

4.2.1 The Effect of the Mead Demand

In this part mean demand of retailer i will vary from 10 to 14 while the mean demand of retailer j will vary from 18 to 2 while keeping the other input parameters unchanged. The results are in the Table 4.3.

Table 4.3

Effect of the Mean Demand

μ_i	Q_i	Q_j	SS_i	SS_j	TC
10	111	99	58	48	57752
11	118	99	62	43	61637
12	125	99	66	39	65646
13	132	99	70	35	69777
14	139	99	73	32	74032
μ_j	Q_i	Q_j	SS_i	SS_j	TC
18	121	89	68	37	57786
19	123	94	67	38	61653
20	125	99	66	39	65646
21	127	103	65	40	69764
22	130	108	64	41	74006

According to the results, it shows that when the mean demand at both retailers increases the total cost of the whole system also increases. Furthermore, increasing the mean demand of retailer will push the system to increase the total the order quantity. When the mean demand increases at retailer i safety stock also increases and retailer j follows the same pattern. These trends are reasonable because when increasing the mean demand, it also increases the total cost of the whole.

4.2.2. The Effect of the Standard Deviation

In this section, the values of standard deviation of demand at retailer i varies from 22 to 26 while the standard deviation of demand at retailer *j* varies from 21 to 25. The other input parameters remain the same. The results of the effect of the standard deviation of the both retailers are presented in the Table 4.4.

Table 4.4

Effect of the Standard Deviation

σ_i	Q_i	Q_j	SS_i	SS_j	TC
22	126	98	65	34	65643
23	126	98	65	37	65644
24	125	99	66	39	65646
25	125	99	67	40	65647
26	124	100	68	42	65649
σ_j	Q_i	Q_j	SS_i	SS_j	TC
21	125	99	64	37	65631
22	125	99	65	38	65639
23	125	99	66	39	65646
24	126	98	67	40	65652
25	126	98	68	42	65659

According to the results it can be observed that the system total cost increases when the standard deviation of demand at both retailers increase. The sum of the order quantities increases according to the constraint 2. When increasing the standard deviation of retailer i the safety stock of the retailer i increases and also when increasing the standard deviation of retailer j the safety stock at retailer j also increases. This trend is reasonable to prevent the shortage cost.

4.2.3. The Effect of the Cycle Time

In this section, the value of the cycle time varies from 5 to 9 while the other input parameters remain the same. The results of the effect of the cycle are presented in the Table 4.5.

Table 4.5

Effect of the Cycle Time

T	Q_i	Q_j	SS_i	SS_j	TC
5	90	70	63	42	47032
6	108	84	64	41	56338
7	125	99	66	39	65646
8	143	113	68	37	74955
9	160	128	69	36	84265

The trend which is shown in the Table 4.5 is reasonable because the total cost of the whole system increases when the cycle time increases. And also, the order quantities have to be increased in order to prevent stockout cost.

4.2.4. The Effect of the Lead Time

In this section, the lead time at both retailers vary from 1 to 4 while the remaining input parameters are kept at initial values. The results are shown in the Table 4.6.

Table 4.6

Effect of the Lead Time

L	Q_i	Q_j	SS_i	SS_j	TC
1	122	102	47	27	65506
2	125	99	66	39	65646
3	129	95	81	48	65755
4	132	92	94	54	65847

According to the results it is noted that the order quantity of retailer i shows a slight increment while the order quantity of the retailer j shows a slight decrement and also safety stock of the both retailers increases. This trend is reasonable because when the lead time increases the total cost of the whole system also increases with the safety stock to prevent the stockout cost.

4.2.5. The Effect of the Holding Cost

In this section, the holding cost of retailer i is varied from 4 to 5.5 $ per unit per time unit while the holding cost at retailer j is varied from 5 to 6.5$ per unit per time unit while the other input parameters are kept at their initial values. The obtained results are presented in Table 4.7.

Table 4.7

Effect of the Holding Cost

h_i	Q_i	Q_j	SS_i	SS_j	TC
4	146	78	86	19	65491
4.5	137	87	77	23	65573
5	125	99	66	39	65646
5.5	112	112	52	53	65707
h_j	Q_i	Q_j	SS_i	SS_j	TC
5	96	128	36	69	65560
5.5	113	111	54	51	65609
6	125	99	66	39	65646
6.5	134	90	75	30	65674

According to the results in the Table 4.7, it is noticed that when the holding cost of one retailer increases while the holding cost of other retailer remains unchanged, the total cost of the whole system also increases. And also, it shows that increase in value of holding cost discourages the vendor to increase the order quantities and allocated safety stock of the two retailers because otherwise it will lead to a high inventory holding cost.

