MITIGATING THE EFFECT OF SUPPLY CHAIN DISRUPTIONS BY USING A COMBINATION OF SALES REBATE CONTRACT AND BI-DIRECTIONAL OPTION CONTRACT

by

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AUTHOR’S DECLARATION

I, Wijekoon Mudiyanselage Arunika Sandani Wijekoon, declare that the research work carried out for this thesis was in accordance with the regulations of the Asian Institute of Technology. The work presented in it are my own and has been generated by me as the result of my own original research, and if external sources were used, such sources have been cited. It is original and has not been submitted to any other institution to obtain another degree or qualification. This is a true copy of the thesis, including final revisions.

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ABSTRACT

We investigate a coordinating mechanism utilizing supply chin contracts in this paper. This supply chain is made up of a single retailer and two distinct suppliers (main supplier and the backup supplier). When the main supplier's is subjected to supply chain disruption, the profit functions for each member are developed. The main supplier is under sales rebate contract, while the backup supplier is under bi-directional option contract. In order to identify optimal ordering quantities, a mathematical model is created, and the impact of variable inputs on decision variables is examined using MATLAB programming. From the derived mathematical model, sensitivity analysis was conducted to investigate both the order quantities and profits of each member when various input parameters were varied. To assess the coordination capabilities of suggested contracts a comparison of centralized and decentralized systems was performed.

Keywords: Supply contracts, supply chain coordination, sales rebate contract, bi-directional option contract, supply disruption.

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CHAPTER 1
INTRODUCTION

1.1 Background of the Study

The supply chain is a combination of many members, like retailer, assembler, distributor and manufacturer. These members take part in different ways to come up with finished products from raw material and to deliver the finished products to end-users. The human & relationship management is a critical point when a company have multiple retailers, multiple assemblers and multiple distributors to make sure there is a smooth supply chain. Likewise, there are many factors affecting the efficiency and performance of the supply chain.

During the end of the 18th century and the beginning of the 19th century came the first industrial revolution which after the proto-industrialization period. This mainly helps for the villagers and people in rural areas. During this period came the mechanization in the industry which led to greater change in the supply. Then comes the second industrial revolution which bought us lot of technological advancements and introducing different types of energy sources like electricity, gas and oil. To this day in history of industrial revolution the second period is known as the most important one because this opens opportunities in new fields of areas. Due to this advancement the transportation, communication, data collection methods, lead time of the supply chain and many wastes in the supply chain were reduced.

After this followed the period where the humans pushed the boundaries of the technological advancement of the second industrial revolution. This period is known as the third industrial revolution. Due to the rise in electronic and telecommunication growth, and the automation of the industry with helps of robotics this was known as era of high-level automation. Then comes the current industrial revolution which is still in debate among some people as to whether this is to be consider revolution or not.

But definitely with the advancement in the internet surely was the evolution of many fields in today's economy. Disruptive technologies are transforming all end-to-end steps in production and business models in most sectors of the economy. Which will in turn re shape the whole supply chain by revolutionizing the ways of creating value. Even though this revolution helps to reduce the wastage and minimize the cost of production or deployment, still these are not enough with advancement of the problems that could affect the supply chain.

The supply chain management has been examined for a very long time in the history. Several definitions have explained the true purpose of supply chain management such as,

1. “The processes from the initial raw materials to the ultimate consumption of the finished product linking across supplier user companies;” and
2. “The functions within and outside a company that enable the value chain to make products and provide services to the customer (Cox et al., 1995).”

The basic idea of managing a supply chain is to transform raw materials into a product (or a service) through several industrial operations. These industrial operations include gathering raw materials, parts manufacturing, assembly, inventory management, order management, distribution, delivery and transportation to the customer. A good coordination of material flow throughout the supply chain brings highest value to the end customer. Researchers found that management of a smooth material flow is a key factor in achieving superior supply chain performance. The main aim of coordination mechanisms is to motivate the supply chain members to make optimal decisions from the view of total supply chain. A successful logistics network can reduce entire supply chain costs, including manufacturing and procurement costs, inventory handling costs, facility costs (fixed costs), labor cost and transportation costs.

Disruptions in any of the member in supply chain can affect the total supply chain and can cause a huge loss to the company. When major disruptions occur, many supply chains tend to break down and take a long time to recover. That is one of the good reasons to their competitors to rise up and expand their boundaries in the industry. Many researches have been done in order to deal with such situations. As a recent example; with the global crisis novel coronavirus outbreak most of the companies had to stop the manufacturing processes and because of that so many employees became unemployed.

The coronavirus pandemic has caused delays and other frustrations in businesses’ global supply chains, highlighting how vulnerable many are to unexpected disruption. To mitigate the supply chain disruption past researchers have been following different strategies such as, risk management strategies, inventory management techniques, supply chain contracts based techniques, multi sourcing and many more. In this proposed research, multi sourcing engaged with different supply contracts will be further investigated.

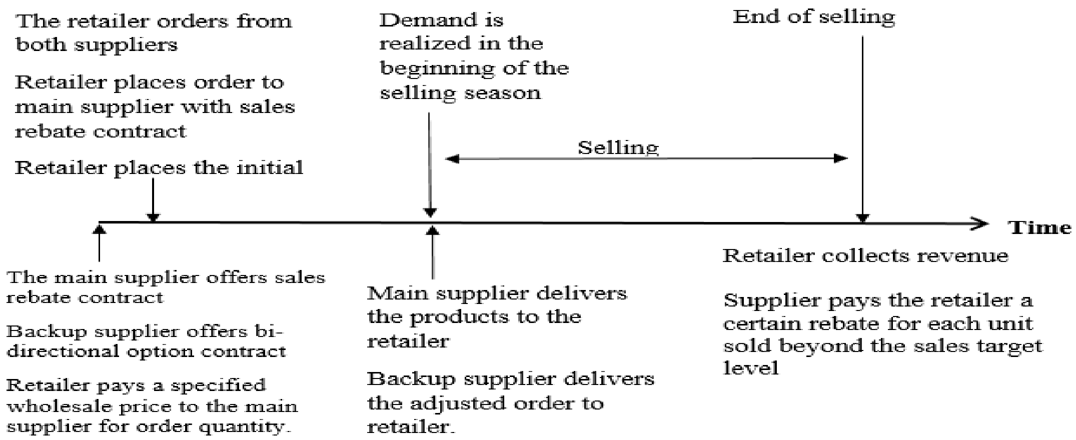
1.2 Statement of the Problem

With the raising technologies and information sharing among supply chain members, minimizing waste, optimizing production costs and improving productivity have been taken into consideration in the supply chain history. Supply chain disruption can lead to customer dissatisfaction in all business sectors. The direct relationship between supplier failure and customer satisfaction got less attention throughout the past. If the supplier fails to provide needed amount of raw materials, assembly parts or essential services, company and the final product may receive a negative response from the customer. Continued coordination among chain members and uninterrupted supply of raw materials and services will be able to improve the productivity and at the same time will help to maximize the profits of supply chain members.

The dual sourcing system is known to be more effective on mitigating supply disruptions due to the existence of secondary supplier (emergency supplier) when the main supplier fails to deliver the committed amount of products. The approach can further be explored when applying different contracts to the suppliers. In this research, we try to analyze the problem by considering a supply network with a single buyer and two suppliers.

Figure 1.1

Timeline of the System



The main supplier is under Sales rebate contract and in the coordination literature, two main types of channel rebates have been defined. (Taylor, 2002)

1. Linear sales rebate
2. Target sales rebate

Firstly, the linear sales rebate is direct and simple in definition where supplier pays the rebate for the items sold by retailer and on the other hand, target sales rebate has a condition that supplier pays retailer for each unit sold above a targeted sales level.

Target sales rebate affects the order quantity directly and it helps to overcome the problem of double marginalization in supply chain which reduces the profits of chain members and also gives disadvantage to the customers. Target sales rebate approach advises retailer to supply sales effort, discounted retail prices and promotions to draw customers attention on the product and increase the sales amount.

Retailers sales effort is important in influencing demand. The activities done such as, hiring more sales people, training them, making attractive shelf space and increasing advertising to increase sales effort usually put a high cost on retailer. This creates a conflict between manufacturer and retailer because those activities benefit both firms but are costly to only one. As explained by (Cachon, 2003), if the demand is considered to be dependent only on retailers' sales effort, both return policy and sales rebate contract fail to coordinate a supply chain.

As the main supplier will be under sales rebate contract and is subjected to supply chain disruptions, using only sales rebate contract seems not the best idea for the retailer. Hence a backup supplier which is perfectly reliable and able to get last minute supplies via the use of a bidirectional option contract should be considered.

1.3 Objectives of the Study

The aim of this research is to manage the supply chain with one retailer and two suppliers using contract agreements in order to maximize the retailer's profits under the circumstance where the main supplier is subject to supply disruption. A comparison of centralized and decentralized systems will be carried out in order to examine the coordination ability of proposed contracts.

1.4 Scope and Limitations

The system with the following characteristics will be examined throughout the research,

1. The supply chain consists of one buyer and two suppliers, i.e., one main supplier and one backup supplier
2. The main supplier is subject to supply chain disruptions
3. The planning period of this research is a single selling season.
4. Demand is uniformly distributed
5. The main supplier will be under sales rebate contract.
6. The backup supplier will be under bidirectional option contract.

CHAPTER 2

LITERATURE REVIEW

This research focuses to explore the possibilities of reducing disturbances in supply chain via a combination of rebate contract and bi directional option contract with a single buyer sourcing from two suppliers in the supply chain contracting system. The applicability of mentioned supply contracts to maximize the profits of the members is the aim of this study. The literature review presented in this section will present some of the previous studies done regarding supply chain coordination, supply disruption, sales rebate contract and bi directional option contract.

2.1 Supply Chain Coordination

Large number of studies have been conducted to examine supply chain coordination for years in the past recent history. Majority of the researches have used the prominent supply contracts such as quantity discount contract, wholesale price contract, buy back contract, revenue sharing contract, quantity flexibility contract to coordinate a supply chain. Wholesale contract is an agreement between the wholesaler and the retailer to buy the products at a wholesale price. Another commonly used contract is quantity discount contract which benefits both seller and the customer when buying goods above certain quantities, the wholesale price of a unit reduces and buyer can support the business by purchasing large number of products that directly benefits the seller.

Buyback and revenue sharing contracts have two different ideas where buyback contract allows the buyer to return unsold goods to seller. The seller usually offers to repurchase an item to boost the sale or to alleviate a buyer's concerns. A buyback usually has a set period of time or takes place under certain conditions. On the other hand, the revenue sharing contract talks about how the revenue is shared among buyer and supplier. Quantity flexibility contract, which increases the customer's commitment to purchase no less than a certain percentage below the forecast with the supplier's guarantee to deliver up to a certain percentage and above.

With wholesale contract and drop shipping contract, Shi et al. (2020) conducted study to investigate the corresponding applicable conditions and the most profitable power structure under each contract, then compare the profits between the two contracts of an online retailer, a manufacture and a physical store. With quantity flexibility contract and capacity reservation contract Li et al. (2020) tried to coordinate a supply chain with two supply chain members (a manufacturer and retailer) to uplift the manufacturer and expand the capacity and improve the overall performance.

With revenue sharing contract, the fashion firms share the retail revenue of the downstream retailers, who can order the products with lower wholesale price. (Ji et al., 2019). As Cachon, (2003) explains, the revenue sharing contract allows a supplier and a single retailer to coordinate their supply chain effectively; that is, the retailer can choose the optimal actions with respect to quantity and price to maximize profit. The supplier sells at a wholesale price below marginal cost (the cost of production, royalties, transportation, and handling), but sharing the retailer's revenue to offset its loss on sales.

Supply coordination is mainly preferring to improve the performance of chain members. Gan et al. (2009) has studied about risk averse agent situations because in real life it is not always a risk natural situation even though some researches take it as an assumption. Pareto-optimal solution is invented in order to be accepted by each agent in 3 different scenarios. They found the procedure to get Pareto-optimal solutions and designed a contract to achieve solutions. Wu, (2013) studied about buyback and news-vendor contract coordination to show buyback contract can increase the profits of chain members in comparison to non -buyback strategies. Vertical integration (VI) and manufacture's Stackelberg (MS) two channel policies were used to achieve the tasks.

2.2 Supply Chain Disruptions

Specifically, mitigating supply chain disruption describes successfully minimizing, and also avoiding the risks that can happen to lead major breakdowns in an active supply chain. Earthquakes, tsunami, heavy rain, industrial and road accidents, changes in government regulations, fire and also machine breakdowns can be introduced as several causes of supply disruptions. The recent global pandemic novel coronavirus global is a good example for a cause of supply chain disruptions. Quarantines, lockdowns, and reduced air travel have disrupted normal business operations all over the world and made it difficult for buyers to keep track with their suppliers. For those individual risks there are number of solutions to overcome the situation and stabilize the supply chain.

Craighead et al. (2007) explained that supply chain disruptions are caused by events that are neither planned nor anticipated and that disrupt the normal flow of goods and materials within a supply chain. Hence, not only catastrophic events, but also fluctuations in regular operations may result in a supply disruption. Daohai, Z. (2010) has modeled a computational experiment with case-based reasoning for supply disruption. The main results show that risk assessment, risk identify, risk control and risk evaluation mechanisms based on the case-based reasoning can effectively deal with supply disruption risk, bringing more profit and service level for enterprises.

As Wu et al. (2007) illustrated the modelling approach via the disruption analysis is a way of providing insights into managing supply chain systems that face disruptions and can allow a company to offer quicker response times to the customer, lower costs, higher levels of flexibility and agility, lower inventories, lower levels of obsolescence, and reduced demand amplification throughout the chain. With the use of Revenue Sharing contract(RSC) and Linear Quantity Discount contract(LQDC), Zhao et al. (2020) developed a model to minimize the demand disruption effect. Demand disruption affects the supply co-ordination in such a way that it could cause either increase or decrease in market demand (Shen et al., 2017). The benchmarks for the study was fixed without demand disruptions under both centralized and decentralized decision making processes. Final results came out as LQDC reacts positively for great increases and decreases in demand and RSC reacts when the demand increases slightly.

