An Economic Order Quantity Model with Imperfect Quality

by

Chiewchan Nimitmongkol

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Engineering in Industrial and Manufacturing Engineering

Examination Committee: Dr. Huynh Trung Luong (Chairperson)
Dr. Pisut Koomsap
Prof. Manukid Parnichkun

Nationality: Thai
Previous Degree: Bachelor of Engineering in Industrial Engineering Chulalongkorn University Thailand

Scholarship Donor: Siam Kubota Corporation Co., Ltd

Asian Institute of Technology
School of Engineering and Technology
Thailand
May 2019
ACKNOWLEDGEMENTS

First of all, I would like to thank my thesis advisor Dr. Huynh Trung Luong of ISE faculty at AIT who always sacrificed his time and gave many suggestions and guidance with a clear explanation whenever I got a trouble about my study. Furthermore, I would like to thank my thesis committee members, Dr. Pisut Koomsap and Prof. Manukid Parnichkun for their comments about my thesis.

Secondly, my thanks go to ISE alumni, ISE staffs who gave me rightful direction and suggestions throughout my study in AIT.

Thirdly, I would like to thank Siam Kubota Corporation Co., Ltd. who is my scholarship donor and gives a great opportunity for me to study master degree at AIT.

Lastly, I would like to dedicate this thesis to my family, my wife and thanks for always supporting me throughout my entire life. All my accomplishment cannot be achieved without them.
ABSTRACT

In this research, we study the optimal length of a cycle from the beginning of the production until products are delivered to the customers to find the maximum profit of the production by using economic order quantity model. We consider an imperfect manufacturing process which possibly produces defective products. We analyze two possibilities for dealing with defective products by reworking or sold at discount to gain the maximum profit as possible. Moreover, a case of a safety stock is added to the inventory in case of a fluctuation demand will be another option to be considered.

Keywords: Economic order quantity model, Inventory level, Safety stock, Imperfect quantity
## TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TITLE PAGE</td>
<td>i</td>
</tr>
<tr>
<td></td>
<td>ACKNOWLEDGEMENTS</td>
<td>ii</td>
</tr>
<tr>
<td></td>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td></td>
<td>TABLE OF CONTENTS</td>
<td>iv</td>
</tr>
<tr>
<td></td>
<td>LIST OF FIGURES</td>
<td>v</td>
</tr>
<tr>
<td>1</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1</td>
<td>Overview</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>Problem statement</td>
<td>2</td>
</tr>
<tr>
<td>1.3</td>
<td>Objective of study</td>
<td>2</td>
</tr>
<tr>
<td>1.4</td>
<td>Scope and limitation</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>LITERATURE REVIEW</td>
<td>4</td>
</tr>
<tr>
<td>2.1</td>
<td>An economic order quantity for items with imperfect quality and</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>inspection errors</td>
<td></td>
</tr>
<tr>
<td>2.2</td>
<td>Optimal manufacturing batch size with rework process at a single-stage</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>production system</td>
<td></td>
</tr>
<tr>
<td>2.3</td>
<td>Production lot sizing with quality screening and rework</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>MATHEMATICAL MODEL DEVELOPMENT</td>
<td>10</td>
</tr>
<tr>
<td>3.1</td>
<td>Assumptions and parameters</td>
<td>10</td>
</tr>
<tr>
<td>3.2</td>
<td>Total cost and total profit without safety stock</td>
<td>11</td>
</tr>
<tr>
<td>3.3</td>
<td>Total cost and total profit with safety stock included</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>NUMERICAL EXPERIMENTS AND SENSITIVITY ANALYSIS</td>
<td>21</td>
</tr>
<tr>
<td>4.1</td>
<td>Numerical and sensitivity analysis without safety stock</td>
<td>21</td>
</tr>
<tr>
<td>4.2</td>
<td>Numerical and sensitivity with safety stock included</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>CONCLUSIONS AND DISCUSSIONS</td>
<td>29</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td>Flow process diagram</td>
<td>10</td>
</tr>
<tr>
<td>Figure 2</td>
<td>Inventory of good products</td>
<td>11</td>
</tr>
<tr>
<td>Figure 3</td>
<td>Inventory of non-reworked defective</td>
<td>13</td>
</tr>
<tr>
<td>Figure 4</td>
<td>Inventory of good products with safety stock</td>
<td>16</td>
</tr>
<tr>
<td>Figure 5</td>
<td>Inventory of safety stock</td>
<td>17</td>
</tr>
<tr>
<td>Figure 6</td>
<td>The sensitivity of total profit and total cost versus length of a cycle</td>
<td>22</td>
</tr>
<tr>
<td>Figure 7</td>
<td>The sensitivity of total profit and total cost versus demand rate</td>
<td>22</td>
</tr>
<tr>
<td>Figure 8</td>
<td>The sensitivity of total profit and total cost versus production rate</td>
<td>23</td>
</tr>
<tr>
<td>Figure 9</td>
<td>The sensitivity of total profit and total cost versus percent defective</td>
<td>23</td>
</tr>
<tr>
<td>Figure 10</td>
<td>The sensitivity of total profit and total cost versus percent rework</td>
<td>24</td>
</tr>
<tr>
<td>Figure 11</td>
<td>Bisection method</td>
<td>24</td>
</tr>
<tr>
<td>Figure 12</td>
<td>The sensitivity of total profit and total cost versus length of a cycle when safety stock included</td>
<td>25</td>
</tr>
<tr>
<td>Figure 13</td>
<td>The sensitivity of total profit and total cost versus demand rate when safety stock included</td>
<td>26</td>
</tr>
<tr>
<td>Figure 14</td>
<td>The sensitivity of total profit and total cost versus production rate when safety stock included</td>
<td>26</td>
</tr>
<tr>
<td>Figure 15</td>
<td>The sensitivity of total profit and total cost versus percent defective when safety stock included</td>
<td>27</td>
</tr>
<tr>
<td>Figure 16</td>
<td>The sensitivity of total profit and total cost versus percent rework when safety stock included</td>
<td>27</td>
</tr>
<tr>
<td>Figure 17</td>
<td>The sensitivity of total profit and total cost versus service level when safety stock included</td>
<td>28</td>
</tr>
<tr>
<td>Figure 18</td>
<td>The sensitivity of total profit and total cost versus standard deviation when safety stock included</td>
<td>28</td>
</tr>
</tbody>
</table>
CHAPTER 1
INTRODUCTION

1.1 Overview

Nowadays, most of the manufacturers use EOQ (Economic Order Quantity) calculation to find the lowest number of inventory which must be ordered to meet the customer demand without shortage of stock and obsolete inventory. The main objective is to reduce the inventory as much as possible in order to make the lowest cost of inventory so as to maximize profit. In general, EOQ model assumes that demand is constant and the depletion of inventory can be predicted while it doesn’t work for many businesses.

