STOCHASTIC DEMAND OPTIMAL INVENTORY MODEL CONSIDERING CUSTOMER'S PURCHASING BEHAVIOR FOR DETERIORATING PRODUCTS

by

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AUTHOR’S DECLARATION

I, Jutamas Boonsri, declare that the research work carried out for this thesis was in accordance with the regulations of the Asian Institute of Technology. The work presented in it are my own and has been generated by me as the result of my own original research, and if external sources were used, such sources have been cited. It is original and has not been submitted to any other institution to obtain another degree or qualification. This is a true copy of the thesis, including final revisions.

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ABSTRACT

Inventory management is important for businesses of any size to reduce the cost as much as possible. Especially, many industries deal with shorter product life cycles due to the deterioration rate in inventory. Moreover, the uncertainties in customer demand are challenging for the replenishment decisions for deteriorating products. In the same way, the demand of customer is a key parameter of any inventory model. This research focus on development of mathematical models for the development of an inventory model for deterioration products under replenishment policy considering customer purchasing behavior. To consider the consumers purchasing behavior, the customers are classified into two groups. The first group will always buy the product regardless of whether the remaining lifetime is very short while the second group will buy only when the remaining lifetime of the product is larger than a predefined value. The mathematical model was developed found application in determining order quantity for reducing the total cost of the system and the cycle length, aiming to reduce the expected total inventory cost within the cycle. Numerical experiments and sensitivity analyses were conducted to analyze how changes in input parameters of model affect the optimal solutions. The last is the conclusion outlines future directions for research.

Keywords: Inventory management, Deterioration product, Replenishment policy, Customer purchasing behavior.

CONTENTS

	Page
AUTHOR'S DECLARATION	**ii**
ACKNOWLEDGMENTS	**iii**
ABSTRACT	**iv**
LIST OF TABLES	**vii**
LIST OF FIGURES	**viii**
LIST OF ABBREVIATIONS	**ix**
CHAPTER 1 INTRODUCTION	**1**
1.1 Background of the Study	1
1.2 Statement of the Problems	4
1.3 Objective of the Study	5
1.4 Scope and Limitations	5
CHAPTER 2 LITERATURE REVIEW	**6**
2.1 Inventory model for Deteriorating Products	6
2.1.1 Inventory Model with Stochastic Demand for Deteriorating Products	7
2.2 Inventory Replenishment Policy for Deteriorating Products	8
2.3 Inventory Policy with Customer Purchasing	9
CHAPTER 3 MODELFORMULATION	**17**
3.1 Assumption	17
3.2 Notations	17
3.3 Structure of Inventory	19
CHAPTER 4 NUMERICAL EXPERIMENTS AND SENSITIVITY ANALYSES	**26**
4.1 Numerical Experiments	26
4.2 Sensitivity Analysis	27
4.2.1 Effect of Customer's Demand Rate	27
4.2.2 Effect of the Time at which the Remaining Lifetime of the Product is Considered to be Short	31
4.2.3 Effect of Holding Cost per Unit per Time Unit	33

4.2.4 Effect of Unit Shortage Cost per Unit 34
CHAPTER 5 CONCLUSIONS AND RECOMMENDATIONS 35
5.1 Conclusions 35
5.2 Recommendations 36
REFERENCES 37
APPENDIX: MATLAB PROGRAMMING CODES 40

LIST OF TABLES

Tables		Page
Table 2.1	The Summarization of Past Research Works of Inventory Model for Deteriorating Products	11
Table 4.1	Input Parameters in the Base Case	26
Table 4.2	Optimal Solution	26
Table 4.3	Effect of Customer's Demand Rate	27
Table 4.4	Effect of the Time at which the Remaining Lifetime of the Product is Considered to be Short	31
Table 4.5	Effect of Holding Cost per Unit per Time Unit	33
Table 4.6	Effect of Unit Shortage Cost per Unit	34