2.3 Sales Rebate Contract

Chiu et al. (2012) stated that the optimal parameters of the sales rebate contract should hence be determined with a good balance between the benefit (expected profit) and the risk (variance of profit) which means inter communication among the chain members should be strong enough to make a win-win situation for both retailer and the manufacture. It can help not only to improve the profits but also to lower the risk of both parties. Heydari & Najafi, (2016) have mentioned in a recent study that sales rebate contract is capable of coordinating a decentralized supply chain with uncertain market demand. In the model they assigned certain rebate for each item sold beyond the target level in order to increase the profit of the chain up to centralized decision making mode and that cheers up the retailer to increase the sales volume and order quantity.

Wong et al. (2009) investigated the effects of sales rebate on coordinating a two-echelon supply chain with a single supplier and multiple retailers in vendor-managed inventory (VMI) environment. He et al. (2009) studied the coordination of a supply chain with uncertain demands which depends on sales effort and retail price. They demonstrated that the combination of sales rebate and return contract is able to coordinate the defined supply chain. Taylor & Xiao, (2009) investigated the situation of retailers' awareness from the market demand by forecasting decisions and proposed a model by using the sales rebate and buyback contracts. Chiu et al. (2011) studied a case with risk sensitive agents in the coordination models. They considered a two-echelon supply chain with a single supplier and a single risk-averse retailer and carry out a mean-variance analysis through the target sales rebate contract.

Chiu et al. (2011) studied the sales rebate contract when demand depends on the retail price. They showed that the sales rebate cannot coordinate the supply chain itself; therefore, they considered a return policy beside the sales rebate. Hu et al. (2013) presented the flexible ordering policy in which, the order quantity varies in a specific range. First, they investigated the optimal flexible ordering policy and calculated the optimal production quantity in the centralized decision making structure and then proposed a coordination plan by implementing the revenue sharing and sales rebate contracts. Huang et al. (2014) investigated the model that the consumer returns are allowed. They studied the impacts of the secondary market on the supply chain coordination under the buyback and sales rebate contracts.

2.4 Bi-directional Option Contract

To make proper communication between retailer and manufacture regarding order quantity, Zhao et al. (2010) conducted a corporative game theory approach and tried to coordinate the supply chain by option contracts with the benchmarks of wholesale price mechanism. Padilla & Mishina (2009) simulated a model which helps retailer to maintain a flexible order quantity and it shows how it is beneficial for both retailer and supplier. Considering the types of option contract, Yang et al. (2017) conducted a study in agricultural supply chain with sales effort. Call, put and bi-directional contracts are used to coordinate the supply chain where the three option contracts are beneficial in different ways. Shortage risk, inventory risk and bilateral risk can be reduced by the mentioned option contracts accordingly. With the use of bi directional option contract, Wang & Tsao, (2006) tried to improve buyers profit by allowing him to adjust the initial order placed.

Some of the prior studies have often considered only the buyers optimal order decisions with bidirectional option contract. As an example for buyer's perspective of flexible supply contracts, Milner & Rosenblatt, (2002) conducted a two-period supply contract allowing bidirectional order adjustment for short life-cycle goods, where the buyer places initial orders for two periods before the planning horizon and can adjust the second order both upwards and downwards with penalties after observing the first period's demand. Wang & Tsao, (2006) developed a single-period two-stage contract with bidirectional option contract. But unlike Milner & Rosenblatt, (2002), they assumed that the buyer places an initial order and an option order for the single sales period. After updating the demand forecast, the buyer can exercise options to bi-directionally adjust the initial order up to the option quantity purchased.

Burnetas & Ritchken, (2005) investigated the role of option contracts (including unidirectional call option and bidirectional option) in a supply chain with downward-sloping demand curve. They analyzed the impact of the introduction of option contracts on the option pricing and retail pricing, and found that the effect of option contracts on the retailer's performance varied with the degree of demand uncertainty. Padilla & Mishina, (2009) discussed the influence of bidirectional option contracts on a two-tier supply chain formed by one retailer and one supplier in the multi-period setting, and concluded through simulation that bidirectional option contracts could obtain performance improvement for both the members as well as for the whole chain. Zhao et al. (2013) developed a supply contract for a two-tier supply chain of perishable products with a bidirectional option, derived closed-form expressions for the retailer's optimal order strategies, and found an elaborate bidirectional option contract can attain supply chain coordination.

CHAPTER 3

MATHEMATICAL MODEL DEVELOPMENT

In this chapter, we are going to develop the mathematical model of the problem based on several assumptions. The supply chain consists of two suppliers and one retailer. The main supplier offers low wholesale price but it is less reliable which is under sales rebate contract while the backup supplier is fully reliable but offers higher wholesale price and it will be under bi directional option contract. Mathematical model will be derived considering the time frame from the beginning of production period and to the end of selling season.

Disruptions will occur only to the main supplier so the model will consider both disruption and non-disruption scenarios. In both scenarios, the initial period before the selling season will be the same where the retailer will place their orders with both suppliers. The retailer will place the initial order Q_1 with the main supplier under sales rebate contract at the beginning of the production period. Retailer will pay w_1 to the main supplier. With the backup supplier the retailer will order Q_2 at whole sale price w_2 and decide the option quantity q_o and pay a premium of w_0 per option unit.

When demand exceeds the target sales level T, retailer gets a rebate for $(x - T)$ units from main supplier. Then the amount retailer earns from main supplier will be fixed as for $(Q_1 - T)$ units once demand exceeds initial order quantity with the main supplier (Q_1). Retailer decides the option quantities based on the realized demand and the current condition of the main supplier. The profit functions at the beginning of selling season is also analyzed based on the realized demand. At the end of the selling season, retailer gets to salvage any remaining products. If supply disruption occurs on main supplier, retailer will figure out the exercised option quantity as there won't be any supply from the main supplier.

When the put option is exercised, the backup supplier pays retailer w_{pp} for each exercised unit up to the maximum quantity q_0. When the call option is exercised, the retailer pays w_{cp} for each exercised unit up to the maximum quantity q_0 to the backup supplier. The probability of supplier disruption at the main supplier to occur will be denoted as P_d and the demand will be assumed to be a random variable with a uniform distribution between $[\gamma - n]$ and $[\gamma + n]$. Note that throughout this research, put option price will be lower than the wholesale price of the main supplier and the call option price will be higher than the wholesale price of the main supplier ($w_{pp} < r < w_{cp}$).

Notations:

x = market demand

R_p = retailer's sales price

Q_1 = initial order quantity with the main supplier

w_1 = whole sale price offered by main supplier

C_1 = unit production cost for main supplier

u = channel rebate (the amount paid by the main supplier to the retailer for each unit retailer sells beyond the target)

T = target sales level at which a unit rebate will be given to the retailer for each unit selling over the target value T

Q_2 = initial order quantity with the backup supplier

w_2 = whole sale price offered by the backup supplier

C_2 = unit production cost for backup supplier

q_o = option quantity

q_e = exercise quantity

w_0 = unit option premium price

w_{pp} = unit exercised put price

w_{cp} = unit exercised call price

v = unit salvage price

s = unit shortage cost

P = penalty cost per unit

P_d = probability that disruption occurs

3.1 Profit Functions Before the Selling Season Without Disruptions

- Profit of the Retailer = - cost of the initial order from the main supplier - cost of the initial order from the backup supplier - cost of option quantity from backup supplier

$$\pi_{Re}^{T1}(Q_1, Q_2, q_o) = -Q_1(w_1) - Q_2(w_2) - q_o(w_o) \quad (1)$$

- Profit of the main supplier = revenue from the initial order - cost of production

$$\pi_{S1}^{T1}(Q_1, Q_2, q_o) = Q_1(w_1 - c_1) \quad (2)$$

- Profit of the backup supplier = revenue from both the initial order and the option order - cost of production

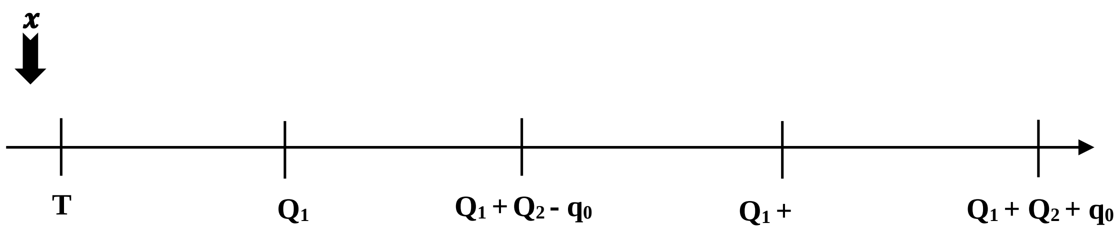
$$\pi_{S2}^{T1}(Q_1, Q_2, q_o) = Q_2(w_2) + q_o(w_o) - C_2(Q_2 + q_o) \quad (3)$$

3.2 Profit Functions During the Selling Season Without Disruption

Case 1: x < T

Figure 3.1

Case 1 Under Without Disruptions, Demand is Less Than T



In this scenario the retailer faces very low demand. The demand hasn't even reached the sales target level. He will exercise the full put option and receive full refunds of option quantity q_o from the backup supplier. Retailer will salvage remaining quantity.

- Profit function of the Retailer = sales revenue + refund from put option + salvage value

$$\pi_{Re}^{\mathrm{NT2}}(Q_1, Q_2, q_o, \mathrm{T}, \mathrm{q_e}) = x(R_p) + q_o(w_{\mathrm{pp}}) + v\,(\mathrm{Q_1} + \mathrm{Q_2} - \mathrm{q_0} - x) \quad (4)$$

- Profit of the main supplier = 0

$$\pi_{S1}^{\mathrm{NT2}}(Q_1, Q_2, q_o, \mathrm{T}, \mathrm{q_e}) = 0 \quad (5)$$

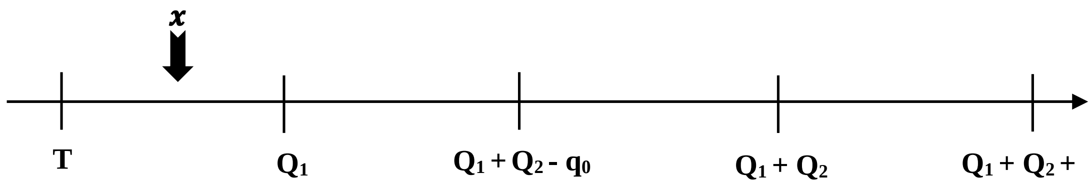
- Profit of the backup supplier = - exercised put option cost + salvage

$$\pi_{S2}^{\mathrm{NT2}}(Q_1, Q_2, q_o, \mathrm{T}, \mathrm{q_e}) = -\, q_o\, w_{\mathrm{pp}} + 2q_o v \quad (6)$$

Case 2: **T** $\leq x <$ Q**1**

Figure 3.2

Case 2 Under Without Disruptions, Demand is Between T and Q_1



In this scenario the demand is more than or equal T but still less than $\mathrm{Q_1}$. Retailer will exercise full put option from the backup supplier and will salvage the remaining quantity of $\mathrm{Q_1} + \mathrm{Q_2} - \mathrm{q_0} - x$. He will also receive the rebate for (x – T) units.

- Profit function of the Retailer = sales revenue + rebate from main supplier + refund from put option + salvage

$$\pi_{Re}^{\mathrm{NT2}}(Q_1, Q_2, q_o, \mathrm{T}, \mathrm{q_e}) = x(R_p) + (x - \mathrm{T})\,(\mathrm{u}) + w_{\mathrm{pp}}\, q_o + v\,(\mathrm{Q_1} + \mathrm{Q_2} - \mathrm{q_0} - x) \quad (7)$$

- Profit of the main supplier = - rebate given to the retailer

$$\pi_{S1}^{\mathrm{NT2}}(Q_1, Q_2, q_o, \mathrm{T}, \mathrm{q_e}) = -\,(x - \mathrm{T})\,(\mathrm{u}) \quad (8)$$

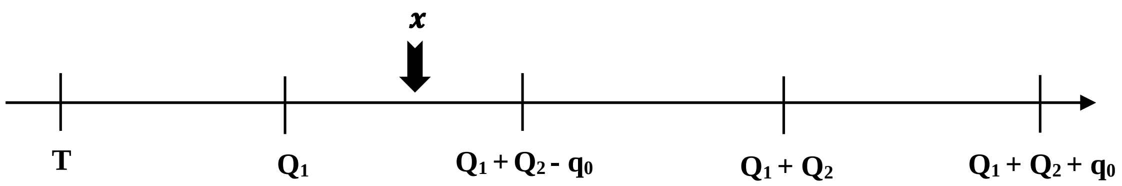
- Profit of the backup supplier = - exercised put option cost + salvage

$$\pi_{S2}^{\mathrm{NT2}}(Q_1, Q_2, q_o, \mathrm{T}, \mathrm{q_e}) = -\, q_o\, w_{\mathrm{pp}} + 2q_o v \quad (9)$$

Case 3: $\boldsymbol{Q_1 \leq x < Q_1 + Q_2 - q_0}$

Figure 3.3

Case 3 Under Without Disruptions, Demand is Between Q_1 and $Q_1 + Q_2 - q_0$



In this scenario the demand is more than or equal to Q_1 but still less than $Q_1 + Q_2 - q_0$. The retailer will exercise the full put option in this scenario and receives rebate for $(Q_1 - T)$ units. Excess inventory at the retailer is $Q_1 + Q_2 - q_0 - x$ and will be salvaged by the retailer.