In basic Economic Order Quantity (EOQ) models, it is assumed that there are no defective items which are delivered from the suppliers. Nevertheless, in some case there is a proportion of defective products. There were two researchers, Porteus (1986) and Rosenblatt and Lee (1986), studied the results of defective products on economic order quantity and economic production quantity models. After that, many researchers tried to studied the results of defective products in to different economic order quantity and economic production quantity models. It was found that the research that studied an average percentage of defective products when the items are ordered is the most well-known research was done by Salameh and Jaber (2000).

Huang (2002, 2004) and Goyal et al. (2003), investigated defective products from a relationship between buyer and supplier. for instance, created models to consider the optimal combination between buyer and supplier inventory policy for defective products and found that mutual decision-making can greatly reduce the expected total inventory cost. Chen and Kang (2007, 2010) invented the problem after determining that the payment is delay. In order to keep up the long-term relationship between buyer and supplier, the supplier can increase the warranty cost. Rezaei and Davoodi (2008) invented a model to define the optimal lot-size which contain defective products and choose the suppliers at the same time, while Lin (2009) invented a model for a single-supplier/single-buyer relationship to define the optimal lot-size when there are some defective products. The received products that delivered from suppliers must have high quality is one of the most important criteria to choose a supplier. (see for example Rezaei and Davoodi, 2008, 2011). Seeing more details and extensions of EOQ model about defective products in Khan et al. (2011).

It is very interesting to notice that, at present the EOQ model becomes a central model in inventory, logistics management and supply chain. Many researchers still tried to make EOQ models work in reality. Just for example, one of the most interesting difference of the EOQ model is the different approach to create inventory model problems by using the first and second laws of thermodynamics, where the flow of commodity or rate of demand is set similar to the heat flow in thermodynamic systems so as to calculate the clutter or
entropy cost created when controlling the flow of commodity from the system to the market (see Jabber et al., 2004; 2009).

Overall, we can see that EOQ is widely used in many businesses and there are many criteria to explore in order to make EOQ works in reality and in specific situations.

1.2 Problem statement

Normally, the aim of model which associate with inventory is to gain an economic production quantity or economic order quantity based on the cost when order is placed and the inventory holding cost. Some suppositions are usually made on the most frugal lot size for an inventory or a production facility. The general supposition is that there are only good products come out after the production. Another assumption is that the screening process which identifies the defective products is perfect. It means that all defective products from a production lot will be screened out by one hundred percent inspection method. However, in reality, the production may contain some amount of defective products, because the control process during production may not be good enough, inefficient maintenance planning, incomplete work instruction or damaged from transportation (Rahim and Ben-Daya, 2001). Moreover, it is impossible to screen all defective products at the end of production line. Thus, it is very important to define an optimal order quantity when the process of inspection may cause error while screening defective items in a lot.

In general, when the defective items cannot be controlled, it will affect to the total profit of the company. First thing to consider is when defective items are sent to the customer, the company must pay for extra transportation cost to resend good quality items to replace the defective ones. Moreover, QC team may be assigned to go to investigate at shop floor. Another thing to consider is cost of production, when the defective items happened. The extra production will be added which costs to the extra utility, workforce, set up cost and more material cost. Screening out all defective items from production lot is preferred but it will take a lot of time and resources. Knowing the number of defective items which will be produced, thus, will help to determine additional production volume to cover the defectives which will be produced. Nonetheless, it is impossible to do that. Another approach to prevent the extra production is to rework defective items. Rework cost is actually cheaper than production cost due to the less processes but if the defective item occurs more than expected, it may affect to the available resources because manufacturing facilities mostly have limited resources. However, the percentage of defective items that will to be reworked items can be fixed to suit available workforce. By doing so, lacking of resources problem will be solved. Moreover, the rest of defective items which are not reworked will be sold at discount to the market to maximize total profit.

1.3 Objective of study

This research aims at developing an economics order quantity (EOQ) model to help maximize total profit when defective items exist in a lot under the supposition that not all defective products can be detected through inspection but only a fix amount of defective items that are detected can be reworked due to the limited resources.
1.4 Scope and limitation

The model with the following characteristics will be studied throughout the research:

- A defective can be detected with a known probability.
- Only the fix percentage of defectives will be reworked.
- Non-reworked items will be sold to the market at lower price.
- Demand is met from non-defective products only.
CHAPTER 2
LITERATURE REVIEW

2.1 An economic order quantity for items with imperfect quality and inspection errors

General models about inventory which based on the cost when the order is placed and holding cost expect to gain an economic order quantity or economic production quantity. Many of them make a lot of suppositions when the idea comes with the clarification for the economical production lot size in an inventory. One supposition is that there are only perfect quality items after production. Another is that there are no defective items when identify by the screening process. That is, when 100% inspection method is taken, all the defective will be screened out. This method is impossible to happen. In practically, during the production, defective products always happen. These may occur due to the weak of process control or inefficient work instruction. Also, the screening process, for instant, it is impossible that at the final inspection at the end of assembly line will never make mistake. Therefore, when the final inspection cannot screen out all defective products, defining an optimal order quantity is necessary to obtain higher profit.

The non-defective supposition has never been used by most of researchers. The basic EOQ model was researched by Porteus (1986) in order to find the result of defective products. He made an assumption that if process can go out of control, it always happens with a constant probability. Rosenblatt and Lee (1986) supposed that the defective products should be reworks instantly if the period between the in control and out of control state of process follows an exponential distribution. The researchers advised to produce the smaller product lot size when the process cannot be controlled to produce only good products. In a few years later, Lee and Rosenblatt (1987) researched on a mutual lot sizing and policy which concern about the inventory for an EOQ model when a percentage of defective products is constant.