4.2.6. The Effect of the Ordering Cost

The sensitivity analysis is conducted to examine the effects of retailers' ordering and the vendor's ordering cost when the other input parameters are kept at their initial values. The obtained results are presented in Table 4.8.

Table 4.8

Effect of the Ordering Cost

O_V	Q_i	Q_j	SS_i	SS_j	TC
100	125	99	66	39	65639
148	125	99	66	39	65646
200	125	99	66	39	65654
250	125	99	66	39	65661
O_i	Q_i	Q_j	SS_i	SS_j	TC
150	125	99	66	39	65639
200	125	99	66	39	65646
250	125	99	66	39	65653
300	125	99	66	39	65660
O_j	Q_i	Q_j	SS_i	SS_j	TC
200	125	99	66	39	65639
250	125	99	66	39	65646
300	125	99	66	39	65653
350	125	99	66	39	65660

According to the results in the Table 4.8 the total cost of the system increases when the ordering cost increases. However, the change of the ordering cost of retailer has no effect on the safety stock for both retailers. So, the other output parameters such as ordering quantities of retailer and also the vendor remain unchanged. But there is a slight increase in the total cost of the whole supply chain.

4.2.7. The Effect of the Lost Sale Cost

In this section, sensitivity analysis is performed by changing the lost sale cost for both retailers from 6 to 9$ while the other input parameters are remained unchanged.

Table 4.9

Effect of the Ordering Cost

ls_i	Q_i	Q_j	SS_i	SS_j	TC
6	125	99	65	40	65644
7	125	99	66	39	65646
8	125	99	67	38	65647
9	125	99	68	37	65649
ls_i	Q_i	Q_j	SS_i	SS_j	TC
6	125	99	68	37	65644
7	125	99	67	38	65645
8	125	99	66	39	65646
9	125	99	65	40	65647

From the results, it is shown that, increase in lost sale cost of retailer will lead to increasing the total cost of the whole system. When increasing the lost sale cost at retailer i, the safety stock at retailer i increases and retailer j also follows the same trend. This trend shows that it encourages the vendor to allocate more safety stock quantity for the retailer when increasing the lost sale cost.

4.2.8. The Effect of the Transportation Cost

In this section the effect of transportation cost at retailer is examined by changing the values from 7 to 10$ per unit while the other input parameters are kept unchanged. The results are presented in the Table 4.10.

Table 4.10

Effect of the Transportation Cost

S_r	Q_i	Q_j	SS_i	SS_j	TC
7	125	99	66	39	51310
8	125	99	66	39	58478
9	125	99	66	39	65646
10	125	99	66	39	72814

According to the results in the Table 4.10, when the transportation cost increases the total cost of the whole system also increases. Also, it might discourage vendor to allocate more quantity to the retailers when the transportation cost is so high.

4.2.9. The Effect of the Safety Factor

In this section, the safety factor associated with stock-out probability α is changed examine its effect. The other input parameters are remained unchanged. The optimal results are illustrated in the Table 4.11.

Table 4.11

Effect of the Safety Factor

$Zalpha$	Q_i	Q_j	SS_i	SS_j	TC
1.751	123	101	52	30	65542
1.881	123	101	56	33	65569
2.054	124	100	61	36	65606
2.236	125	99	66	39	65646

It can be observed that the increase in safety factor will lead to increase in the total cost of the whole system. And also, it encourages vendor to allocate more safety stock to retailers.

CHAPTER 5
CONCLUSIONS AND RECCOMENDATIONS

5.1 Conclusions

For a supply chain, engaging a good inventory policy is a must to reduce the total cost of the supply chain while improving the service level. In this research, VMI model with one vendor and two retailers under continuous review policy was developed to derive the optimal values for the allocated delivery quantities and the safety stocks for each retailer so as to minimize the total cost of the whole supply chain.

Firstly, a mathematical model was developed when the demand is stochastic and also the lost sales are considered. Then numerical experiments and the sensitivity analyses were conducted to illustrate to identify the impact of the changes in the input parameters while showing the applicability of the proposed model. According to the results it is noticed that the optimal values of decision variables (delivery quantity to the retailers (Q_i & Q_j) and allocated safety stocks (SS_i & SS_j))can be determined while minimizing the total expected cost per a time unit. The important findings can be summarized below.