LIST OF FIGURES

Figures		Page
Figure 3.1	Inventory Level for Case 1 with Positive Ending Inventory	21
Figure 3.2	Inventory Level for Case 1 with Negative Ending Inventory	22
Figure 3.3	Inventory Level for Case 2 with Negative Ending Inventory	24
Figure 4.1	The Total Cost for the Value of λ_1=140 to 180 and the Values λ_2=140	28
Figure 4.2	The Total Cost for the Value of λ_1=140 to 180 and the Values λ_2=150	29
Figure 4.3	The Total Cost for the Value of λ_1=140 to 180 and the Values λ_2=160	29
Figure 4.4	The Total Cost for the Value of λ_1=140 to 180 and the Values λ_2=170	30
Figure 4.5	The Total Cost for the Value of λ_1=140 to 180 and the Values λ_2=180	30
Figure 4.6	The Total Inventory Cost with the Time at which the Lifetime of the Product	32
Figure 4.7	The Total Inventory Cost with the Holding Cost	33
Figure 4.8	The Total Inventory Cost with the Shortage Cost	34

LIST OF ABBREVIATIONS

MAP	= Markovian customer Arrival Process
RFID	= Radio Frequency Identification
EOQ	= Economic Order Quantity
EPQ	= Extended Project Qualification
TTI	= Time to interactive
FIFO	= First in, First out
LIFO	= Last in, First out
MIP	= a Mixed-integer Programming

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

Organizations have tackled with inventory problems throughout their existence until analytical methods for examining these substances were established during the twentieth century. The beginning drives for analysis derived from the manufacturing industries. In a manufacturing industry, inventory is not only the final product that is manufactured and ready for sale but also includes raw materials, in-process goods, and finished products. The inventory exists because supply and demand differ in the rates, and hence, stocks are required. The target of inventory management is to maintain suitable quantities of raw materials, supplies, and finished goods in a low cost, the right place, and at the right time.

The costs of inventory occur from the decisions or exemption made by management in establishing the inventory system. The inventory system cost factors include the costs associated with Purchase cost, Order/setup cost, Holding cost, and Stockout cost. The primary objectives of materials management include minimizing inventory investment, maximizing customer service, and ensuring efficient plant operation. Subgoals often include achieving low unit costs, high inventory turnover, maintaining consistency in quality, fostering positive supplier relations, and ensuring a continuous and reliable supply.

Inventories play a crucial role in business, serving not only operational needs but also contributing significantly to customer satisfaction. Inventory decisions become particularly critical in-service organizations. For instance, hospitals stock a variety of drugs and blood supplies that might be required on short notice. However, many of these items have a limited shelf life, and maintaining large quantities could lead to the disposal of unused and costly supplies. Thus, the most important inventory functions are the following:

- *To fulfill expected customer demand.*

A customer can be an individual walking in off the street to purchase a new telephone, a mechanic seeking a tool from a tool barrel, or a part of a manufacturing operation. These inventories are commonly known as anticipation stocks, as they are kept in anticipation of satisfying anticipated demand.

- *To even out production requirements.*

Companies facing seasonal demand patterns often accumulate stocks during preseason periods in preparation for addressing heightened demand during the seasonal period. These inventories are appropriately called seasonal inventories.

- *To decouple operations.*

Companies have considered buffer inventories more closely, acknowledging the costs and area incurred. They recognize that identifying and mitigating sources of disruptions can significantly reduce the necessity for decoupling operations. Thorough analysis can identify optimal points for implementing buffers where they are most beneficial and points where they may increase costs without adding significant value.

- *To against the risk of stockouts.*

Delayed deliveries and unanticipated in demand elevated the risk of shortages. To mitigate this risk, companies often maintain safety stocks inventory levels exceeding maintain an inventory level based on median of the demand to account for variations in both demand and lead time.

- *To strength order cycles.*

Inventory keeping facilitates a company to gain and manufacture in economical lot sizes without the immediate necessity to make happen at the same time for purchases or production with short-term demand requirements by minimizing purchasing and inventory costs. This practice gives rise to periodic order cycles. In certain situations, it is practical or cost-effective to combine orders or place them at fixed intervals.

- *To against the price increases.*

A company may anticipate a significant rapid price increase and decide to purchase larger-than-normal quantities to prevent the increase and capitalize on potential price discounts offered for larger orders.

- *To make the most of quantity discounts.*

Suppliers may offer discounts for bulk orders.