In the situation that many literatures in this field tackles with deterministic problems, a lot of researchers suggested production rate and demand rates follow the stochastics. Gerchak et al. (1988) examined a problem about a single period production and widened it to the multi-period problem where the production process has a non-constant yield and the demand is variable. Yano and Lee (1995) reviews and identified the defective products in the paper that tackle with defining production batch sizes where production process or purchasing methods are stochastics. Grosfeld-Nir and Gerchak (2004) tried to inspect the research paper on a single stage and multistage production systems the produce defective products that has random yield and inspection process. Inderfurth (2004) defined a policy that concern about optimal production for a demand that distributed uniformly and yield rate and advised some management perspectives of this policy. The two cases in the paper of Inderfurth (2004) was researched by Rekik et al. (2007). The first case is an additive errors case where the variability of errors is independent of the order quantity, and the second case is multiplicative errors case where the variability of errors is proportional to the order quantity.

Lately, the researchers paid attention to the research of Salameh and Jaber (2000). They focused on an EOQ model with a mutual lot sizing and policy that concern about inspection about when the defective occurs in a non-constant percentage of the products
in a production batch. The researchers supposed that the workforce will never make mistake when doing one hundred percent screening process. They suggested that when one hundred percent screening process is done; all the defective products should be scraped as a single batch. The model mentioned earlier can be referred to the Salameh and Jaber (2000) model for the entire research. Goyal and Cardenas-Barron (2002) advised an easier method to the Salameh and Jaber (2000) model, which Goyal et al. (2003) used to define as the policy that concern with the integrated seller-buyer inventory for a product that has a bad quality. The researchers discussed that the ordering quantity should be dispatched in multi sub-lots. Chiu (2003) investigated the defective products that need to be reworked that cause to the economic production quantity model which could have backordering. He commended that we should rework only some part of all defective products while another part should be destroyed. Chang (2004) researched the Salameh and Jaber (2000) model under a fluctuated defective rate and a fluctuated total demand. An integrated seller-buyer inventory model for an unreliable process was demonstrated by Huang (2004). He derived a systematic result for the optimal order quantity and the amount of products which were delivered every time the purchase order was placed. Papachristos and Kontantaras (2006) suggested the inventory models that shortage is not allowed where the percentage of defective products can occur randomly. This is offered to be the option for the S&J model. Liao (2007) researched about the production process that is not always perfect and always needs to be maintained. Liao (2007) determined two conditions, called in-control and out-of-control state of the processes. When the maintenance takes place, it can bring the process of production even worse or becomes to a brand-new condition. Before the production process is becoming to the brand-new condition, the researchers depicted that there is a unique optimal number of maintenance processes (N) which is not always perfect. Maddah and Jaber (2008) studied renewal theory to correct a defective product in the Salameh and Jaber (2000) model. They showed the basic mathematical model to solve expected profit and the quantity to be ordered.

The Salameh and Jaber (2000) model proposed that the defective products will be removed from the production lot and not reworked. It can be supposed that in the screening process, there is no human error. Raouf et al. (1983) tried to research inside inspection planning that is there any human error happen in this section. They showed the idea about a person who made mistake inside inspection planning section for multi-characteristic critical components. They proposed to ensure the quality of every products and defined an optimal length of times in the inspection cycle according to the budget of inspection and misclassification by repeating the cycle. Duffuu and Khan (2002) proposed a plan for inspection section for these critical materials which an inspector can confirm a frequency of misclassifications. They prolong the Raouf et al. (1983) plan that involve with the inspection for the situation that there is a frequency of misclassification. This was a method that happen in reality where an inspector can categorize a product to be non-defective product, rework able or should be scrapped. A few years later, Duffuu and Khan (2005) demonstrated a sensitivity analysis to learn the result of misclassifications in many different types on the inspection plan with optimization.

The inspection might be error due to many conditions; it is possible that the record in the past is not accuracy. Kok and Shang (2007) advised about the record data of inventory in the past may not accuracy enough. K&S showed that an inspection should be adjusted according to the level of stock policy is prefer for a single period situation, where inspection is executed if the record of inventory is less than an expected level. Atali et al. (2009) also created a model with the differences between recorded inventories in the past
and actual inventories in retail and distribution situations. They depicted the worth of RFID (radio-frequency identification) the reduced a number of the differences.

Moreover, it can say that the part that talk about the inspection in the Salameh and Jaber (2000) model is appropriate for purchasers who have the robotic system in order to screen out the defective products where no one will make any mistake. Otherwise, if it is not possible to screen the characteristic of interest by a robot and is found by workforce, the screening process will always have an error. For instance, a supermarket will check the delivered products and dispatch the non-defective products to the customers. When the items are low quality, it will be sold at lower price. The returned defective items, that were misclassified, would also be sold at lower price. The wrong categorized non-defective products might encounter the loss of profit. The similar method brought to use in an industrial company that try to make supplier products to be in perfect condition during assembly process and if the defective products are produced, they will be screened out and will be sold to another market after inspection. These wrong categorizations will be danger if the components of an aircraft are under inspection, a space shuttle or a complex gas ignition system (Raouf et al., 1983). The results of wrong categorization are the reason in this kind of systems can make someone die. That is the reason why the parts of this kind of work should be high quality and cannot let any defective get through the screening process, (Zunzanyika and Drury, 1975). A similar case in the manufacturing field is to assign many inspections in order to lower a chance of mistaken when producing these parts (Chandra and Schall, 1988). Thus, it is very important for a purchaser that not only check the potential of the inspector who can screen out all the defectives but also seek the methods to eliminate these errors.

Finally, this research paper determines a policy that concern about the inventory for defective products obtained by a purchaser. A screening method is applied to use in reality. To do so, an inspector will categorize a non-defective product to be flaw product (Type I error) and it will categorize a flaw product to be non-defective (Type II error). The flaw product which categorized by the inspector and some of them sent back from customers are accumulated and brought to sell at a lower rate when the purchasing cycle ends.

This research paper determines that the total profit at the end of a year that includes an error in inspection process will be concave with respect to the order lot size. Seeing at the results coming out from Salameeh and Jaber (2000). The total profit at the end of a year is lower but the optimal order quantity is mostly the indifference. The out out shows the result when the errors occur during the inspection. The growing number of defectives will always decrease the total profit.

2.2 Optimal manufacturing batch size with rework process at a single-stage production system

In present year, many researchers pay attention and try to investigate the ways to make production lot size during a production process to be optimality. Various lot size model that was made to be optimality were developed by many researchers in many situations to maximize the total profit. The appropriate evaluates a policy which concern about an inventory system considers an inventory carrying cost, cost that happens due to shortage and, cost when process is running and set up cost that included in a lot-sizing model with
optimality. The rate of demand and the rate of production of the system will lead a lot size quantity or economic order quantity to be optimality.