- Profit function of the Retailer = sales revenue + refund from the main supplier + refund from backup supplier + salvage

$$\pi_{Re}^{NT2}(Q_1, Q_2, q_o, T, q_e) = x(R_p) + (Q_1 - T)(u) + w_{pp}\, q_o + v\,(Q_1 + Q_2 - q_0 - x) \qquad (10)$$

- Profit of the main supplier = - rebate given to the retailer

$$\pi_{S1}^{NT2}(Q_1, Q_2, q_o, T, q_e) = -(Q_1 - T)(u) \qquad (11)$$

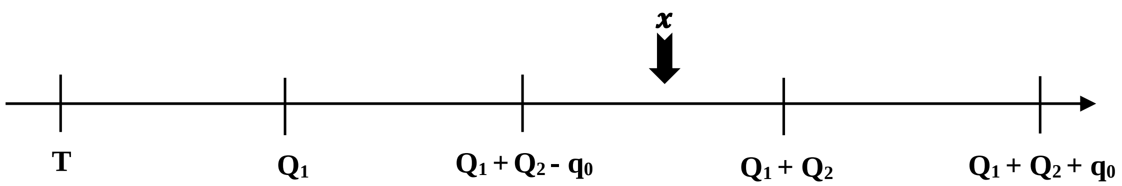
- Profit of the backup supplier = - exercised put option cost + salvage

$$\pi_{S2}^{NT2}(Q_1, Q_2, q_o, T, q_e) = -\, q_o\, w_{pp} + 2 q_o v \qquad (12)$$

Case 4: $\boldsymbol{Q_1 + Q_2 - q_0 \leq x < Q_1 + Q_2}$

Figure 3.4

Case 4 Under Without Disruptions, Demand is Between $Q_1 + Q_2 - q_0$ and $Q_1 + Q_2$



In this scenario the demand is more than or equal to $Q_1 + Q_2$ - q_0 but still less than $Q_1 + Q_2$. In this case the retailer will exercise partial put option at an amount ($Q_1 + Q_2 - x$) with the backup supplier and receives rebate for (Q_1 – T) units

- Profit function of the Retailer = sales revenue + refund from the main supplier + refund from backup supplier

$$\pi_{Re}^{NT2}(Q_1, Q_2, q_o, T, q_e) = x(R_p) + (Q_1 - T)\,u + w_{pp}\,(Q_1 + Q_2 - x) \quad (13)$$

- Profit of the main supplier = - rebate given to the retailer

$$\pi_{S1}^{NT2}(Q_1, Q_2, q_o, T, q_e) = -\,u\,(Q_1 - T) \quad (14)$$

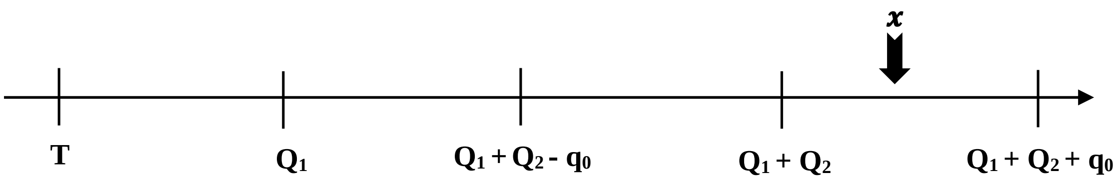
- Profit of the backup supplier = - exercised put option cost + salvage

$$\pi_{S2}^{NT2}(Q_1, Q_2, q_o, T, q_e) = -\,w_{pp}\,(Q_1 + Q_2 - x) + v\,(Q_1 + Q_2 - x + q_0) \quad (15)$$

Case 5: $Q_1 + Q_2 \leq x < Q_1 + Q_2 + q_0$

Figure 3.5

Case 5 Under Without Disruptions, Demand is Between $Q_1 + Q_2$ and $Q_1 + Q_2 + q_0$



In this scenario, the demand is more than or equal $Q_1 + Q_2$ and less than to $Q_1 + Q_2 + q_0$, which means the demand is greater than total initial orders. Therefore, the retailer will partially exercise the call option at an amount ($x - Q_1 - Q_2$) with the backup supplier and receives rebate for (Q_1 – T) units.

- Profit function of the Retailer = sales revenue + rebate from the main supplier – cost of exercised call option quantity

$$\pi_{Re}^{NT2}(Q_1, Q_2, q_o, T, q_e) = x(R_p) + (Q_1 - T)\,u - w_{cp}\,(x - Q_1 - Q_2) \quad (16)$$

- Profit of the main supplier = - rebate given to the retailer

$$\pi_{S1}^{NT2}(Q_1, Q_2, q_o, T, q_e) = -(Q_1 - T)\, u \tag{17}$$

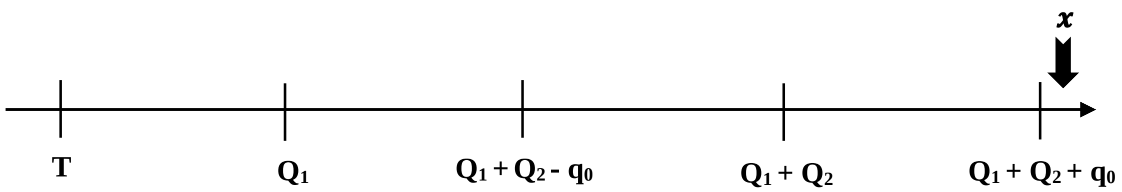
- Profit of the backup supplier = income from exercised call option + salvage

$$\pi_{S2}^{NT2}(Q_1, Q_2, q_o, T, q_e) = w_{cp}(x - Q_1 - Q_2) + v\{q_0 - [x - (Q_1 + Q_2)]\} \tag{18}$$

Case 6: $\boldsymbol{x > Q_1 + Q_2 + q_0}$

Figure 3.6

Case 6 Under Without Disruptions, Demand is Greater Than $Q_1 + Q_2 + q_0$



This scenario has the highest realized demand in the system. The demand is more than $Q_1 + Q_2 + q_0$ and the retailer will not have to salvage any products at all, but he will have to face some shortage costs even though the maximum call option is being exercised with the backup supplier.

- Profit function of the Retailer = sales revenue + rebate from the main supplier – cost of exercised call option quantity – shortage cost

$$\pi_{Re}^{NT2}(Q_1, Q_2, q_o, T, q_e) = (Q_1 + Q_2 + q_0)(R_p) + (Q_1 - T)\, u - q_0 (w_{cp}) - s\,[x - (Q_1 + Q_2 + q_0)] \tag{19}$$

- Profit of the main supplier = - rebate given to the retailer

$$\pi_{S1}^{NT2}(Q_1, Q_2, q_o, T, q_e) = -(Q_1 - T)\, u \tag{20}$$

- Profit of the backup supplier = income from exercised maximum call option

$$\pi_{S2}^{NT2}(Q_1, Q_2, q_o, T, q_e) = q_0\, w_{cp} \tag{21}$$

3.3 Profit functions before the selling season with disruptions

- Profit of the Retailer = - cost of the initial order from the main supplier - cost of the initial order from the backup supplier - cost of option quantity from backup supplier + penalty paid by the manufacturer

$$\pi_{Re}^{T1}(Q_1, Q_2, q_o) = -Q_1(w_1) - Q_2(w_2) - q_o(w_o) + Q_1(P) \quad (22)$$

- Profit of the main supplier = revenue from the initial order – penalty cost

$$\pi_{S1}^{T1}\ (Q_1, Q_2, q_o) = Q_1(w_1 - P) \quad (23)$$

- Profit of the backup supplier = revenue from both the initial order and the option order - cost of production

$$\pi_{S2}^{T1}\ (Q_1, Q_2, q_o) = Q_2(w_2) + q_o(w_o) - C_2(Q_2 + q_o) \quad (24)$$

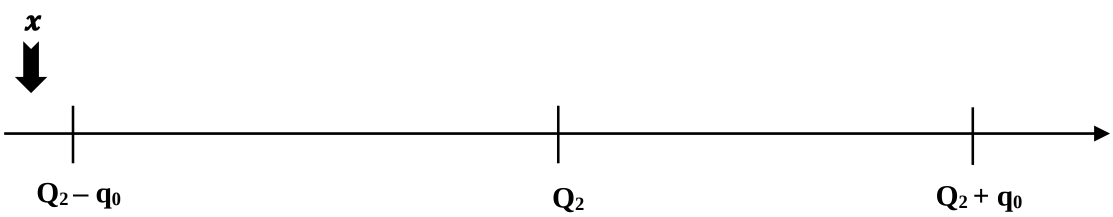
3.4 Profit functions during the selling season with disruptions

In the following situations where supply chain disruption occurs, the retailer will not receive any amount of goods from the main supplier. So that the retailer has to rely only on the backup supplier. When the retailer realizes the actual demand, he decides the exercised option quantity (q_e). The following cases are analyzed separately based on the realized demand (x).

Case 1: $\boldsymbol{x < Q_2 - q_o}$

Figure 3.7

Case 1 Under with Disruptions, Demand is Less Than $Q_2 - q_0$



In this case the realized demand is less than $Q_2 - q_0$. So, the retailer will have to exercise maximum put option and also will receive full refunds of q_o. The backup supplier and the retailer will be left with extra inventory which can be solved as salvage.

- Profit of the retailer = sales revenue + refund from exercised put option + salvage

$$\pi_{Re}^{\mathrm{DT2}}(Q_1, Q_2, q_o, \mathrm{T}, \mathrm{q_e}) = x(R_p) + q_o(w_{\mathrm{pp}}) + v\,[(Q_2 - q_o) - x] \quad (25)$$

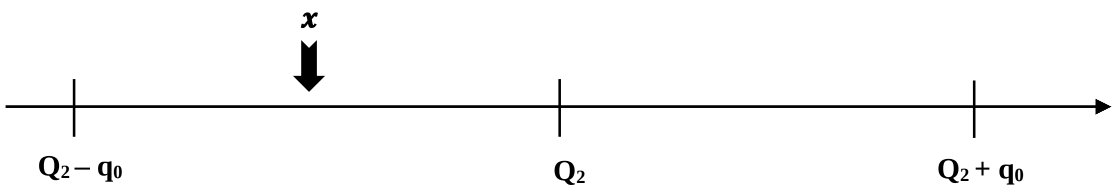
- Profit of the backup supplier = - cost of exercised put option + salvage

$$\pi_{S2}^{\mathrm{DT2}}(Q_1, Q_2, q_o, \mathrm{T}, \mathrm{q_e}) = -\,q_o(w_{\mathrm{pp}}) + 2q_o(v) \quad (26)$$

Case 2: $Q_2 - q_o \leq x < Q_2$

Figure 3.8

Case 2 Under with Disruptions, Demand is Between $Q_2 - q_0$ and Q_2



In this case the realized demand is more than or equal to $Q_2 - q_0$ but still less than Q_2. The retailer will partially exercise the put option. The backup supplier will be left with extra inventory which can be solved as salvage.

- Profit of the retailer = sales revenue + refund from exercised put option

$$\pi_{Re}^{\mathrm{DT2}}(Q_1, Q_2, q_o, \mathrm{T}, \mathrm{q_e}) = x(R_p) + w_{\mathrm{pp}}\,(Q_2 - x) \quad (27)$$

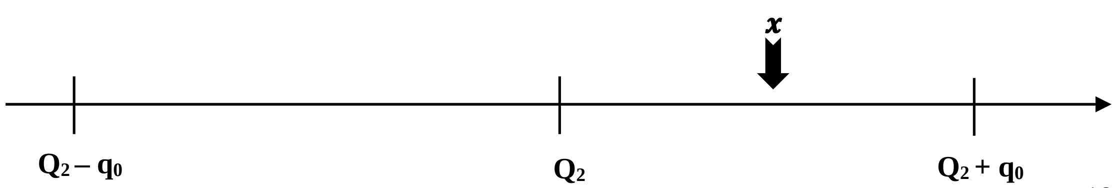
- Profit of the backup supplier = - cost of exercised put option + salvage

$$\pi_{S2}^{\mathrm{DT2}}(Q_1, Q_2, q_o, \mathrm{T}, \mathrm{q_e}) = -\,w_{\mathrm{pp}}\,(Q_2 - x) + v\,[q_o + (Q_2 - x)] \quad (28)$$

Case 3: $Q_2 \leq x < Q_2 + q_o$

Figure 3.9

Case 3 Under with Disruptions, Demand is Between Q_2 and $Q_2 + q_0$



In this case the realized demand is more than or equal to Q_2 but still less than $Q_2 + q_0$. The retailer will have to partially exercise the call option. The backup supplier will salvage any remaining goods.

- Profit of the retailer = sales revenue – cost of exercised call option

$$\pi_{Re}^{DT2}(Q_1, Q_2, q_o, T, q_e) = x(R_p) - w_{cp}\,(x - Q_2) \tag{29}$$

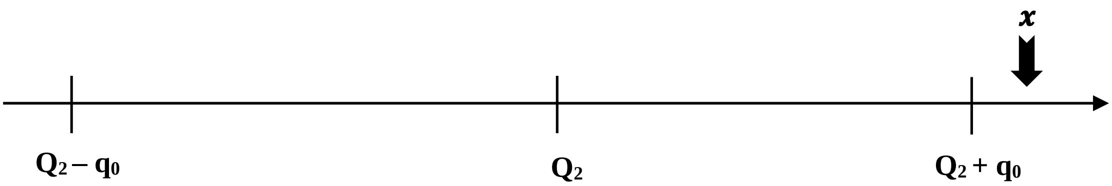
- Profit of the backup supplier = income from exercised call option + salvage

$$\pi_{S2}^{DT2}(Q_1, Q_2, q_o, T, q_e) = w_{cp}\,(x - Q_2) + v\,[q_o - (x - Q_2)] \tag{30}$$

Case 4: $x \geq Q_2 + q_o$

Figure 3.10

Case 4 Under with Disruptions, Demand is More Than $Q_2 + q_0$



In this case the realized demand is more than or equal $Q_2 + q_0$. The retailer will have to exercise the maximum call option. The retailer will have to face some shortage cost because the demand still not be satisfied.