The research work was inspired by local bakeries and groceries that sell a wide range of products. Each product has a different expiration date. Therefore, it is an interesting problem that should be studied because the manager faces decision-making challenges in its day-to-day operations. Any unsold items at the end of the day are sent back to the manufactory for displacement. To reduce the number of products disposed of, discounted prices every evening have occurred. Thus, the manager's consideration is to decide on determining the quantity to be sold at a discounted price, and adjudge the amount to order for the following day involves crucial decision making in the daily operations to reduce the cost of deteriorating products.

Deteriorating products create a dual concern, as they not only cause a risk to the security of those in need of distribution but also meet additional costs for proper disposal. Deterioration is characterized as any form of damage or spoilage usefulness that happens to products such as vegetables, drugs, blood, chemicals, and so forth. Due to the product's deterioration, the firm faces a constraint on storing it for an extended period. In addition, determining the quantity of products to be sold at a markdown price as the expiration date approaches lead to another challenge. The manager must carefully consider customer behavior when coordinating discount sales with deciding to order inventory. Understanding how customers respond to markdowns is crucial in optimizing inventory management strategies.

To study the topic, The complexity of the model is conditional on the assumptions that encompass demands, cost structures, and the physical attributes of the system. Initially, demand is classified into two types: deterministic demand and stochastic demand. Consideration of deterioration rate for perishable products add on complexity to the model. It becomes even more confused when addressing demand uncertainty. Sensitivity analyses can give a crucial role in providing reliable estimates when managers confront real world challenges.

1.2 Statement of the Problem

This research considers a stochastic inventory model of deteriorating products. Various industries are dealing with limited product life cycles, and they employ price markdowns to reduce losses from deteriorating goods. Examples of companies implementing this strategy include Publix, Walmart, and Foodstuffs. Bulk disposal of expired products showed the inefficient management of the industries. Moreover, making replenishment decisions for deteriorating items create challenges because of variability in customer demand. The unpredictability in demand expanded complexity to managing the inventory of perishable or deteriorating goods

The concern from many retailers is the use of clearance sales serves as a strategy to sell items approaching their expiration dates at a reduced price. It is crucial for retailers to determine the optimal timing for such sales. Proper timing ensures that the discounts attract customers effectively and help in minimizing losses from unsold perishable items. Furthermore, the retailer should understand the customer purchasing behavior to reduce the risk that may occur in the future.

Therefore, dealing with the inventory of deteriorated products is an interesting research direction. Moreover, none of the past research works considered simultaneously the inventory problem and consumer purchasing behavior of the firm for deterioration products. To deal with this problem, this research will classify customers into two groups. The first group includes of customers that are not concerned about the expiry date. This group prefers to buy lower price products, and they will buy the product at a higher price only when the product with a lower price does not exist. The second group consists of customers who care about the expiry date. This group will only buy the product with the highest remaining lifetime if it still exists in the store. Otherwise, they will not buy. To fulfill the above gap, this research also considers the fluctuation of demand under uncertainty in deriving a replenishment policy for deteriorating products. Due to uncertainties of customer demand for deteriorating products and limited product shelf life, markdown price will be considered when the product's remaining lifetime is nearly close.

1.3 Objective of the Study

This overarching goal is to expand an inventory model for deteriorating products under a replenishment policy considering customer purchasing behavior. The specific objectives are as follows:

- To develop an inventory model for deteriorating products under a fixed order quantity replenishment policy that shortages with be lost in order to minimize the total cost of the supply Chain.
- To determine the order quantity in an (T, S) policy for reducing the total cost of the system. *(The order to place in every T cycle length and the inventory level is the order-up-to level S)*
- To consider the consumers buying behavior under expiration date condition and price markdown condition.
- To develop obvious direction for store managers to control sales for remainder products that are near expiration.

1.4 Scope and Limitation

This work is to develop an inventory model under replenishment decisions for deterioration products considering stochastic demand. The research will conduct based on the following assumptions:

1. The demand is a random variable.
2. The product has a fixed expiry date.
3. The replenishment rate is assumed to be infinite.
4. The customers are classified into two groups. The first group will always buy the product regardless of whether the remaining lifetime is very short while the second group will buy only when the remaining lifetime of the product is larger than a predefined value.
5. When the remaining lifetime of the product is short, a markdown price will be applied.
6. The product does not have any salvage value at the end of its lifetime.