In a production system that has only one state, a confident number of flaw products out puts according to many causes by the uncontrollable quality from a production process and low quality raw materials, and following some percentage of the defective products will be eliminated. If there are a lot of percentage of defective products, many factor that involve with costs such as setup cost, processing cost, and inventory holding cost will have a direct effect to a lot size that needs to be optimality. Most of manufacturers normally have a rework or repair unit in order to fix or rework defective products to be good products. The policy for an industrial engineering, zero inventory and zero defect policy should be applied to remove wastes. But in fact, the economist and technician do not want to eliminate the wastes. In order to make them satisfy, reworking defective products still exists but there must be a plan to maximize total profit. All the defective products will be sent to eliminate at scrap yard if a rework unit is not available in an industry. If there are a lot of defective products produced in every production period, it is a very bad production system, and if the manufacturers do not have rework unit in a plant. Their total profit will be very low. In some case, if there is a few defective happens in production process, rework unit may not necessary.

Goyal and Gunashekharan (1990) studied the situation that there are no defective products occur and find out the result of it. Some researchers stated because we cannot ignore the batch-sizing has a problem, a bad quality control and a production which contain the defective product are still an issue. Lee, Chandra, and Deleveaux (1997) invented a model in a situation that defective products will be produced in every production cycle under the n-stage production process and assumed that all defective products that produced during a process will be made to a good condition after a cycle ends.

Liu and Yang (1996) stated a production batch-sizing model that can produce defective products and the defective products can be reworked uniformly. Minner and Kleber (2001) invented a method of using linear cost functions with a production and remanufacturing model with optimality for a model that can be recovery and Teunter and van der Laan (2002) tried to discover the result when an inventory model is in a condition of non-optimality with remanufacturing. Richter and Sombrutzki (2000) and Richter and Weber (2001) try to joined an originally Wagner/Whitin model and a pure reverse Wagner/Whitin model with given returns of used products which they termed ‘variable manufacturing and remanufacturing cost.’

Chakrabarty and Rao (1988) considered when the defective products is possible to be reworked to determine an n-stage production system with optimal order quantity in a lot. Gupta and Chakraborty (1984) studied the situation when the defective products occur and being reworked later. The rework process starts when the first defective product produced and keep reworking until the end of the production cycle. Moreover, they tried to formulated the EOQ model in this situation. In their research paper, all the penalty cost that happen because of producing defective product will be ignored. Shih (1980) studied about the defective products may cause to a shortage situation.

This research determines the defective items will be reworked in a production system that has only one stage. In order to maximize the total profit, a production system that has only one state will be considered to invented the economic order quantity models by
considering two policies. First of all, in a situation that all the defective products that produced will be reworked in the same cycle will be formulated to be an economic order quantity model. Secondly, all the defective products occur in each cycle will be collected and reworked when the total defective products reach to the target.

Finally, in order to reworked defective products, this paper invented the optimal production order quantity according to a procurement models. In the first policy, during a production cycle, the defective products will be reworked before a cycle ends. In the second policy, the defective products will be collected until the number of defective products reach to the target number, and then, those defective products will be reworked. However, the manufacturing lot size raises to make to total profit as high as possible in the first policy. On the other hand, in the second policy, the production lot raises only until it reach to a confident level of defectiveness.

2.3 Production lot sizing with quality screening and rework

This paper randomly selected items when a production cycle finished and find out how many defective items occur. After that, the defective products will be sent to rework unit to be reworked and sent to be the inventory. However, it is not easy to find out because the proportion of defective is low but the rate of a production quite high, and result to the cost will be expensive. To make it simple, this paper including the screening system into the production order quantity model with defective products can be reworked. It means that the screening rate must be equal to the demand rate and this process begins before production ends and also during production. In this paper, the researcher analyzed two models: the first one, defective products that screened out during the screening process are sent to storage area and sold at a lower price. Secondly, all defective products will be screened out and reworked at a steady rate. And a shortage is not permitted.

This paper assumed that defective items are repairable for the model with rework. However, it is true for many manufacturing fields such as automobile and furniture, etc. Scrapping is also possible but its cost is quite high. Thus, repairing is a better choice.

In case of food, retailers must inspect the received items before committing because fresh product is impossible to be reworked, it has to be thrown away or returned to supplier.

The model in this paper demonstrates three simple perspectives:
1. When a production process ends, the screening process will be started but screening during production may be difficult due to the fast production process.
2. Demand during production is met from non-defective items only.
3. When identify, defective items are brought to storage and scrapped when a production cycle ends or they will be reworked at the steady rate.

In previous papers the following topics below were developed:
1. Effect to the EOQ model when there are defective products contain in a lot and investigate the quality of products when lot size is increasing.
2. A mutual lot sizing and policy that concern about the inventory for EOQ model included defective.
3. EPQ with defective items.
4. Assume the demand to be stochastic, delivery lead time is steady and the amount of defective products to be random variable.
5. Screening system can detect defective products and the defective products will be sold at lower price or replaced when a cycle ends. Moreover, in case of items are irreplaceable from original supplier, the items are sent to be repaired or replaced from local supplier at higher cost.
6. Determine the optimal lot sizes of a producer and a purchaser with quantity discount, price when selling retail, mark-up rate and how many shipments should occur per production run from the supplier to the buyers.
7. Determine the product reliability to find its optimality and rate of production to minimize cost.
8. Manufacturing process produces only perfect quality products up to the point then the defective products will be produced until the production cycle ends and backordering is not allowed.

This paper is closest to an economic production quantity model but there are defective products can be detected when a production is running and all of defective products will be reworked when the production stops. In case where the defective products are not possible be reworked or repaired, they will be sold in other industries at lower cost. Furthermore, to find defective products most of the paper tries to avoid the screening time but this paper provides the method to tackle with this trouble by including the screening time for calculation and also includes rework process of defectives in a model.

In conclusion, this paper focused on the cases where defective products can be reworked and scrapped. The model that use to calculate the optimal total profit per unit time and economic order quantities contains of a group of number to test the sensitivity which include the rate of demand, production capacity, defective products percentage and rate of screening defective products.

This paper also considered the screening process right after the production stops to screen out the defective products that need to be reworked. Furthermore, this work is integrated the screening time and rework process into the production mathematical model and also screening the items as they are being sold, suppose that all defective products can be repaired.