- Profit of the retailer = sales revenue – cost of exercised call option– shortage cost

$$\pi_{Re}^{DT2}(Q_1, Q_2, q_o, T, q_e) = x(R_p) - w_{cp}\,(q_o) - s\,[\,x - (Q_2 + q_0)] \tag{31}$$

- Profit of the backup supplier = income from maximum exercised call option

$$\pi_{S2}^{DT2}(Q_1, Q_2, q_o, T, q_e) = w_{cp}\,(q_o) \tag{32}$$

3.5 Expected Profits

Expected profit functions for each supply chain member of supply chain, with disruptions and without disruption scenarios are developed in 3.5.1 and 3.5.2 subsections.

3.5.1 Expected Profits Without Disruption

In order to develop the expected profits for each member of supply chain in no disruption situation, we will have to consider all possible demand scenarios in selling season. For that, assuming that the demand is uniformly distributed between $[\gamma - n]$ and $[\gamma + n]$.

- Retailer's profit function without disruption

Retailer's profit function = Profit function before the selling season + Expected profit function during the selling season

$$\pi_{Re}^{\mathrm{N}}(Q_1, Q_2, q_o) = [-Q_1(w_1) - Q_2(w_2) - q_o(w_o)] + E[\pi_{Re}^{\mathrm{NT2}}(Q_1, Q_2, q_o, \mathrm{T}, q_e)] \quad (33)$$

Retailers expected profit during the selling season can be determined as,

$$\begin{aligned} E[\pi_{Re}^{\mathrm{NT2}}(\,(Q_1, Q_2, q_o, \mathrm{T}, q_e)] = & \int_{\gamma - n}^{\mathrm{T}} \pi_{Re}^{\mathrm{NT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx \\ & + \int_{\mathrm{T}}^{Q_1} \pi_{Re}^{\mathrm{NT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx \\ & + \int_{Q_1}^{Q_1 + Q_2 - q_0} \pi_{Re}^{\mathrm{NT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx \\ & + \int_{Q_1 + Q_2 - q_0}^{Q_1 + Q_2} \pi_{Re}^{\mathrm{NT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx \\ & + \int_{Q_1 + Q_2}^{Q_1 + Q_2 + q_0} \pi_{Re}^{\mathrm{NT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx \\ & + \int_{Q_1 + Q_2 + q_0}^{\gamma + n} \pi_{Re}^{\mathrm{NT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx \end{aligned}$$

(34)

- Main supplier's profit function without disruption

Main supplier's profit function = Profit function before the selling season + Expected profit function during the selling season

$$\pi_{S1}^{\mathrm{N}}(Q_1, Q_2, q_o) = [Q_1(w_1 - c_1)] + E[\pi_{S1}^{\mathrm{NT2}}\,(Q_1, Q_2, q_o, \mathrm{T}, q_e)] \quad (35)$$

The main suppliers expected profit function during the selling season can be obtained as follows,

$$E[\pi_{S1}^{\mathrm{NT2}}((Q_1, Q_2, q_o, \mathrm{T}, q_e)] = \int_{\gamma - n}^{\mathrm{T}} \pi_{S1}^{\mathrm{NT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx$$

$$+ \int_{\mathrm{T}}^{Q_1} \pi_{S1}^{\mathrm{NT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx$$

$$+ \int_{Q_1}^{Q_1 + Q_2 - q_0} \pi_{S1}^{\mathrm{NT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx$$

$$+ \int_{Q_1 + Q_2 - q_0}^{Q_1 + Q_2} \pi_{S1}^{\mathrm{NT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx$$

$$+ \int_{Q_1 + Q_2}^{Q_1 + Q_2 + q_0} \pi_{S1}^{\mathrm{NT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx$$

$$+ \int_{Q_1 + Q_2 + q_0}^{\gamma + n} \pi_{S1}^{\mathrm{NT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx$$

(36)

- Backup supplier's profit function without disruption

Backup supplier's profit function = Profit function before the selling season + Expected profit function during the selling season

$$\pi_{S2}^{\mathrm{N}}(Q_1, Q_2, q_o) = [Q_2(w_2) + q_o(w_o) - C_2(Q_2 + q_o)] + E[\pi_{S2}^{\mathrm{NT2}}\ (Q_1, Q_2, q_o, \mathrm{T}, q_e)] \quad (37)$$

The backup suppliers expected profit function during the selling season can be obtained as follows,

$$E[\pi_{S2}^{\mathrm{NT2}}((Q_1, Q_2, q_o, \mathrm{T}, q_e)] = \int_{\gamma - n}^{\mathrm{T}} \pi_{S2}^{\mathrm{NT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx$$

$$+ \int_{\mathrm{T}}^{Q_1} \pi_{S2}^{\mathrm{NT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx$$

$$+ \int_{Q_1}^{Q_1 + Q_2 - q_0} \pi_{S2}^{\mathrm{NT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx$$

$$+ \int_{Q_1 + Q_2 - q_0}^{Q_1 + Q_2} \pi_{S2}^{\mathrm{NT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx$$

$$+ \int_{Q_1 + Q_2}^{Q_1 + Q_2 + q_0} \pi_{S2}^{\mathrm{NT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx$$

$$+ \int_{Q_1 + Q_2 + q_0}^{\gamma + n} \pi_{S2}^{\mathrm{NT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx$$

(38)

3.5.2 Expected Profits with Disruption

As we build the expected profits for all member of the supply chain with disruption situation here, all possible demand scenarios in the selling season should be taken into consideration. For this assuming that demand is uniformly distributed between [γ − n] and [γ + n].

- Retailer's profit function with disruption

Retailer's profit function = Profit function before the selling season + Expected profit function during the selling season

$$\pi_{Re}^{\mathrm{D}}(Q_1, Q_2, q_o) = [-Q_1(w_1) - Q_2(w_2) - q_o(w_o) + Q_1(P)\ ] + E[\pi_{Re}^{\mathrm{DT2}}(Q_1, Q_2, q_o, \mathrm{T}, q_e)] \quad (39)$$

Retailers expected profit during the selling season can be determined as,

$$E[\pi_{Re}^{\mathrm{DT2}}(\ (Q_1, Q_2, q_o, \mathrm{T}, q_e)] = \int_{\gamma - n}^{Q_2 - q_0} \pi_{Re}^{\mathrm{DT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx$$
$$+ \int_{Q_2 - q_0}^{Q_2} \pi_{Re}^{\mathrm{DT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx$$
$$+ \int_{Q_2}^{Q_2 + q_0} \pi_{Re}^{\mathrm{DT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx$$
$$+ \int_{Q_2 + q_0}^{\gamma + n} \pi_{Re}^{\mathrm{DT2}}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx \quad (40)$$

- Main supplier's profit function with disruption

Main supplier's profit function = Profit function before the selling season

$$\pi_{S1}^{\mathrm{D}}(Q_1, Q_2, q_o) = [Q_1(w_1 - P)] \quad (41)$$

- Backup supplier's profit function with disruption

Backup supplier's profit function = Profit function before the selling season + Expected profit function during the selling season

$$\pi_{S2}^{\mathrm{N}}(Q_1, Q_2, q_o) = [Q_2(w_2) + q_o(w_o) - C_2(Q_2 + q_o)] + E[\pi_{S2}^{\mathrm{DT2}}\ (Q_1, Q_2, q_o, \mathrm{T}, q_e)] \quad (42)$$

The backup suppliers expected profit function during the selling season can be obtained as follows,

$$E[\pi_{S2}^{DT2}((Q_1, Q_2, q_o, T, q_e)] = \int_{\gamma - n}^{Q_2 - q_0} \pi_{S2}^{DT2}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx$$

$$+ \int_{Q_2 - q_0}^{Q_2} \pi_{S2}^{DT2}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx$$

$$+ \int_{Q_2}^{Q_2 + q_0} \pi_{S2}^{DT2}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx$$

$$+ \int_{Q_2 + q_0}^{\gamma + n} \pi_{S2}^{DT2}(Q_1, Q_2, q_o, T, q_e)\, f(x)dx \qquad (43)$$

The final expressions of profits for all members in the supply chain can be developed by combining the expressions for disruption and non-disruption cases by the use of probability of disruption P_d.

- Retailer's Final profit function = Retailer Expected Profit with Disruption *[Probability that disruption occurs] + Retailer Expected Profit without Disruption * [Probability that disruption does not occur]

$$\pi_{Re} = [\ \pi_{Re}^{D} * P_d\,] + [\ \pi_{Re}^{N} *(1- P_d\,)\,] \qquad (44)$$

- Main supplier's profit function = Main supplier's Expected Profit with Disruption *[Probability that disruption occurs] + Main supplier's Expected Profit without Disruption * [Probability that disruption does not occur]

$$\pi_{S1} = [\ \pi_{S1}^{D} * P_d\,] + [\ \pi_{S1}^{N} *(1- P_d\,)\,] \qquad (45)$$

- Backup supplier's profit function = Backup supplier's Expected Profit with Disruption *[Probability that disruption occurs] + Backup supplier's Expected Profit without Disruption * [Probability that disruption does not occur]

$$\pi_{S2} = [\ \pi_{S2}^{D} * P_d\,] + [\ \pi_{S2}^{N} *(1- P_d\,)\,] \qquad (46)$$

CHAPTER 4

NUMERICAL EXPERIMENTS

4.1 Numerical Experiments

In this chapter, numerical experiments are carried out using MATLAB software to demonstrate the realistic implementation and significance of proposed contracts. The experiments are being carried out to see how the proposed contracts coordinate where the main supplier is subjected to supply chain disruptions. In this research main supplier is under sales rebate contract and the backup supplier is under bidirectional option contract. Before considering in depth analysis, first a comparison of retailer's profit is conducted to prove that it is beneficial to use a backup supplier in the system. For that retailer's profit is compared under the conditions with backup supplier and without backup supplier. This chapter is conducted in three main parts which are basic numerical experiments, sensitivity analysis and coordination analysis.

As a first step for the experiments a few assumptions have been made in order to avoid certain unrealistic and meaningless final results. Those assumptions are as below,

1. $P < R_p$: To make sure that the retailer's sales price is greater than the penalty cost per unit

2. $w_{pp} < w_2 < w_{cp}$: To guarantee that both the retailer and backup supplier profit from option contract.

3. $u < w_1 - C_1$: To assure that retailer gets the advantage out of rebate contract

4. $w_1 < w_2$: To ensure that the retailer will use the main supplier as the main source to fulfil demand
5. $v < C_1 < C_2$: To make sure salvaging is the only solution for the retailer if any goods remain unsold

The following values were used when conducting the numerical experiments.

R_p = retailer's sales price	= 25$
w_1 = whole sale price offered by main supplier	= 10$
C_1 = unit production cost for main supplier	= 6$
u = channel rebate	= 3$
T = target sales level	= 700
w_2 = whole sale price offered by the backup supplier	= 12$
C_2 = unit production cost for backup supplier	= 7$
w_0 = unit option premium price	= 5$
w_{pp} = unit put exercised price	= 10$
w_{cp} = unit call exercised price	= 13$
v = unit salvage price	= 4$
s = unit shortage cost	= 3$
P = penalty cost per unit	= 10$
P_d = probability that disruption occurs	= 0.1

r = 800

n = 300

The first test compares the profits of all members with and without a backup supplier. The purpose of this experiment is to evaluate if adding a backup supplier to the system is beneficial. The table below expresses the results.

Table 4.1

Comparison of Retailer's Profit with and Without Backup Supplier

Optimal Results	Without backup supplier	With backup supplier
Q_1	1099	1684
Q_2	0	843
q_o	0	95
Retailer's profit	6522$	12967$
Normal supplier's profit	10118$	6829$
Backup supplier's profit	0$	3802$
Supply chain's profit	16640$	23598$

When comparing the results of the two situations in table 4.1, retailer's profit is higher when there is a backup supplier involved in the supply chain. Results shows that retailer's profit increases from 6522 to 12967 with backup supplier in the system. Moreover, the total supply chain profit increases from 16640 to 23598 when a backup supplier is introduced. According to the observed results, introducing a backup supplier when the main supplier is under supply chain disruptions, is beneficial.

4.2 Sensitivity Analysis

Under sensitivity analysis, a few key parameters in the study will be varied to identify how it affects the order quantities as well as the profits of all supply chain members. The impact on total supply chain profit will also be observed.

4.2.1 Sensitivity Analysis with Respect to Channel Rebate (u)

In this subsection, channel rebate (u) is varied from 4 to 3 in step sizes of 0.2. The results are shown in Table 4.2 and graphically demonstrated as line graphs in Figure 4.1 and Figure 4.2.