- One of the main objectives of this thesis is to obtain the reorder point of the whole supply chain. In the presence of random demand, reorder point usually includes a safety stock, in addition to the expected demand during the lead time. Therefore, it can be determined as follows.
  $ROP = Lead\ time\ demand + Safety\ Stock = (\mu_i + \mu_j) * L + SS_i + SS_j$
- Increasing the holding cost of the retailers will encourages the vendor to reduce the allocated delivery quantity and the safety stock to retailers.
- Changing the transportation cost and the ordering cost also shows the same trend. It is noticed that it discourages the vendor to allocate more delivery quantities for each retailer.
- Increasing the lost sales cost will lead to increase in the safety stock of each retailer so as to prevent from the stock out cost.
- It can be observed that the increase in safety factor will lead to increase in the total cost of the whole system. And also, it encourages vendor to allocate more quantities to retailers as well as the safety stock.

- Increasing the cycle time will lead to allocate more delivery quantities while increasing the lead time will encourage vendor to allocate more safety stock for each retailer.
- Increasing the mean demand will increase the delivery quantity to each retailer and also the standard deviation will affect to the safety stock of each retailer.

5.2 Recommendations

In this research, the mathematical model was developed for the emergency lateral transshipment in the case of a single supplier – two retailers under the Vendor Managed Inventory system. For further research directions, this can be expanded to a VMI system with single-supplier and multiple retailers or multiple suppliers - multiple retailer system where the preventive lateral transshipment can occur in in response to the stockout risk.