The optimal profit and quantity with respect to the demand, production capacity, percent of flaw products, and screening rate is depicted as the sensitivity analysis. In case of excluding rework and including rework process, it can be observed that the optimality of cost is reducing and production quantity are growing along with the demand rate and declining with the production rate offset high carrying costs. The screening rate make the optimality of cost to be low but quantity to be high. When the defective products percentage is raising, the optimal production quantity is dropping. In case of excluding rework process, a higher percentage of flaw products resulted in a higher production batch quantity to compensate for the eliminated defective products which are not utilized to satisfy the demand.
CHAPTER 3
MATHEMATICAL MODEL DEVELOPMENT

3.1 Assumptions and parameters

In this part, a mathematical model of a production system according to the assumption which mention earlier will be derived. Assume that this is a single model production, and this manufacturing facility follows made-to-order policy. We consider this situation in a machining line. A loop of production will be considered as a cycle \( T \), which starts when a production begins and stops when all good products are delivered to customers.

Before the production starts, the amount of products which customer requires or demand rate \( D \) will be confirmed and data will be sent to machining line. A person in charge will check the capacity of the machining line, production rate \( p \), set up machines, and starts a production \( t_1 \). Every unit that produced has its cost \( C_p \). However, there will be some defective products occur during production. In this case the defective products occur will be known as a percentage \( y \) according to a recorded data in the past. Moreover, only a fix percentage of defective products will be reworked \( \delta \) due to a limited resource in a period. There is a cost when rework defective products \( C_w \). After production period ends, all products must be waited before distributed to a customer. The good products will be kept in one area with holding cost for good products \( H \). Defective products will be kept separately with holding cost for defective \( H_d \) which is a bit cheaper. When a cycle ends, good products can be sold to a direct customer at, selling price \( S \). Defective products will be sold to other market at discount price \( S_d \).

![Flow Process Diagram](image_url)

**Figure 1: Flow process diagram**

The input notations below will be used in EOQ model in this research.

\[
TC = \text{Total cost in a cycle}
\]
\( TP = \) Total profit in a cycle
\( TR = \) Total revenue in a cycle
\( D = \) Demand rate per unit time
\( H = \) Good products inventory holding cost per time unit
\( H_d = \) Non-reworked defective products inventory holding cost per time unit
\( \gamma = \) Percentage of defectives produced
\( \delta = \) Percentage of defectives reworked
\( p = \) Production rate per unit time
\( \bar{p} = \) Rate at which good products (non-defective & reworked) accumulated in the system
\( t_1 = \) Production time in a cycle
\( T = \) Length in a cycle
\( U = \) Set up cost
\( C_p = \) Cost of production per unit
\( C_w = \) Rework cost per unit
\( S = \) Good products selling price
\( S_d = \) Non-reworked defective selling price (discounted price)
\( Z = \) Service level
\( \sigma_{T-t_1} = \) Standard deviation of demand during lead time

### 3.2 Total cost and total profit without safety stock

According to the manufacturing process, there are two phases contained in a cycle. The first phase is production phase and another one is non-production phase. In a production phase, there are many processes such as production, set up and rework processes. In a non-production phase, all products will be transferred to storage area and prepared for delivery to a customer.

![Figure 2: Inventory of good products](image)

First of all, we focus on events that can happen during production time. The main objective in this period is to produce good products in order to satisfy customer’s demand.
• Total production quantity in a cycle = Production rate per unit time multiply by production time in a cycle

\[ pt_1 \] (1)

In reality, defective products can be produced during this period, and a number of defective products that could be produced is known by recording data.

• Total amount of non-defective products produced in a cycle = Total production quantity in a cycle multiply by percentage of non-defective items

\[ pt_1(1 - \gamma) \] (2)

• Total amount of defective products produced in a cycle = Total production quantity in a cycle multiply by percentage of defective items

\[ pt_1 \gamma \] (3)

Due to limited resources, only a fix percentage of defective products will be reworked.

• Total amount of defective products reworked in a cycle = Total amount of defective products produced in a cycle multiply by percentage of defective products which can be reworked

\[ pt_1 \gamma \delta \] (4)

In order to satisfy customer satisfaction, only good products will be delivered to a customer.

• Total amount of products that can be used to fulfill demand = Total amount of non-defective products produced in a cycle + total amount of defective products reworked in a cycle

\[ pt_1 (1 - \gamma) + pt_1 \gamma \delta = p(1 - \gamma + \gamma \delta)t_1 \]

• Rate at which good products (non-defective & reworked) accumulated in the system:

\[ \dot{p} = p(1 - \gamma + \gamma \delta) - D \] (5)

• Thus, total inventory of good products at time \( t_1 \):

\[ \dot{p}t_1 \] (6)

Refer to Figure 2,

• In order to have enough good products to meet customer requirement, we must have:

\[ \dot{p}t_1 = D(T - t_1) \]
Then, we can find the relationship between production period and length of a cycle by:

\[ t_1 = \frac{DT}{p+d} \]  

(7)

Looking at Figure 2, it can be seen that the level of inventory during production time is rising. It can say that at time \( t_1 = 0 \), the total good products will be zero but at time \( t_1 \), the inventory will be \( pt_1 \), which is a maximum amount of good products that produced, and finally at time \( T \), the inventory of good products will drop to zero again.

- Time-weighted inventory holding of good products in a cycle = A half of the maximum level of good inventory multiply by a length of a cycle.

\[ \frac{pt_1T}{2} \]  

(8)

- Inventory holding cost of good products in a cycle:

\[ H \left( \frac{pt_1T}{2} \right) \]  

(9)

Moreover, during the production time, defective products are produced and reworked but some percentage of them are not reworked. The amount of non-reworked defective products keeps growing until time \( t_1 \), and all of them will be sold at discounted at the end of a cycle.

Looking at Figure 3, it is clearly seen that the level of non-reworked defective products during production period is raising over the time from zero and reach to the
maximum level at time $t_1$. After that, the level of non-reworked defective products is stable until a cycle ends.

- Time-weighted inventory holding of non-reworked products in a cycle = A half of the maximum level of non-reworked defective products multiply by a production time in a cycle + the maximum level of non-reworked defective products multiply by non-production time.

$$p\gamma(1 - \delta)(Tt_1 - \frac{t_1^2}{2})$$  \hspace{1cm} (10)

- Inventory holding cost of non-reworked products in a cycle:

$$H_d \left[p\gamma(1 - \delta)(Tt_1 - \frac{t_1^2}{2})\right]$$  \hspace{1cm} (11)

Every process during production has its cost. In this case, we will mention about production cost and rework cost. In manufacturing, production cost and rework cost can be contained of raw material cost, labor cost, utility cost and tool cost.