Table 4.2

Optimal Order Quantities and Profit Variation with Respect to Value of u

u	Q_1	Q_2	q_o	Retailer's Profit	Normal Supplier's Profit	Backup Supplier's Profit	Supply chain's Profit
3$	1684	843	95	12967$	6829$	3802$	23598$
3.2$	1645	803	94	12977$	6598$	3614$	23189$
3.4$	1611	768	93	12985$	6394$	3449$	22828$
3.6$	1580	737	93	13006$	6205$	3297$	22508$
3.8$	1553	710	92	13018$	6037$	3169$	22224$
4$	1529	685	92	13035$	5886$	3050$	21971$

Figure 4.1

Optimal Order Quantity Variation With Respect to Channel Rebate (u)

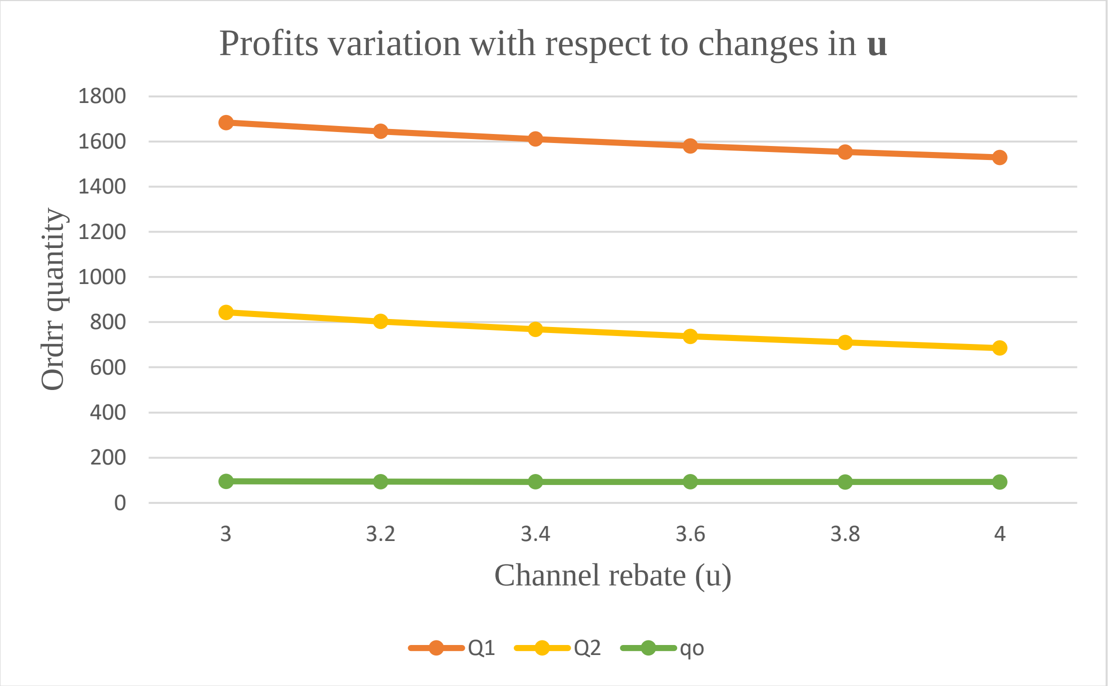
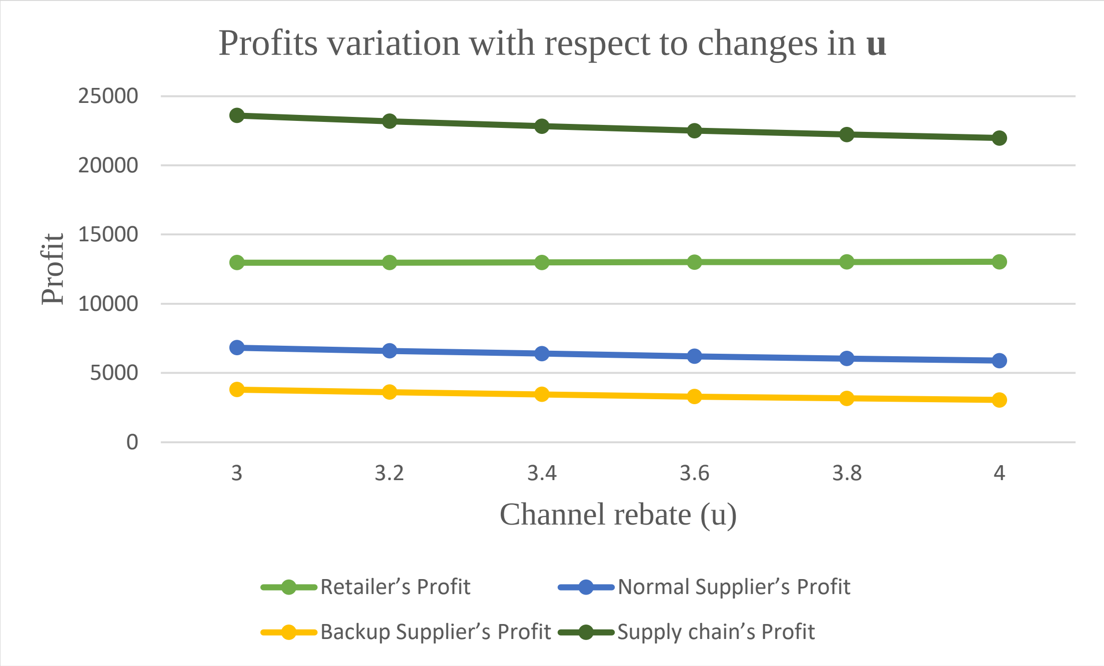


Figure 4.2

Profits Variation With Respect to Channel Rebate (u)



From the results in table 4.2 we can observe that when u increases Q_1, Q_2 and q_o decreases. Due to decreasing order quantities, main supplier's profit and backup supplier's profit decreases. Another observation is that retailer's profit tends to increase even though ordering quantities decreases. These results are reasonable because in spite of ordering quantities the channel rebate gives more benefit to the retailer in terms of profit. As an example, if we consider the difference in initial order quantities of main supplier and backup supplier, the normal suppliers order quantity is twice as the backup supplier's initial order quantity. So once the quantity reaches T (target sales level) for each unit retailer orders beyond that, he receives an extra amount (channel rebate). That benefits the retailer to increase his profit despite the fact that order quantities are decreasing. Due to decreasing profits of main supplier and backup supplier, total supply chain's profit tends to decrease.

4.2.2 Sensitivity Analysis with Respect to Target Sales Level (T)

In this subsection, target sales level (T) is varied from 700 to 1000 in step sizes of 50. The results are shown in Table 4.3 and graphically demonstrated as line graphs in Figure 4.3 and Figure 4.4.

Table 4.3

Optimal Order Quantities and Profit Variation with Respect to Target Sales Level (T)

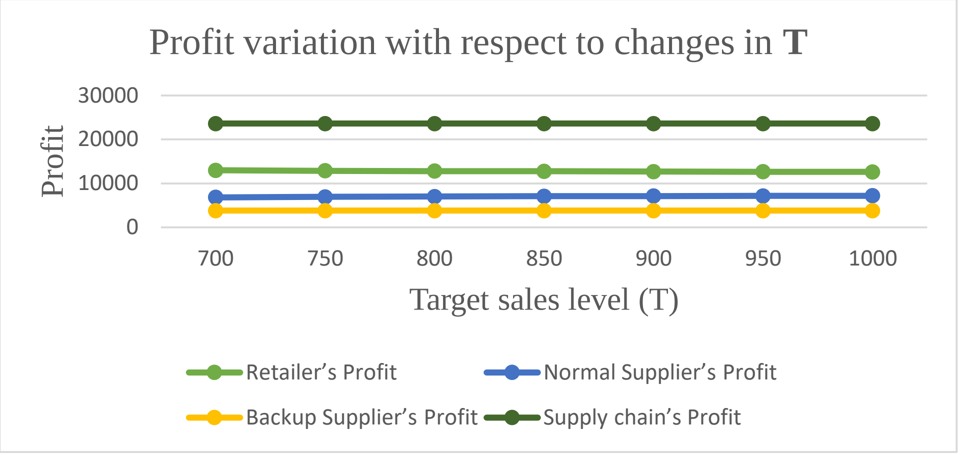
T	Q_1	Q_2	q_o	Retailer's Profit	Normal Supplier's Profit	Backup Supplier's Profit	Supply chain's Profit
700	1684	843	95	12967$	6829$	3802$	23598$
750	1684	843	95	12878$	6918$	3802$	23598$
800	1684	843	95	12801$	6995$	3802$	23598$
850	1684	843	95	12735$	7061$	3802$	23598$
900	1684	843	95	12684$	7112$	3802$	23598$
950	1684	843	95	12640$	7156$	3802$	23598$
1000	1684	843	95	12611$	7185$	3802$	23598$

Figure 4.3

Optimal Order Quantity Variation with Respect to Target Sales Level (T)

Figure 4.4

Profit Variation with Respect to Target Sales Level (T)



From the results in table 4.3 we can observe that Q_1, Q_2, q_o does not vary with the change in target sales level. So, it can be confirmed that changes in target sales level does not affect optimal ordering quantities. However, in spite of ordering quantities, retailer's profit decreases when target sales level increases. This is reasonable because the channel rebate amount that retailer received reduces as target sales level increases. Another observation is that the main supplier's profit increases when target sales level increases. This is also expected because the channel rebate that retailer receives from main supplier reduces as T increases. However, backup supplier's profit and total supply chain's profit remain unchanged at this situation.

4.2.3 Sensitivity Analysis with Respect to Shortage Cost (s)

In this subsection, shortage cost (s) is varied from 3 to 4 in step sizes of 0.2. The results are shown in Table 4.4 and graphically demonstrated as line graphs in Figure 4.5 and Figure 4.6

Table 4.4

Optimal Order Quantities and Profit Variation with Respect to Shortage Cost (s)

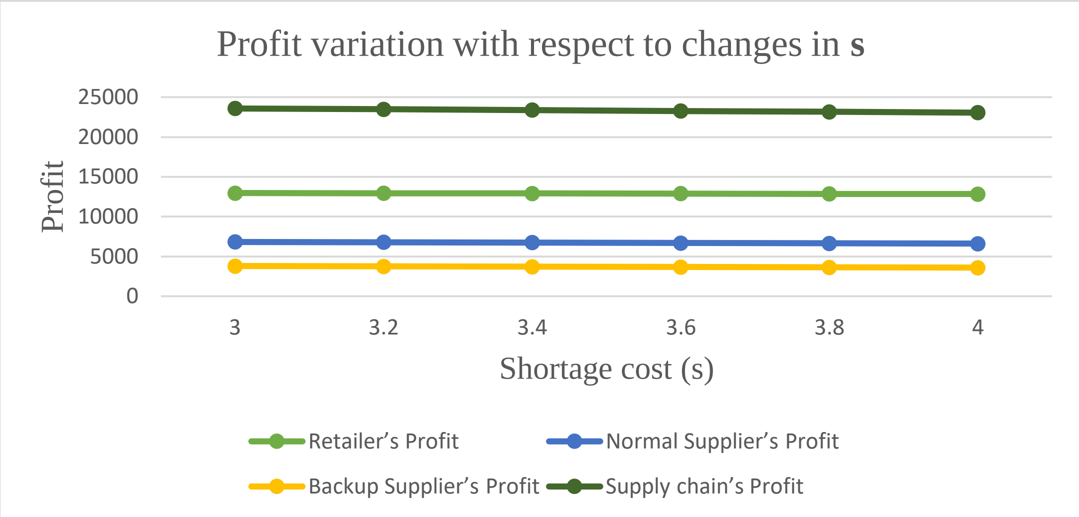
s	Q_1	Q_2	q_o	Retailer's Profit	Normal Supplier's Profit	Backup Supplier's Profit	Supply chain's Profit
3$	1684	843	95	12967$	6829$	3802$	23598$
3.2$	1677	834	96	12946$	6783$	3759$	23488$
3.4$	1671	825	96	12913$	6744$	3722$	23379$
3.6$	1664	817	97	12895$	6699$	3681$	23275$
3.8$	1657	808	97	12866$	6653$	3642$	23161$
4$	1650	800	98	12849$	6608$	3601$	23058$

Figure 4.5

Optimal Order Quantity Variation with Respect to Shortage Cost (s)

Figure 4.6

Profit Variation with Respect to Shortage Cost (s)



From the results in table 4.4 we can see that with the increment of shortage cost, normal supplier's order quantity decreases. As a result, normal supplier's profit decreases. Option quantity increases while backup suppliers initial order quantity decreases. This trend is expected because retailer tries to meet the demand requirements by increasing the option quantity. Also the increase in shortage cost has an impact on retailer's profit. Retailer's profit decreases because of retailer's inability to satisfy the demand even though q_o increases. As a result, total supply chain's profit tends to decrease.