CHAPTER 2
LITERATURE REVIEW

This part includes the literature review which proposes the inventory model with stochastic demand for deterioration products and the deterioration inventory under replenishment policy. Moreover, many works of literature consider the inventory policy with customer purchasing behavior. However, previous research has not tackled these certain specific issues. To fill the gap, this research will study a stochastic inventory model and strategic customer buying behavior.

2.1 Inventory Model for Deteriorating Products

Inventory management problem is a big problem in many companies. The appropriate inventory control can reduce the overall cost of the system. Of course, every company desires to reduce costs as much as possible. Perishable inventory models are categorized depend on the product's lifetime and the demand type. (Bakker et al., 2012; Janssen et al., 2016, 2018) They are categorized based on inventory control methods, including periodic or continuous systems. (Tekin et al., 2001) In their investigation, they analyzed the formulations for the primary operational types of a perishable inventory model with lost sales, utilize within the suggested age-based policy. The study considered the responsiveness of optimum policy parameters and demonstrated that the age-based policy surpasses the inventory level strategy, especially in perishable inventory systems characterized by slow-moving items and high service levels. (Goyal & Giri, 2001) This model found in the pertinent literature have been systematically categorized determined by the shelf-life characteristic of the inventoried goods. Furthermore, these models have been sub-classified considering variations in demand and various other conditions or constraints. The literature also encompasses discussions on the stimulations, expansions, and generalizations of different models. (Li et al., 2010) Models designed for deteriorating products must effectively handle two crucial cases: the demand and rate of deterioration. Demand serves as a structural typical of the whole inventory system, while the deterioration rate help in characterizing managing inventoried products and determining the date of expiration. The price discounts, shortages, and the time cost of fund are central determinants that must take into consideration when dealing with perishable products inventory. (Duary et al., 2022) This work is motivated by highly competitive marketing situations. The researchers employed the GRG tools to resolve this issue with employed the suggested algorithm to find out optimal cost from an economic perspective. Additionally, they made modifications to a two-warehouse model mathematically, considering deteriorating products (Abdul Halim et al., 2021).

This work goals to establish a model for manufacture inventory that accounts for the process of overtime production, incorporating a nonlinear pricing structure and linear stock amount correlated with demand. The model allows the manufacturer to adjust the total cost effectively and is applicable to various manufacturing processes. Furthermore, it is extendable under the consideration of nonlinearity.

2.1.1 Inventory Model with Stochastic Demand for Deteriorating Products

Indeed, the stochastic demand model is a mathematical approach that incorporates probability to model uncertain factors. In this context, probability distributions are employed to represent the uncertainty associated with demand. This modeling technique recognizes that demand is subject to variability and uses probability theory to account for the range of possible outcomes, offering a more realistic study of the uncertainties physical in real-world scenarios. (Sivakumar, 2009) The study contributes to the understanding of how a deterioration inventory process with continuous review and the definite sources with demand, an (s, S) operating policy, and specific characteristics such as exponential demand distribution and inventory-dependent deterioration rate can be effectively managed and optimized. (Yadavalli et al., 2011) his research contributes to the understanding of managing perishable inventory within a continuous review system that involves a multi-server service facility, Customer arrival processes following a Markovian model, (s, S) ordering policies, and exponentially distributed lead times. The numerical illustrations enhance the applicability and interpretability of the proposed model's results. (Ozbay & Ozguven, 2007) the research contributes to the development of effective inventory management strategies, particularly in the context of emergency situations. The incorporation of perishability considerations and the utilization of RFID technology enhance the practical relevance of the model in real-world scenarios where timely and accurate resource tracking is crucial. (Janssen et al., 2018) studies mathematical models for deterioration goods and develops introducing the recent age-based inventory model with a near limitation day. The comprehensive model considers various aspects of perishable item inventory management, from capacity constraints to customer service goals, lead times, and dynamic demand patterns. Such a model is valuable for optimizing inventory decisions and ensuring efficient and effective management of perishable items in a dynamic and uncertain environment. (Chatwin, 2000) This work contributes to understanding how continuous-time inventory management, coupled with dynamic pricing strategies and Poisson demand modeling, can influence the retailer's decision-making process. Such models are valuable for businesses seeking to optimize pricing and inventory decisions in dynamic and rapidly changing market environments. (Tunc et al., 2014) By considering these elements, the study contributes to advancing the understanding of addressing the stochastic lot-sizing trouble under service level restrictions, providing a practical approach through mixed-integer reformulation. The focus on nonstationary solutions emphasizes the dynamic nature of demand, adding a realistic dimension to the optimization problem. (Kaya & Ghahroodi, 2018) Overall, this study advocates to the literature by addressing the complex cooperation between the decisions related to inventory management and pricing for perishable goods. The use of dynamic programming and numerical experiments enhances the practical applicability of the findings, providing insights that can inform decision-making strategies for businesses dealing with perishable products.