- Total production cost in a cycle = Cost of production per unit multiply by a total production

$$C_p \cdot pt_1$$  \hspace{1cm} (12)

- Reworked cost in a cycle = Rework cost per unit multiply by a number reworked defective products

$$C_w \cdot pt_1 \gamma \delta$$  \hspace{1cm} (13)

When the production time ends, both good products and defective products will be sold to customers. Good products can be sold at normal price to a main customer but defective products will be sold at discount to other market.

- Revenue from demand fulfilling in a cycle = Selling price multiply by number of good products

$$Sp(1 - \gamma + \gamma \delta)t_1$$  \hspace{1cm} (14)

- Revenue from selling non-reworked defective products in a cycle = Discounted price multiply by number of defective products

$$S_d p\gamma(1 - \delta)t_1$$  \hspace{1cm} (15)

After that, it is simple to find the total revenue and total cost.

- Total revenue in a cycle = the sum of revenue from demand fulfilling and revenue from selling non-reworked defective products

$$TR = Sp(1 - \gamma + \gamma \delta)t_1 + S_d p\gamma(1 - \delta)t_1$$
Total cost in a cycle = Production cost + Holding cost of Inventory + Holding cost for defective items + Rework cost + Set up cost

\[ TC = (C_p pt_1) + H \left( \frac{\dot{p} t_1 T}{2} \right) + H_d \left[ p y (1 - \delta) \left( T t_1 - \frac{t_1^2}{2} \right) \right] + C_w pt_1 y \delta + U \]

(16)

We could find the total profit when the total cost is less than the total revenue, and is given as

Total profit = Total revenue – Total cost

\[ TP = Sp(1 - \gamma + \gamma \delta)t_1 + S_d p y (1 - \delta)t_1 \]

\[ - \left\{ (C_p pt_1) + H \left( \frac{\dot{p} t_1 T}{2} \right) + H_d \left[ p y (1 - \delta) \left( T t_1 - \frac{t_1^2}{2} \right) \right] + C_w pt_1 y \delta + U \right\} \]

(17)

When

\[ t_1 = \frac{DT}{\bar{p} + D} \]

Then, we can find the total profit per time unit.

\[ \text{Total profit per time unit} = \frac{\text{Total profit in a cycle (TP)}}{\text{Cycle length (T)}} \]

\[ = \frac{Sp(1 - \gamma + \gamma \delta) + S_d p y (1 - \delta)}{\bar{p} + D}D \]

\[ - \left\{ \frac{C_p p D}{\bar{p} + D} + H \left( \frac{\dot{p} D T^2}{(\bar{p} + D)} \right) + H_d \left[ p y (1 - \delta) \left( \frac{DT}{\bar{p} + D} - \frac{1}{2} \left( \frac{DT}{\bar{p} + D} \right)^2 \right) \right] + C_w p y \delta \left( \frac{D}{\bar{p} + D} \right) + U \right\} \]

The first order conditions of TP with respect to T gives the optimal length of a cycle as:
\[
\frac{dTP}{dT} = \frac{H_d p \gamma (1 - \delta)}{2} \left( \frac{D}{\bar{p} + D} \right)^2 - \frac{H}{2} \left( \frac{\bar{p} D}{\bar{p} + D} \right) - H_d p \gamma (1 - \delta) \left( \frac{D}{\bar{p} + D} \right) + \frac{U}{T^2} = 0
\]

(19)

\[
T^* = \frac{U}{\sqrt{H_d p \gamma (1 - \delta)\left( \frac{D}{\bar{p} + D} \right) - \frac{H_d p \gamma (1 - \delta)}{2} \left( \frac{D}{\bar{p} + D} \right)^2 + \frac{H}{2} \left( \frac{\bar{p} D}{\bar{p} + D} \right)}}
\]

(20)

However, we can find the optimal production time in a cycle as:

\[
t_1^* = \sqrt{\frac{2DU}{H_d p \gamma (1 - \delta) (2\bar{p} + D) + \bar{p} (\bar{p} + D)}}
\]

(21)

3.3 Total cost and total profit with safety stock included

Because there will be a fluctuation of the demand during non-production period. A safety stock should be added to the inventory to increase the service level in order to lower shortage. A safety stock always exists under the good products inventory from the beginning of a cycle.

![Figure 4: Inventory of good products with safety stock](image)

- A safety stock can be defined based on
When consider a safety stock. It is a constant number from the start of a cycle until the end.

Inventory level of good products

\[
Z\sigma_{t-t_1} \\
Z\sigma\sqrt{T-t_1}
\]

(22)  

(23)

Figure 5: Inventory of safety stock

- Time-weighted safety stock level in a cycle = Safety stock level multiply by a length of a cycle

\[
(Z\sigma\sqrt{T-t_1})T
\]

(24)

- A holding cost of safety stock in a cycle is

\[
H(Z\sigma\sqrt{T-t_1})T
\]

(25)

Therefore, the result of total cost in a cycle will be changed.

- Total cost in a cycle = Production cost + Holding cost of Inventory + Holding cost for defective items + Rework cost + Set up cost + Safety stock

\[
TC = \left(C_ppt_1\right) + H\left(\frac{pt_1T}{2}\right) + H_d\left[py(1-\delta)(Tt_1 - t_1^2)\right] + C_wpt_1y\delta + U + H(Z\sigma\sqrt{T-t_1})T
\]

(26)

We could find the total profit when the total cost is less than the total revenue, and is given as
Total profit = Total revenue – Total cost

\[
TP = Sp(1 - \gamma + \gamma \delta)t_1 + S_dpy(1 - \delta)t_1 \\
- \left\{ (C_ppt) + H \left( \frac{p_D T}{2} \right) + H_d \left[ py(1 - \delta) \left( T t_1 - \frac{t_1^2}{2} \right) \right] + C_wpt_1 \gamma \delta + U \right. \\
+ H(Z\sigma \sqrt{T - t_1})T \right\}
\]

(27)

When

\[
t_1 = \frac{DT}{\bar{p} + D}
\]

\[
TP = \frac{[Sp(1 - \gamma + \gamma \delta) + S_dpy(1 - \delta)]DT}{\bar{p} + D} \\
- \left\{ \frac{C_p pDT}{\bar{p} + D} + H \left( \frac{\bar{p} DT^2}{2 (\bar{p} + D)} \right) + H_d \left[ py(1 - \delta) \left( \frac{DT^2}{\bar{p} + D} - \frac{1}{2} \left( \frac{DT}{\bar{p} + D} \right)^2 \right) \right] \\
+ C_wpy(1 - \delta) \left( \frac{DT}{\bar{p} + D} \right) + U + H \left( Z\sigma \sqrt{T - \frac{DT}{\bar{p} + D}} \right) \right\}
\]

Then, we can find the total profit per time unit.