4.2.4 Sensitivity Analysis with Respect to Option Premium Price (w_o)

In this subsection, option premium price (w_o)is varied from 1 to 5 in step sizes of 1. The results are shown in Table 4.5 and graphically demonstrated as line graphs in Figure 4.7 and Figure 4.8

Table 4.5

Optimal Order Quantities and Profit Variation with Respect to Option Premium Price (w_o)

w_o	Q_1	Q_2	q_o	Retailer's Profit	Normal Supplier's Profit	Backup Supplier's Profit	Supply Chain's Profit
1$	1702	887	159	14117$	6948$	4299$	25364$
2$	1698	876	143	13794$	6922$	4135$	24851$
3$	1693	865	127	13499$	6889$	3995$	24383$
4$	1689	854	111	13220$	6862$	3886$	23968$
5$	1684	843	95	12967$	6829$	3802$	23598$

Figure 4.7

Optimal Order Quantity Variation with Respect to Option Premium Price (w_o)

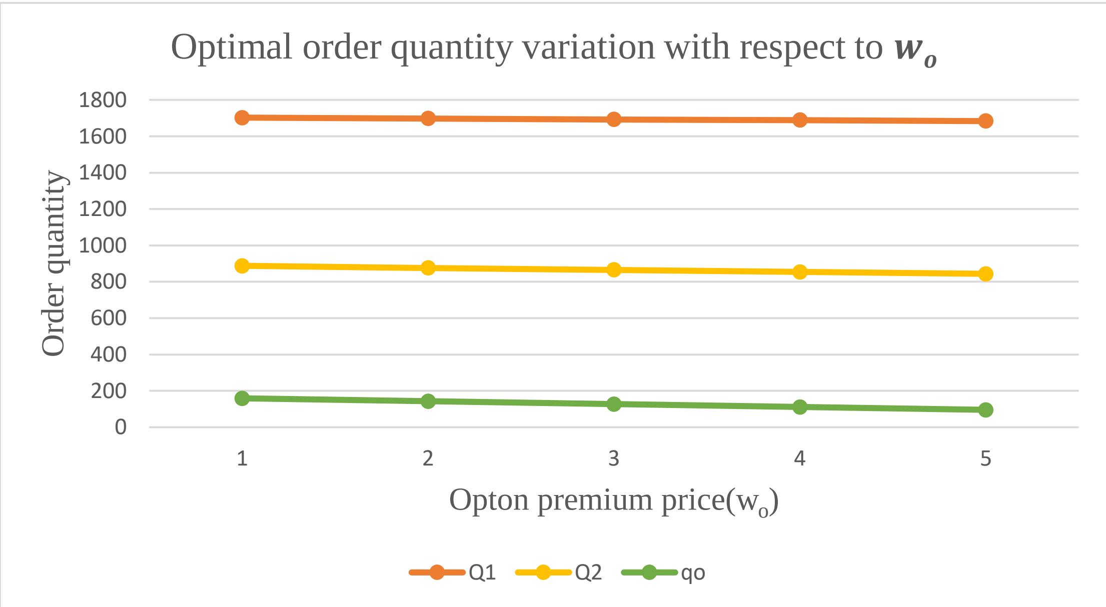
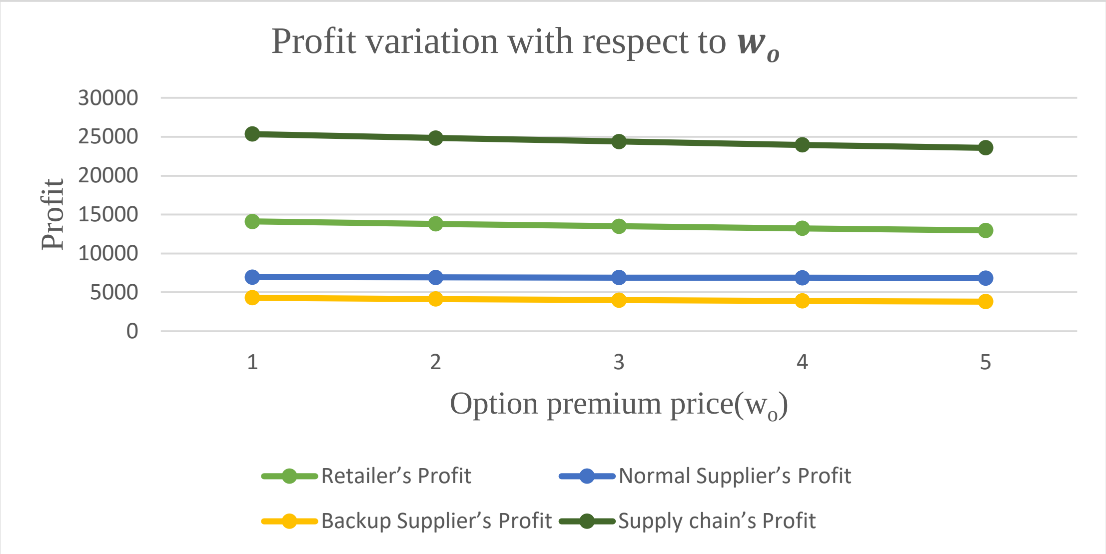


Figure 4.8

Profit Variation with Respect to Option Premium Price (w_o)



According to the results in table 4.5 when option premium price (w_o) increases, Q_1, Q_2, q_o decreases. Compared to the decrease in initial order quantity of backup supplier(Q_2), decrease in the initial order quantity of main supplier (Q_1) is less. So that we can say even though retailer orders less from both suppliers, he favors ordering from main supplier. These trends are reasonable because when w_o increases, it makes the goods from main supplier more attractive than backup supplier. Due to the decrease in order quantities, each member's profit decreases. Also the overall supply chains profit decreases as w_o increases.

4.2.5 Sensitivity Analysis with Respect to Put Exercised Price (w_{pp})

In this subsection, put exercised price (w_{pp}) is varied from 9 to 10 in step sizes of 0.2. The results are shown in Table 4.6 and graphically demonstrated as line graphs in Figure 4.9 and Figure 4.10

Table 4.6

Optimal Order Quantities and Profit Variation with Respect to Put Exercised Price (w_{pp})

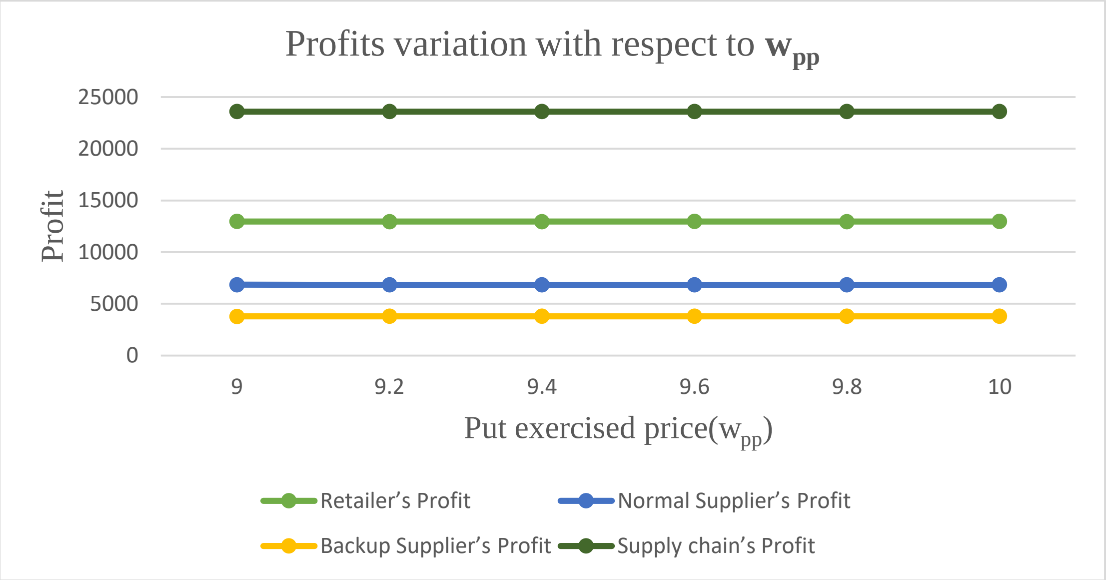
W_{pp}	Q_1	Q_2	q_o	Retailer's Profit	Normal Supplier's Profit	Backup Supplier's Profit	Supply chain's Profit
9$	1687	851	97	12961$	6849$	3789$	23599$
9.2$	1686	849	96	12958$	6842$	3792$	23592$
9.4$	1686	847	96	12959$	6842$	3793$	23594$
9.6$	1685	846	96	12967$	6836$	3795$	23598$
9.8$	1685	844	95	12959$	6836$	3801$	23596$
10$	1684	843	95	12967$	6829$	3802$	23598$

Figure 4.9

Optimal Order Quantity Variation with Respect to Put Exercised Price (w_{pp})

Figure 4.10

Profit Variation with Respect to Put Exercised Price (w_{pp})



From the results in table 4.6, when put exercised price increases, main supplier's initial order quantity and backup supplier's initial order quantity decreases. However, backup supplier's profit increases as w_{pp} increases. This is reasonable because retailer increases the option quantity as put exercised price increases and as well as total cost to exercising option contract is high. Unusual ups and downs in retailer's profit and total supply chain's profit is observed in the results. Those changes can be expected due to the behavior of initial order quantity of backup supplier. Due to reduced order quantity from main supplier his profit decreases.

4.2.6 Sensitivity Analysis with Respect to Call Exercised Price (w_{cp})

In this subsection, call exercised price (w_{cp}) is varied from 18 to 13 in step sizes of 1. The results are shown in Table 4.7 and graphically demonstrated as line graphs in Figure 4.11 and Figure 4.12

Table 4.7

Optimal Order Quantities and Profit Variation with Respect to Call Exercised Price (w_{cp})

W_{cp}	Q_1	Q_2	q_o	Retailer's Profit	Normal Supplier's Profit	Backup Supplier's Profit	Supply chain's Profit
13$	1684	843	95	12967$	6829$	3802$	23598$
14$	1685	839	92	12971$	6836$	3761$	23568$
15$	1686	835	89	12971$	6842$	3724$	23537$
16$	1686	831	87	12981$	6842$	3681$	23504$
17$	1687	828	85	12987$	6849$	3646$	23482$
18$	1687	825	82	12998$	6849$	3617$	23464$

Figure 4.11

Optimal Order Quantity Variation with Respect to Call Exercised Price (w_{cp})

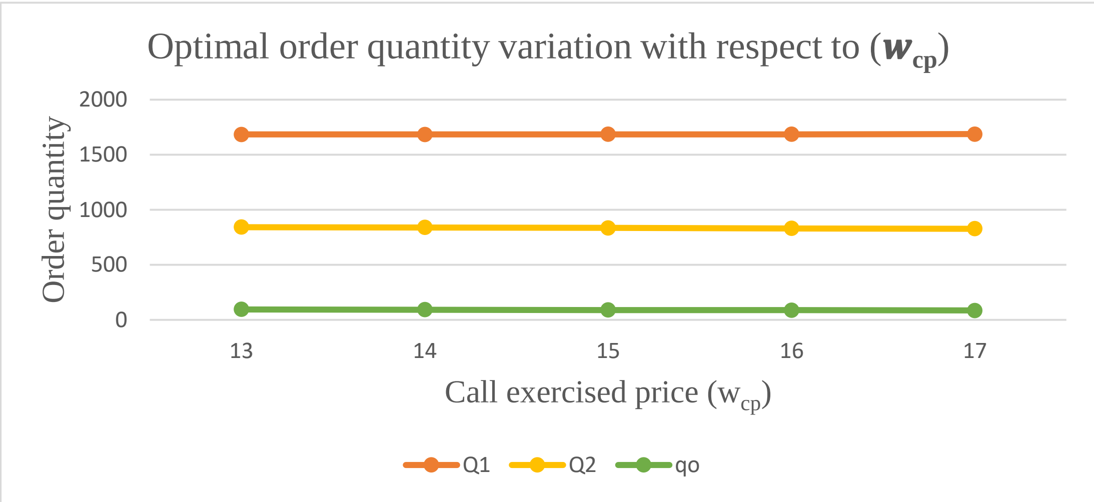
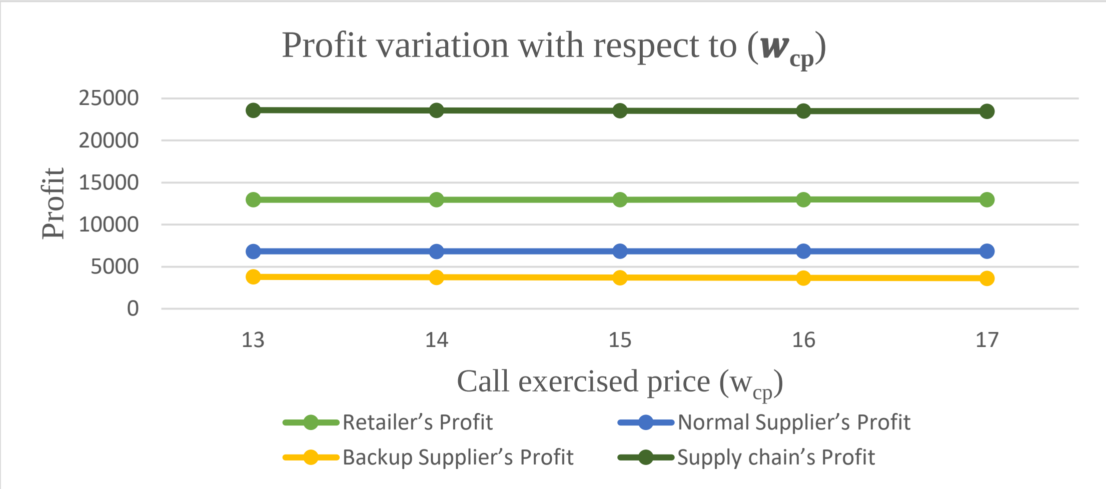


Figure 4.12

Profit Variation with Respect to Call Exercised Price (w_{cp})



According to the results in table 4.7, as call exercised price increases in the system, main supplier's initial ordering quantity increase while backup supplier's initial ordering quantity and option quantity decreases. This trend is reasonable because when w_{cp} increases, exercising call option becomes more expensive to the retailer. An increased w_{cp} and higher ordering quantity becomes more expensive for the retailer, the retailer favors a lower w_{cp} from the backup supplier and high quantity from main supplier to satisfy the demand. Due to the lower quantities from the backup supplier, his profit decreases. As w_{cp} increases main supplier's initial order quantity increases and as a result main supplier's and retailer's profit increases. However, despite the fact that retailer's and main supplier's profits increase, the total supply chain's profit tends to decrease.