2.2 Inventory Replenishment Policy for Deteriorating Products

The replenishing deteriorating products involves a multifaceted approach that considers demand uncertainties, limited shelf life, pricing dynamics, and supply chain uncertainties. Implementing advanced technologies and data-driven strategies can help organizations navigate these challenges more effectively. (Lin & Hung, 2015) the research contributes insights into how a retailer in a single echelon supply chain can strategically manage disruptions by adopting a dual-sourcing policy. The recommendations and mathematical models provide a basis for decision making in the confront of uncertainties and risks for the supply chain. (Haijema, 2013) the statement underscores the complexity and scarcity of models that comprehensively address the intricacies of managing perishable products with specific characteristics, while emphasizing the pivotal role of the customer service level in the factors of the food industry and grocery store. The acknowledgment sets the stage for the importance of developing or finding models that can effectively balance these challenges to optimize performance in these industries. (Ferguson & Ketzenberg, 2006) this exploration underscores the role of data, specifically information about product age, in optimizing the supply chain operations of a retailer dealing with perishable goods from a single supplier. It recognizes the potential benefits of data sharing and seeks to determine the conditions that maximize these benefits. (Dai et al., 2017) the focus of these inventory models is on effectively managing inventory across different demand patterns within a supply chain. The adoption of a centralized policy indicates a coordinated approach to decision-making, and the overarching objective is to decrease the mean overall cost of inventory across all echelons within the system. Each form is tailored to address specific characteristics with ramp-type, invert ramp-type, and trapezoidal-type demand patterns.

2.3 Inventory Policy with Customer Purchasing

Strategic customer behavior is a subject of research that involves analyzing how consumers strategically act in various scenarios. The emphasis is notably on waiting for price discounts, especially those occurring at the end of the day, with comparatively less research on consumer forward-buying behavior. Understanding these behaviors is crucial for businesses to adapt their pricing strategies and meet customer expectations. (Talluri & van Ryzin, 2004) They addressed diversion as customer choice behavior. (Gallego et al., 2020) The statement highlights the focus of research on strategic customer behavior and its evaluation in different situations, with particular emphasis on two main scenarios: waiting for price discounts and forward-buying behavior, the research addresses the complex problem of allocating capacity to sell products at different prices, considering the impact on consumer choices. The extension to a network of resources and the optimization approach through deterministic linear programming are key components. The complexity of the linear program varies based on whether demand is independent or driven by a customer choice model, with the latter posing computational challenges. (Shen & Su, 2007) The statement underscores a critical consideration in modeling customer behavior, highlighting the assumption that customers are often portrayed as not employing decision-making processes in certain modeling approaches. It emphasizes that in reality, customers actively assess alternatives and make choices in various aspects such as pricing, product selection, and timing of purchases. The concept of diversion is mentioned as a primary consideration in modeling customer decision processes, the statement emphasizes the need to move beyond assumptions that customers do not actively engage in decision-making processes. Instead, it suggests that modeling efforts should recognize and incorporate the reality that customers assess alternatives and make choices actively. The concept of diversion is mentioned as a key consideration in this modeling approach, and attention is drawn to understanding the dynamics of customer decision processes. (Anderson & Wilson, 2003) they examination in strategic customer behavior in the context of airline seat pricing explores how customers react to variations in fares for the same seat on comparable flights at different times. This analysis contributes to understanding customer decision-making in response to dynamic pricing strategies implemented by airlines. (Kuksov & Wang, 2014) the research indicates that understanding and incorporating loss-averse behavior into pricing strategies can lead to insights into higher prices and profits, particularly when considering factors such as consumer estimates, search costs, and the impact of price promotions on future profits. This provides valuable information for businesses aiming to optimize their pricing strategies based on consumer behavior and long-term profitability considerations.