Total profit per time unit = \frac{\text{Total profit in a cycle}(TP)}{\text{Cycle length}(T)}

\[
TP = \frac{[Sp(1 - \gamma + \gamma \delta) + S_dpy(1 - \delta)]D}{\bar{p} + D} \\
- \left\{ \frac{C_p pD}{\bar{p} + D} + H \left( \frac{\bar{p} DT}{\bar{p} + D} \right) + H_d \left[ py(1 - \delta) \left( \frac{DT}{\bar{p} + D} - \frac{1}{2} \left( \frac{D}{\bar{p} + D} \right)^2 \right) \right] \\
+ C_wpy(1 - \delta) \left( \frac{D}{\bar{p} + D} \right) + \frac{U}{T} + H \left( Z\sigma \sqrt{T - \frac{DT}{\bar{p} + D}} \right) \right\}
\]

The first order conditions of TP with respect to T gives the optimal length of a cycle as:

\[
\frac{dTP}{dT} = - \frac{H}{2} \left( \frac{\bar{p} D}{\bar{p} + D} \right) - H_d py(1 - \delta) \left( \frac{D}{\bar{p} + D} \right) + \frac{H_d p y(1 - \delta)}{2} \left( \frac{D}{\bar{p} + D} \right)^2 + \frac{U}{T^2} \\
- \frac{1}{2} HZ\sigma \left( \frac{\bar{p}}{\bar{p} + D} \right) \cdot \frac{1}{\sqrt{T}} = 0
\]

(28)

The second order conditions of TP with respect to T gives the optimal length of a cycle as:
\[
\frac{d^2 TP}{dT^2} = -\frac{2U}{T^3} + \frac{1}{4} HZ\sigma \sqrt{\frac{\dot{p}}{\dot{p} + D}} \cdot \frac{1}{T^2}
\]

If \[
\frac{d^2 TP}{dT^2} = -\frac{2U}{T^3} + \frac{1}{4} HZ\sigma \sqrt{\frac{\dot{p}}{\dot{p} + D}} \cdot \frac{1}{T^2} < 0
\]

We can conclude that this is a concave function, and this equation has a unique solution.

**Prove:**

If \( T^* \) is the solution of (28), then

\[
\frac{1}{2} HZ\sigma \sqrt{\frac{\dot{p}}{\dot{p} + D}} \cdot \frac{1}{\sqrt{T}} = -\frac{H}{2} \left( \frac{\dot{p} D}{\dot{p} + D} \right) - H_d p y (1 - \delta) \left( \frac{D}{\dot{p} + D} \right) + \frac{H_d p y (1 - \delta)}{2} \left( \frac{D}{\dot{p} + D} \right)^2
\]

So,

\[
\frac{d^2 TP}{dT^2} = -\frac{2U}{T^3} + \frac{1}{4} HZ\sigma \sqrt{\frac{\dot{p}}{\dot{p} + D}} \cdot \frac{1}{\sqrt{T}} \cdot \frac{1}{T}
\]

\[
\frac{d^2 TP}{dT^2} = -\frac{2U}{T^3} + \left[ -\frac{H}{2} \left( \frac{\dot{p} D}{\dot{p} + D} \right) - H_d p y (1 - \delta) \left( \frac{D}{\dot{p} + D} \right) + \frac{H_d p y (1 - \delta)}{2} \left( \frac{D}{\dot{p} + D} \right)^2 \right]
\]

\[
\frac{d^2 TP}{dT^2} = -\frac{3U}{2T^3} + \frac{1}{2T} \left[ -\frac{H}{2} \left( \frac{\dot{p} D}{\dot{p} + D} \right) - H_d p y (1 - \delta) \left( \frac{D}{\dot{p} + D} \right) + \frac{H_d p y (1 - \delta)}{2} \left( \frac{D}{\dot{p} + D} \right)^2 \right]
\]

Considering,

\[-H_d p y (1 - \delta) \left( \frac{D}{\dot{p} + D} \right) + \frac{H_d p y (1 - \delta)}{2} \left( \frac{D}{\dot{p} + D} \right)^2\]

We can see that,
\[
\left( \frac{D}{\dot{p} + D} \right) > \left( \frac{D}{\dot{p} + D} \right)^2
\]

Then,
\[
H_d p \gamma (1 - \delta) \left( \frac{D}{\dot{p} + D} \right) > \frac{H_d p \gamma (1 - \delta)}{2} \left( \frac{D}{\dot{p} + D} \right)^2
\]

Thus,
\[
-H_d p \gamma (1 - \delta) \left( \frac{D}{\dot{p} + D} \right) + \frac{H_d p \gamma (1 - \delta)}{2} \left( \frac{D}{\dot{p} + D} \right)^2 < 0
\]

Finally, we can conclude that
\[
\frac{d^2 T_P}{dT^2} = -\frac{2U}{T^3} + \frac{1}{4} HZ \sigma \sqrt{\frac{\dot{p}}{\dot{p} + D}} \cdot \frac{1}{T^3} < 0 : Concave \ function
\]

Noted that \( TP \) is concave at solution of (28), if (28) has solution and that solution is the maximum value.

It means that this equation has a unique solution. In order to solve this problem, we need to use bisection method to find \( T^* \).
4.1 Numerical experiments and sensitivity analysis without safety stock

In this chapter, we will analyze how the optimal length of the cycle and optimal profit get along with the model parameters that mentioned in the previous chapter. First of all, we consider constant demand rate \( D = 220 \) units/day, and a production rate \( p = 480 \) units/day. The defective products \( \gamma \) will occur approximately 20%, and it will be reworked with parameter \( \delta \) only 65%. The production cost per unit is \( C_p = 150 \)$, selling price of good products is \( S = 350 \$/unit, and discounted price of non-reworked defective items \( S_d = 40 \$/unit. The holding cost of good products is \( H = 30 \$/unit/day, and the holding cost of non-reworked defective products is \( H_d = 20 \$/unit/day. The rework cost \( C_w = 40 \$/unit/day, and the set up cost \( U = 2500 \$.