4.2.7 Sensitivity Analysis with Respect to Probability That Disruption Occurs (P_d) in Decentralized System

In this subsection, probability that disruption occurs (P_d) in decentralized system is varied from 0.05 to 0.1 in step sizes of 0.01. The results are shown in Table 4.8 and graphically demonstrated as line graphs in Figure 4.13 and Figure 4.14

Table 4.8

Optimal Order Quantities and Profit Variation with Respect to Probability That Disruption Occurs (P_d) in Decentralized System

P_d	Q_1	Q_2	q_o	Retailer's Profit	Normal Supplier's Profit	Backup Supplier's Profit	Supply chain's Profit
0.05	1958	1167	138	12955$	8345$	4693$	25993$
0.06	1893	1090	128	13025$	7957$	4507$	25489$
0.07	1834	1020	119	13053$	7620$	4324$	24997$
0.08	1780	956	110	13044$	7325$	4149$	24518$
0.09	1730	897	102	13013$	7062$	3973$	24048$
0.1	1684	843	95	12967$	6829$	3802$	23598$

Figure 4.13

Optimal Order Quantity Variation with Respect to Probability That Disruption Occurs (P_d) in Decentralized System

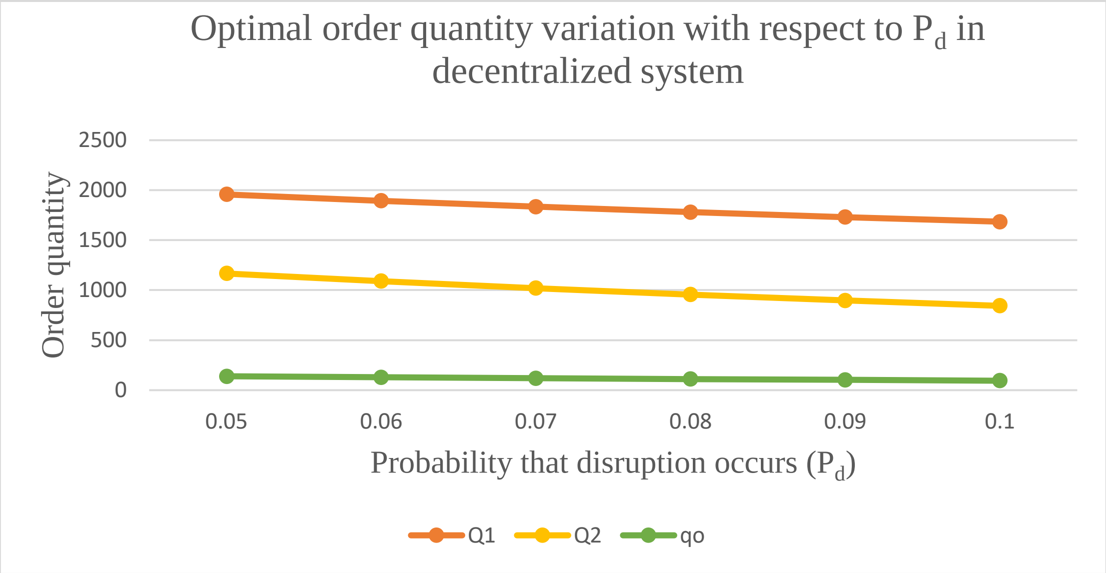
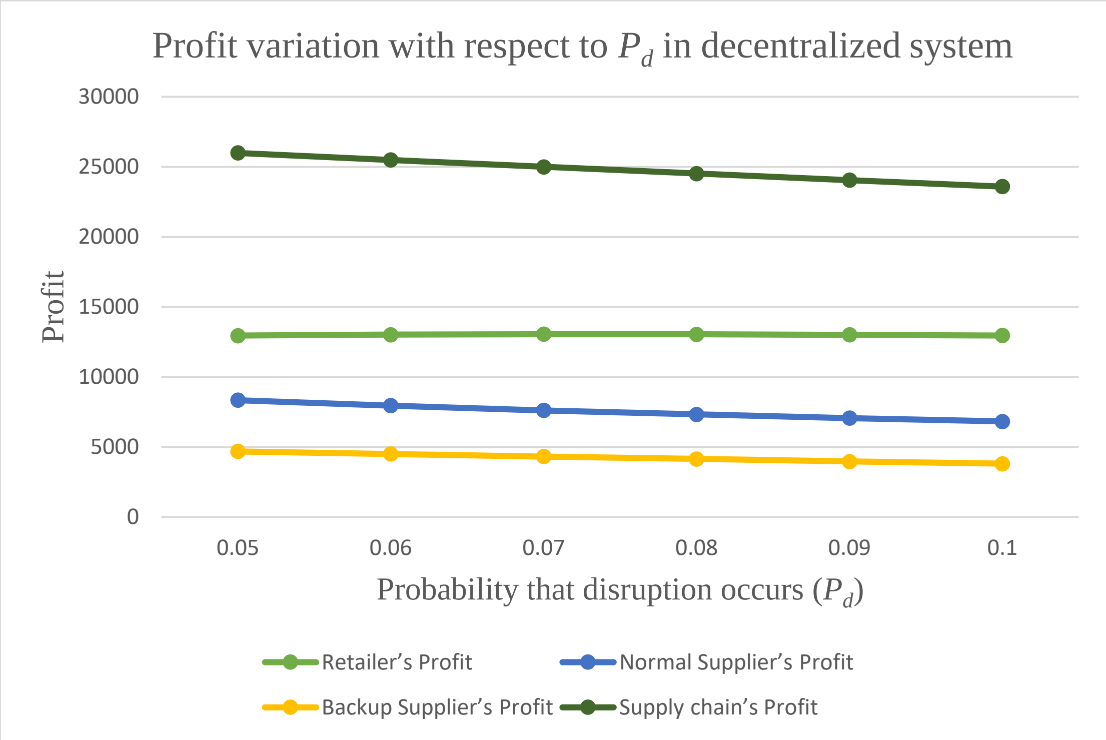


Figure 4.14

Profit Variation with Respect to Probability That Disruption Occurs (P_d) in Decentralized System



4.2.8 Sensitivity Analysis with Respect to Probability That Disruption Occurs (P_d) in Centralized System

In this subsection, probability that disruption occurs (***P_d***) in centralized system is varied from 0.05 to 0.1 in step sizes of 0.01. The results are shown in Table 4.9 and graphically demonstrated as line graphs in Figure 4.15 and Figure 4.16

Table 4.9

Optimal Order Quantities and Profit Variation with Respect to Probability That Disruption Occurs (P_d) in Centralized System

P_d	Q_1	Q_2	q_o	Retailer's Profit	Normal Supplier's Profit	Backup Supplier's Profit	Supply chain's Profit
0.05	75	531	245	8062$	2586$	5374$	16022$
0.06	72	532	246	8035$	2564$	5387$	15986$
0.07	68	534	246	7992$	2542$	5398$	15932$

0.08	65	535	247	7965$	2519$	5399$	15883$
0.09	62	537	247	7925$	2499$	5415$	15839$
0.1	59	538	247	7875$	2476$	5416$	15767$

Figure 4.15

Optimal Order Quantity Variation with Respect to Probability That Disruption Occurs (P_d) in Centralized System

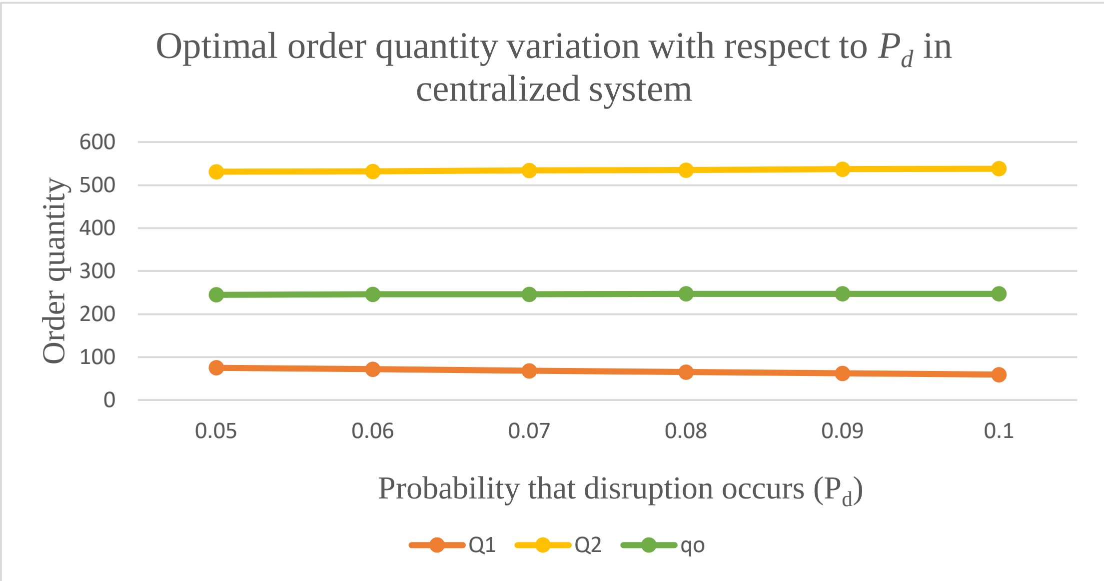
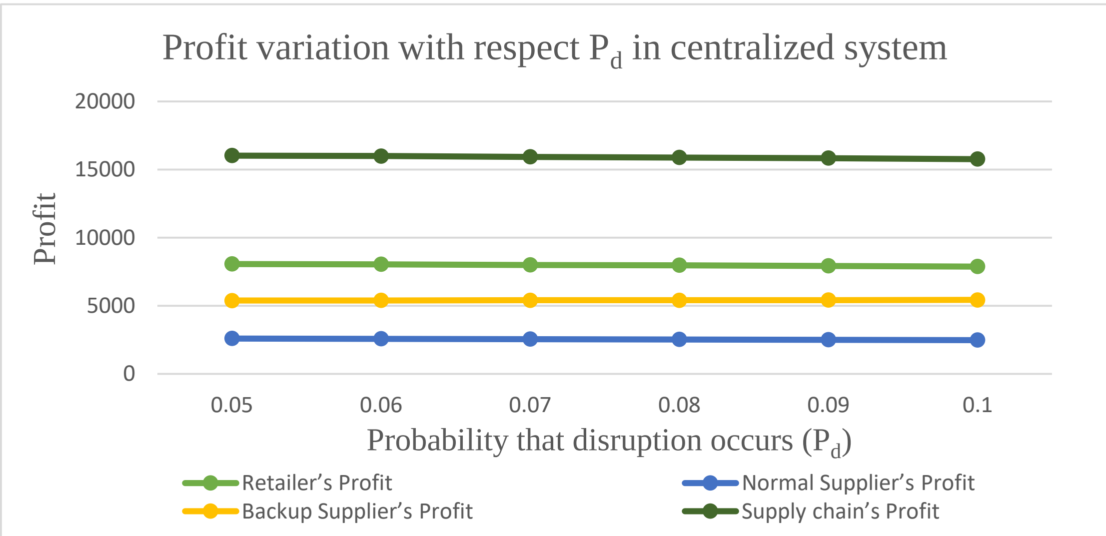


Figure 4.16

Profit Variation with Respect to Probability That Disruption Occurs (P_d) in Centralized System



According to tables 4.8 and 4.9, when probability of disruption increases, main supplier's initial order quantity reduces in both centralized and decentralized situations. The results show that in the centralized system, an increasing trend in backup supplier's initial order quantity is observed when P_d increases. That is reasonable because in order to satisfy the demand and lower the risk of disruption, retailer orders more from backup supplier. Also in both situations total supply chain's profit decreases as probability of disruption increases.

4.2.9 Supply Chain Coordination Analysis with Changes in Probability That Disruption Occurs (P_d)

Table 4.10

Comparison of Supply Chain Profit Changes as (P_d) Changes

P_d	Supply chain's Profit (Centralized system)	Supply chain's Profit (Decentralized system)
0.05	16022$	25993$
0.06	15986$	25489$
0.07	15932$	24997$
0.08	15883$	24518$
0.09	15839$	24048$
0.1	15767$	23598$

From table 4.10 we can see that for all values of disruption probability, the decentralized system has a higher supply chain profit than centralized system. This confirms that proposed sales rebate contract and bi-directional option contract reaches a high level of coordination in the supply chain.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

This research study dealt with the case of one retailer who seeks the supply from two different suppliers (i.e., a normal supplier and a backup supplier). The main supplier is subjected to supply chain disruption and the backup supplier is used to help fulfill the demand requirements. The main supplier is under target sales rebate contract and the backup supplier is under bi-directional option contract. The reason to choose these two contracts is from their own characteristics and abilities to coordinate the supply chain in regular non-disruptive environment, as well as their ability to supply to the retailer when demand is realized.

A mathematical model was built to illustrate the system and MATLAB is utilized to obtain optimal ordering quantities. At the beginning of the analysis, it has been proven that the use of a backup supplier can help to enhance profits of the retailer and the whole supply chain when the main supplier is subjected to supply chain disruption in comparison to the case without a backup supplier. Moreover, coordinating ability of proposed contracts have been analyzed by comparing the total profits of the supply chain under decentralized and centralized systems.

From sensitivity analysis, the effect of each parameter was analyzed by varying one at a time. Channel rebate, target sales level, shortage cost, option premium price, exercises put price, exercised call price and probability of disruption occurrence were the parameters varied to identify the impact on initial ordering quantities Q1, Q_2 and option quantity q_o. In the coordination analysis, it has been illustrated that the system is well coordinated under the proposed supply chain contracts.

Further research on this area can be explored when the suppliers are under different supply chain contracts and subjected to supply chain disruptions. Another future research recommendation would be using the same sales rebate contract for main supplier but with return policy. Supply chain disruption occurrences can also be considered for the backup supplier and investigate would the contract combinations be able to handle such a situation.