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APPENDIX

Computer Codes in Python:

```
import numpy as np
from scipy.optimize import minimize
from scipy.integrate import quad

m_i = 12
m_j = 20
sd_i = 24
sd_j = 23
O_v = 148
O_i = 200
O_j = 250
ls_i = 7
ls_j = 8
h_i = 5
h_j = 6
S_r = 9
T =7
L = 2
Zalpha = 2.236
pi = 22/7

def objective(X):
    from scipy.integrate import quad
    X1 = X[0]
    X2 = X[1]
    X3 = X[2]
    X4 = X[3]

    #formulating probabilities
    #P(Di(T)<=Qi+SSi)
```

```
def f1(x):
    m1 = T*m_i
    v1 = T**0.5*sd_i
    return np.exp(-0.5*((x-m1)/(v1))**2)/((2*pi)**0.5*v1)
b1 = quad(f1,0,X1+X3+1)
A = b1[0]

#P(Dj(T)<=Qj+SSj)
def f2(y)
    m2 = T*m_j
    v2 = T**0.5*sd_j
    return np.exp(-0.5*((y-m2)/(v2))**2)/((2*pi)**0.5*v2)
b2 = quad(f2,0,X2+X4+1)
B = b2[0]
#P(Dj(T)> Qj+SSj
def f3(y):
    m3 = T*m_j
    v3 = T**0.5*sd_j
    return np.exp(-0.5*((y-m3)/(v3))**2)/((2*pi)**0.5*v3
b3 = quad(f3,X2+X4,np.inf)
C = b3[0]

#P(Di(T)> Qi)
def f4(x):
    m4 = T*m_i
    v4 = T**0.5*sd_i
    return np.exp(-0.5*((x-m4)/(v4))**2)/((2*pi)**0.5*v4)
b4 = quad(f4,X1+X3,np.inf)
E= b4[0]

#PROBABILITY FOR EACH CASE
P1 = A*B
P2 = A*C
P3 = E*B
```

```
P4 = E*C


#FORMULATING EXPEXTATIONS
#CASE 1
def f5(x):
    m5 = T*m_i
    v5 = T**0.5*sd_i
    return x*np.exp(-0.5*((x-m5)/(v5))**2)/((2*pi)**0.5*v5)
b5 = quad(f5,0,X1+X3+1)
F = float(b5[0])/(A)

def f6(y):
    m6 = T*m_j
    v6 = T**0.5*sd_j
    return y*np.exp(-0.5*((y-m6)/(v6))**2)/((2*pi)**0.5*v6)
b6 = quad(f6,0,X2+X4+1)
G = b6[0]/(B)

#CASE
def f7(x):
    m7 = T*m_i
    v7 = T**0.5*sd_i
    return x*np.exp(-0.5*((x-m7)/(v7))**2)/((2*pi)**0.5*v7)
b7 = quad(f7,0,X1+X3+1)
H = b7[0]/(A)
def f8(y):
    m8 = T*m_j
    v8 = T**0.5*sd_j
    return y*np.exp(-0.5*((y-m8)/(v8))**2)/((2*pi)**0.5*v8)
b8 = quad(f8,X2+X4,np.inf)
I = b8[0]/(C)

#CASE
```

```
def f9(x):
    m9 = T*m_i
    v9 = T**0.5*sd_i
    return x*np.exp(-0.5*((x-m9)/(v9))**2)/((2*pi)**0.5*v9)
b9 = quad(f9,X1+X3,np.inf)
J = float(b9[0])/(E)

def f10(y):
    m10 = T*m_j
    v10 = T**0.5*sd_j
    return y*np.exp(-0.5*((y-m10)/(v10))**2)/((2*pi)**0.5*v10)
b10 = quad(f10,0,X2+X4+1)
K = b10[0]/(B)



#CASE 4
def f11(x):
    m11 = T*m_i
    v11 = T**0.5*sd_i
    return x*np.exp(-0.5*((x-m11)/(v11))**2)/((2*pi)**0.5*v11)
b11 = quad(f11,X1+X3,np.inf)
L = b11[0]/(1-A)

def f12(y):
    m12 = T*m_j
    v12 = T**0.5*sd_j
    return y*np.exp(-0.5*((y-m12)/(v12))**2)/((2*pi)**0.5*v12)
b12 = quad(f12,X2+X4,np.inf)
M = b12[0]/(1-B)

#FORMULATING TEETA FOR EACH CASE
#CASE 1
tta1_i = F/T
tta1_j = G/T
```

```
#CASE 2
tta2_i = H/T
tta2_j = I/T
#CASE 3
tta3_i = J/T
tta3_j = K/T

#CASE 4
tta4_i = L/T
tta4_j = M/T

#TOTAL COST AT VENDOR
TCv = O_v*(m_i+m_j)/(X1+X2)

#COST AT RETAILER
#CASE 1
HC1_i = h_i*((2*X1+2*X3-tta1_i*T)/(2))
HC1_j = h_j*((2*X2+2*X4-tta1_j*T)/(2))
TCr_1 = HC1_i+HC1_j

#CASE 2
HC2_i = h_i*((2*X1+2*X3-tta2_i*T))/(2.0)
HC2_j = h_j*(((X2+X4)**2)/(2*tta2_j*T))
T_2   = min((X1+X3-tta2_i*T),(tta2_j*T-(X2+X4)))
N2_j  = ls_j*(tta2_j*T-(X2+X4)-T_2)/T
TCr_2 = HC2_i+HC2_j+N2_j
#CASE 3
HC3_i = h_i*(((X1+X3)**2)/(2*tta3_i*T))
HC3_j = h_j*((2*X2+2*X4-tta3_j*T))/(2)
T_3   = min((tta3_i*(T)-(X1+X3)),(X2+X4-tta3_j*T))
N3_i  = ls_i*(tta3_i*(T)-(X1+X3)-T_3)/T
TCr_3 = HC3_i+HC3_j+N3_i
```

```
    #CASE 4
    HC4_i = h_i*(((X1+X3)**2)/(2*tta4_i*(T)))
    HC4_j = h_j*(((X2+X4)**2)/(2*tta4_j*(T)))
    N4_i  = ls_i*(tta4_i*(T)-(X1+X3))/T
    N4_j  = ls_j*(tta4_j*(T)-(X2+X4))/T
    TCr_4 = HC4_i+HC4_j+N4_i+N4_j

    # ORDERING COST AT RETAILER
    OCr = (O_i+O_j)*(m_i+m_j)/(X1+X2)

    #TRANSPORTATION COST AT RETAILER
    TPr = S_r*(X1+X2)*(m_i+m_j)

    #TOTAL COST AT RETAILER
    TCr = OCr+TPr+TCr_1*P1+TCr_2*P2+TCr_3*P3+TCr_4*P4

    return TCv+TCr

#CONSTRAINT 1
def constraint1(X):
    X3 = X[2]
    X4 = X[3]
    return (X3+X4)-(Zalpha*(L**0.5)*(((sd_i**2)+(sd_j**2))**0.5))

#CONSTRAINT 2
def constraint2(X):
    X1 = X[0]
    X2 = X[1]
    return (X1+X2)-(T*(m_i+m_j))
X0 = [1,1,1,1]
b = (1,250)
bnds = (b,b,b,b)
con1 = {'type': 'eq','fun':constraint1}
con2 = {'type': 'eq','fun':constraint2}
```

```
cons=[con1,con2]
sol = minimize(objective,X0,method='SLSQP',bounds=bnds,constraints=cons)
print(sol)
```