When there are no defective products occur \( \gamma = 0 \), the optimal length of the cycle, \( T^* = 1.0242 \) days and the optimal Total Profit and Total Cost per unit time \( TP = 39,118.06 \$ \) and \( TC = 37,881.94 \) per day respectively. It is clearly seen that \( T^* \) increases as the holding cost \( H \) and \( H_d \) decreases or set up cost increases. However, the effects of others parameter will be illustrated through sensitivity analysis. Noted that, to avoid endless production system situation during production, the amount of good production should meet the demand rate during the cycle, in which the condition is

\[ \gamma \leq \frac{1 - \frac{D}{p}}{1 - \delta} \]

So that, the maximum Demand rate which we can assume is

\[ D \leq (1 - \gamma + \gamma \delta)p = 446.4 = 446 \text{ units per unit time} \]

And the minimum production rate which we can assume is

\[ p \geq \frac{D}{1 - \gamma + \gamma \delta} = 236.6 = 237 \text{ units per unit time} \]

And also, the maximum percentage of defective products after reworked should not exceed

\[ \gamma(1 - \delta) \leq 1 - \frac{D}{p} = 0.5417 = 54.17\% \]

Furthermore, we can see the relationship of Total profit, Total cost and Length of a cycle of Demand rate, Production rate, Percentage of defective and Percentage of reworked defective as below.

The result of Total cost and Total Profit when \( T \) is shifted is shown below.
Figure 6: The sensitivity of total profit and total cost versus length of a cycle

It can be seen that the result of Length of a cycle at optimal point of $T^* = 1.0038$ days and the optimal point of a production time $t_1^* = 0.4947$ days.

Figure 7: The sensitivity of total profit and total cost versus demand rate

When considering the demand rate per unit time, total cost and total profit are increasing when the demand increases. However, the length of a cycle keeps decreasing to the lowest point at 1.0026 days and then increases.
When the production rate increases, total cost keeps increasing while total profit decreases. It can also see that if the production rate reaches close to the minimum value, the length of a cycle will be very high.

When the percentage of defective product ($\gamma$) increases, total cost will be increasing while total profit and length of a cycle are decreasing.
Figure 10: The sensitivity of total profit and total cost versus percent rework

When the percentage of defective reworked ($\delta$) increases, total cost will be decreased but total profit is increasing. However, length of a cycle keeps raising when percent rework increases.

4.2 Numerical and sensitivity analysis with safety stock included

We consider an average demand rate $D = 220$ units/day with standard deviation $\sigma = 20$ units/day, and a production rate $p = 480$ units/day. The defective products $\gamma$ will occur approximately 20%, and it will be reworked $\delta$ only 65%. The production cost per unit is $C_p = 150$ $\$, selling price of good products is $S = 350$ $\$/unit, and discounted price of non-reworked defective items $S_d = 40$ $\$/unit. The holding cost of good products is $H = 30$ $\$/unit/day, and the holding cost of non-reworked defective products is $H_d = 20$ $\$/unit/day. The rework cost $C_w = 40$ $\$/unit/day, and the set up cost $U = 2500$ $\$ with the service level at 95%.

Figure 11: Bisection method
By using the bisection method to find the optimal solution. It is found that the optimal length $T^* = 0.9174 \text{ days}$ and optimal production time $t_1^* = 0.4521 \text{ days}$ and Total Profit $TP = 37,698.92 \text{ $ per day.}$

When there are no defective products occur $\gamma = 0$, the optimal length of the cycle, $T^* = 0.9308 \text{ days}$, optimal production time $t_1^* = 0.4266 \text{ days}$, the optimal Total Profit and Total Cost per unit time $TP = 38,161.29 \text{$ and } TC = 35,049.46 \text{ per day respectively.}$

It can be seen that $T^*$ raises when the holding cost $H$ and $H_d$ decreases or set up cost increases. However, the others parameter will be depicted through sensitivity analysis. Noted that, to prevent endless production system situation during production, the amount of good production should meet the demand rate during the cycle, and its condition is as same as safety stock excluded case.

Furthermore, we can see the relationship of Total profit, Total cost and Length of a cycle of Demand rate, Production rate, Percentage of defective and Percentage of reworked defective as below.

![Figure 12: The sensitivity of total profit and total cost versus length of a cycle when safety stock included](image)

The result of Total cost and Total Profit when $T$ is shifted is shown above. It shows that when the length of a cycle shifts from optimal point, total cost starts to increase while total profit decreases.
When considering the demand rate per unit time, total cost and total profit are increasing when the demand increases. However, the length of a cycle keeps decreasing to the lowest point at $T = 0.9174$ days and then increases.

When the production rate increases, total cost keeps growing while total profit reduces. It can also see that if the production rate reaches close to the minimum value, the length of a cycle will be very high.
Figure 15: The sensitivity of total profit and total cost versus percent defective when safety stock included

When the percentage of defective product ($\gamma$) increases, total cost will be increasing while total profit and length of a cycle are declining.

Figure 16: The sensitivity of total profit and total cost versus percent rework when safety stock included

When the percentage of defective reworked ($\delta$) rises, there is a reduction in total cost but total profit is increasing. However, length of a cycle keeps raising when percent rework increases.
When a service level increases, total cost will be higher while total profit will be less. However, length of a cycle will be shorter to make as much as profit.

When a standard deviation (σ) is rising, there is a decline of total profit and the length of a cycle but total cost keeps growing.
CHAPTER 5
CONCLUSIONS AND DISCUSSIONS

This research studies an economics order quantity model in a situation that there are some defective products contained in a production lot. In order to fulfill a customer’s demand, there are many factors need to be considered before a production begins. The most important factor is the production rate; it must be higher than demand rate in order to fulfill the customer’s demand. The imperfection process or bad work instruction will make defective products occur. When there are a lot with defective products happens, total revenue will be less. Another way to increase total revenue if the defective products cannot be controlled is rework. Reworking is the option, but there is a cost of rework also. It means that total revenue cannot reach to the maximum if the defective products still occur.

In reality, there is a 50% chance that a shortage will happen when there is a fluctuation of demand during non-production period if there is no safety stock in an inventory. Safety stock is a way to increase the service level to lower shortage. If a service level is very high, a shortage will be very low. However, a safety stock has a drawback, the inventory will be larger and there will be more cost. The fluctuation of demand or standard deviation has a direct effect to a safety stock level, if standard deviation is huge, a safety stock will increase also.
REFERENCES