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APPENDIX

MATLAB CODE FOR CONDUCTING THE NUMERICAL EXPERIMENTS

```
function [] = test_015()
syms Q_1 Q_2 q_o x positive
assume (Q_2 > q_o)
global R_p w_1 C_1 u T P w_2 C_2 w_o w_pp w_cp v s P_d r n
R_p = 25; w_1 = 10; C_1 = 6; u = 3;
T = 700; P = 10; w_2 = 12; C_2 = 7;
w_o = 5; w_pp = 10; w_cp = 13;
v = 4; s = 3; P_d = 0.1;
r = 800; n = 300; fd = 1/(2*n);


Retailer_Non_c1 = (x*R_p)+(w_pp*q_o)+(v*(Q_2-q_o+Q_1-x));
Int_Retailer_Non_c1 = int((Retailer_Non_c1*fd),x,(r-n),T);
Normal_Non_c1 = 0;
Int_Normal_Non_c1 = int((Normal_Non_c1*fd),x,(r-n),T);
Backup_Non_c1 = (2*q_o*v)-(q_o*w_pp);
Int_Bakcup_Non_c1 = int((Backup_Non_c1*fd),x,(r-n),T);
Retailer_Non_c2 = (x*R_p)+ (x-T)*u +(w_pp*q_o)+ (v*(Q_2+Q_1-q_o-x));
Int_Retailer_Non_c2 = int((Retailer_Non_c2*fd),x,T,Q_1);
Normal_Non_c2 = -(x-T)*u ;
Int_Normal_Non_c2 = int((Normal_Non_c2*fd),x,T,Q_1);
Backup_Non_c2 = (2*q_o*v)-(q_o*w_pp);
Int_Backup_Non_c2 = int((Backup_Non_c2*fd),x,T,Q_1);
Retailer_Non_c3 = (x*R_p) + (Q_1-T)*u +(w_pp*q_o)+(v*(Q_2+Q_1-q_o-
x));
Int_Retailer_Non_c3 = int((Retailer_Non_c3*fd),x,Q_1,(Q_2+Q_1-q_o));
Normal_Non_c3 = -(Q_1-T)*u ;
Int_Normal_Non_c3 = int((Normal_Non_c3*fd),x,Q_1,(Q_2+Q_1-q_o));
Backup_Non_c3 = -(q_o*w_pp) +(2*v*q_o);
Int_Backup_Non_c3 = int((Backup_Non_c3*fd),x,Q_1,(Q_2+Q_1-q_o));
Retailer_Non_c4 = (x*R_p ) + (Q_1-T)*u -(w_pp*(Q_2+Q_1-x));
Int_Retailer_Non_c4 = int((Retailer_Non_c4*fd),x,(Q_2+Q_1-
q_o),(Q_2+Q_1));
Normal_Non_c4 = -(Q_1-T)*u;
Int_Normal_Non_c4 = int((Normal_Non_c4*fd),x,(Q_2+Q_1-
q_o),(Q_2+Q_1));
Backup_Non_c4 = -(w_pp*(Q_2+Q_1-x))+(v*(q_o+Q_2+Q_1-x));
```

```
Int_Backup_Non_c4 = int((Backup_Non_c4*fd),x,(Q_2+Q_1-
q_o),(Q_2+Q_1));
Retailer_Non_c5 = (x*R_p ) + (Q_1-T)*u -(w_cp*(x-Q_2-Q_1));
Int_Retailer_Non_c5 =
int((Retailer_Non_c5*fd),x,(Q_2+Q_1),(Q_2+Q_1+q_o));
Normal_Non_c5 = -(Q_1-T)*u;
Int_Normal_Non_c5 =
int((Normal_Non_c5*fd),x,(Q_2+Q_1),(Q_2+Q_1+q_o));
Backup_Non_c5 = -(w_cp*(x-Q_2-Q_1))+(v*(q_o-(x-(Q_2+Q_1))));
Int_Backup_Non_c5 =
int((Backup_Non_c5*fd),x,(Q_2+Q_1),(Q_2+Q_1+q_o));
Retailer_Non_c6 = (R_p*(Q_2+q_o+Q_1))+ (Q_1-T)*u -(q_o*w_cp)-(s*(x-
(Q_2+q_o+Q_1)));
Int_Retailer_Non_c6 =
int((Retailer_Non_c6*fd),x,(Q_2+Q_1+q_o),(r+n));
Normal_Non_c6 = -(Q_1-T)*u;
Int_Normal_Non_c6 = int((Normal_Non_c6*fd),x,(Q_2+Q_1+q_o),(r+n));
Backup_Non_c6 = w_cp*(q_o);
Int_Backup_Non_c6 = int((Backup_Non_c6*fd),x,(Q_2+Q_1+q_o),(r+n));
Exp_Retailet_Non_T2 =
Int_Retailer_Non_c1+Int_Retailer_Non_c2+Int_Retailer_Non_c3+Int_Retai
ler_Non_c4+Int_Retailer_Non_c5+Int_Retailer_Non_c6;
Retailer_Non = (-(Q_1*w_1)-(Q_2*w_2)-(q_o*w_o))+Exp_Retailet_Non_T2;
Exp_Normal_Non_T2 =
Int_Normal_Non_c1+Int_Normal_Non_c2+Int_Normal_Non_c3+Int_Normal_Non_
c4+Int_Normal_Non_c5+Int_Normal_Non_c6;
Normal_Non = (Q_1*(w_1-C_1))+(Exp_Normal_Non_T2);
Exp_Backup_Non_T2 =
Int_Bakcup_Non_c1+Int_Backup_Non_c2+Int_Backup_Non_c3+Int_Backup_Non_
c4+Int_Backup_Non_c5+Int_Backup_Non_c6;
Backup_Non = (Q_2*w_2)+(q_o*w_o)-(C_2*(Q_2+q_o))+Exp_Backup_Non_T2;

Retailer_Dis_c1 = (x*R_p)+(q_o*w_pp)+(v*(Q_2-q_o)-x);
Int_Retailer_Dis_c1 = int((Retailer_Dis_c1*fd),x,(r-n),(Q_2-q_o));
Backup_Dis_c1 = -(q_o*w_pp) + (2*q_o*v);
Int_Backup_Dis_c1 = int((Backup_Dis_c1*fd),x,(r-n),(Q_2-q_o));
Retailer_Dis_c2 = (x*R_p)+(w_pp*(Q_2-x));
Int_Retailer_Dis_c2 = int((Retailer_Dis_c2*fd),x,(Q_2-q_o),(Q_2));
Backup_Dis_c2 = -(w_pp*(Q_2-x))+(v*(q_o+(Q_2-x)));
Int_Backup_Dis_c2 = int((Backup_Dis_c2*fd),x,(Q_2-q_o),(Q_2));
Rretailer_Dis_c3 = (x*R_p)-(w_cp*(x-Q_2));
```

```
Int_Retailer_Dis_c3 = int((Rretailer_Dis_c3fd),x,(Q_2),(Q_2+q_o));
Backup_Dis_c3 = (w_cp*(x-Q_2))+(v*(q_o-(x-Q_2)));
Int_Backup_Dis_c3 = int((Backup_Dis_c3*fd),x,(Q_2),(Q_2+q_o));
Retailer_Dis_c4 = (x*R_p)-(q_o*w_cp)-(s*(x-(Q_2+q_o)));
Int_Retailer_Dis_c4 = int((Retailer_Dis_c4*fd),x,(Q_2+q_o),(r+n));
Backup_Dis_c4 = (q_o*w_cp);
Int_Backup_Dis_c4 = int((Backup_Dis_c4*fd),x,(Q_2+q_o),(r+n));
Exp_Retailer_Dis_T2 =
Int_Retailer_Dis_c1+Int_Retailer_Dis_c2+Int_Retailer_Dis_c3+Int_Retai
ler_Dis_c4;
Retailer_Dis = (-(Q_1*w_1)-(Q_2*w_2)-
(q_o*w_o))+(P*Q_1)+(Exp_Retailer_Dis_T2);
Normal_Dis = (Q_1*(w_1-P));
Exp_Backup_Dis_T2 = Int_Backup_Dis_c1 + Int_Backup_Dis_c2 +
Int_Backup_Dis_c3 + Int_Backup_Dis_c4;
Backup_Dis = (((Q_2*w_2)+(q_o*w_o))-
(C_2*(Q_2+q_o)))+Exp_Backup_Dis_T2;
R = (Retailer_Non*(1-P_d))+ (Retailer_Dis*P_d);
N = (Normal_Non*(1-P_d))+ (Normal_Dis*P_d);
B = (Backup_Non*(1-P_d))+ (Backup_Dis*P_d);

Diff1 = diff(R, 'Q_1');
Diff2 = diff(R, 'Q_2');
Diff3 = diff(R, 'q_o');
S = vpasolve([Diff1==0, Diff2==0, Diff3==0], [Q_1, Q_2, q_o]);
Q1 = round(S.Q_1);
Q2 = round(S.Q_2);
qo = round(S.q_o);

R_N1 = (x*R_p)+(w_pp*qo)+(v*(Q2-qo+Q1-x));
Int_R_N1 = int((R_N1*fd),x,(r-n),T);
N_N1 = 0;
Int_N_N1 = int((N_N1*fd),x,(r-n),T);
B_N1 = (2*qo*v)-(qo*w_pp);
Int_B_N1 = int((B_N1*fd),x,(r-n),T);
R_N2 = (x*R_p)+((x-T)*u)+(w_pp*(qo))+(v*(Q1+Q2-qo-x));
Int_R_N2 = int((R_N2*fd),x,T,Q1);
N_N2 = -(x-T)*u;
Int_N_N2 = int((N_N2*fd),x,T,Q1);
B_N2 = (2*qo*v)-(qo*w_pp);
Int_B_N2 = int((B_N2*fd),x,T,Q1);
```

```
R_N3 = (x*R_p)+((Q1-T)*u)+(w_pp*(qo))+(v*(Q1+Q2-qo-x));
Int_R_N3 = int((R_N3*fd),x,Q1,(Q2+Q1-qo));
N_N3 =  -((Q1-T)*u);
Int_N_Pf_N3 = int((N_N3*fd),x,Q1,(Q2+Q1-qo));
B_N3 = (2*qo*v)-(qo*w_pp);
Int_B_N3 = int((B_N3*fd),x,Q1,(Q2+Q1-qo));
R_N4 = (x*R_p)+((Q1-T)*u)+(w_pp*(Q1+Q2-x));
Int_R_N4 = int((R_N4*fd),x,(Q2+Q1-qo),(Q2+Q1));
N_N4 =-((Q1-T)*u);
Int_N_N4 = int((N_N4*fd),x,(Q2+Q1-qo),(Q2+Q1));
B_N4 = -(w_pp*(Q1+Q2-x))+(v*(Q1+Q2+qo-x));
Int_B_N4 = int((B_N4*fd),x,(Q2+Q1-qo),(Q2+Q1));
R_N5 = (x*R_p)+((Q1-T)*u)+(w_cp*(x-Q1-Q2));
Int_R_N5 = int((R_N5*fd),x,(Q2+Q1),(Q2+qo+Q1));
N_N5 = -((Q1-T)*u);
Int_N_N5 = int((N_N5*fd),x,(Q2+Q1),(Q2+qo+Q1));
B_N5 = w_cp*(x-Q1-Q2)+(v*(qo-(x-(Q1+Q2))));
Int_B_N5 = int((B_N5*fd),x,(Q2+Q1),(Q2+qo+Q1));
R_N6 = ((Q1+Q2+qo)*R_p)+((Q1-T)*u)+(w_cp*(qo))-s*(x-(Q1+Q2+qo));
Int_R_N6 = int((R_N6*fd),x,(Q2+Q1+qo),(r+n));
N_N6 = -((Q1-T)*u);
Int_N_N6 = int((N_N6*fd),x,(Q2+Q1+qo),(r+n));
B_N6 = w_cp*(qo);
Int_B_N6 = int((B_N6*fd),x,(Q2+Q1+qo),(r+n));

Exp_R_NT2 = Int_R_N1+Int_R_N2+Int_R_N3+Int_R_N4+Int_R_N5+Int_R_N6;
R_N = (-(Q1*w_1)-(Q2*w_2)-(qo*w_o))+Exp_R_NT2;
Exp_N_NT2 = Int_N_N1+ Int_N_N2+Int_N_Pf_N3+Int_N_N4+Int_N_N5+Int_N_N6
;
N_N = (Q1*(w_1-C_1))+Exp_N_NT2;
Exp_B_NT2 = Int_B_N1+Int_B_N2+Int_B_N3+Int_B_N4+Int_B_N5+Int_B_N6;
B_N = (((Q2*w_2)+(qo*w_o))-(C_2*(Q2+qo)))+Exp_B_NT2;

R_D1 = (x*R_p)+(qo*w_pp)-(v*((Q2-qo)-x));
Int_R_D1 = int((R_D1*fd),x,(r-n),(Q2-qo));
B_D1 = -(qo*w_pp)+(2*qo*v);
Int_B_D1 = int((B_D1*fd),x,(r-n),(Q2-qo));
R_D2 = (x*R_p)+(w_pp*(Q2-x));
Int_R_D2 = int((R_D2*fd),x,(Q2-qo),(Q2));
B_D2 = -(w_pp*(Q2-x))+(v*(qo+(Q2-x)));
Int_B_D2 = int((B_D2*fd),x,(Q2-qo),(Q2));
```

```
R_D3 = (x*R_p)-(w_cp*(x-Q2));
Int_R_D3 = int((R_D3*fd),x,(Q2),(Q2+qo));
B_D3 = (w_cp*(x-Q2))+(v*(qo-(x-Q2)));
Int_B_D3 = int((B_D3*fd),x,(Q2),(Q2+qo));
R_D4 = (R_p*x)+(qo*w_cp)-(s*(x-(Q2+qo)));
Int_R_D4 = int((R_D4*fd),x,(Q2+qo),(r+n));
B_D4 = (qo*w_cp);
Int_B_D4 = int((B_D4*fd),x,(Q2+qo),(r+n));
Exp_R_DT2 = Int_R_D1+Int_R_D2+Int_R_D3+Int_R_D4;
R_D = (-(Q1*w_1)-(Q2*w_2)-(qo*w_o))+Q1*P +Exp_R_DT2;
N_D = (Q1*(w_1-P));
Exp_B_DT2 = Int_B_D1 + Int_B_D2 + Int_B_D3 + Int_B_D4;
B_D = (((Q2*w_2)+(qo*w_o))-(C_2*(Q2+qo)))+Exp_B_DT2;
Q1 = abs(Q1)
Q2 = abs(Q2)
qo = abs(qo)
R_fin = (R_N*(1-P_d)) + (R_D*P_d);
R_final = abs( round (R_fin))
N_fin = (N_N*(1-P_d)) + (N_D*P_d);
N_final = abs(round (N_fin))
B_fin = (B_N*(1-P_d)) + (B_D*P_d);
B_final = abs(round (B_fin))


end
```