In their past literature review, none of these works consider the stochastic demand for deteriorated products and classified customer buying behavior by considering the price and expiration date condition. Therefore, the research will consider these points to the perfectly of our knowledge. Most of the companies have aimed to minimize the supply chain cost. Thus, sensitive analyses will show the reader understands or can apply it to future problems. Moreover, the summarization of past research works is assigned in Table 2.1

Table 2.1

The Summarization of Past Research Works of Inventory Model for Deteriorating Products

Authors	Research topic	Shortages	Demand	Deteriorating product	Consider customer purchasing	Model /Policy	Software
(Abdul Halim et al., 2021)	Developing an inventory model for overtime manufacturing, focusing on perishable products with nonlinear price and stock-dependent demand.	Not allowed due to over time product	Determinants of demand	✓		An overtime production inventory model	lingo software
(Anderson & Wilson, 2003)	Analyzing the Strategic Consumer: Implications for Pricing and Profit				✓	Prices using yield-management	
(Bakker et al., 2012)	Review of Deteriorating Inventory Systems: A Comprehensive Analysis Since 2001	Allowing shortages and backordering	Stochastic demand	✓		EOQ model	
(Chatwin, 2000)	Optimum dynamic pricing strategy for deterioration goods under stochastic demand, utilizing a fixed set of costs.	Allowing backordering	Stochastic demand	✓		A continuous-time dynamic programming model	

Authors	Research topic	Shortages	Demand	Deteriorating product	Consider customer purchasing	Model /Policy	Software
(Dai et al., 2017)	Optimal Multi-Echelon Inventory with Three Types of Demand in the Supply Chain	Shortages and backordering are allowed	Stochastic demand			Multi-echelon inventory models	GA, Lingo, and Baron
(Duary et al., 2022)	Integrating upfront and deferred payment considerations into the price-discount inventory model for perishable products while operating under capacity constraints	Backlogged shortage is allowed	Stochastic demand	✓		Economic Order Quantity (EOQ) model	GRG technique
(Ferguson & Ketzenberg , 2006)	Enhancing Retail Product Freshness of Perishables through Information Sharing	Allowing shortages	Stochastic demand	✓	✓	The value of information (VOI) (replenishmen t policies)	RFID
Gallego et al., 2020)	Optimizing the Management of Flexible Products within a Network		Determinants of demand		✓	A consumer choice model	Column-generation

Authors	Research topic	Shortages	Demand	Deteriorating product	Consider customer purchasing	Model /Policy	Software
(Goyal & Giri, 2001)	Current Trends in Modeling Deteriorating Inventory processes	Allowing shortages and backordering	Determinants demand and stochastic demand	✓		a Markov processes (EOQ, EPQ, etc.)	
(Haijema, 2013)	Suggesting a category of ordering policies dependent on stock levels designed for perishable items with a brief maximum shelf life.	Allowing shortages	Stochastic demand	✓		Markov decision problem (MDP)	Dynamic programm ing
(Janssen et al., 2016)	Comprehensive of Deteriorating Inventory Models by major headings from 2012 to 2015		Determinants demand and Stochastic demand	✓		Deteriorating inventory (FIFO, LIFO, etc.)	RFID, TTI
(Janssen et al., 2018)	Developing and simulating a deteriorating inventory model incorporating a constraint on near days under non fixed stochastic demand.	Allowing shortages	Stochastic demand	✓	✓	FIFO and LIFO	IBM ILOG Solver

Authors	Research topic	Shortages	Demand	Deteriorating product	Consider customer purchasing	Model /Policy	Software
(Kaya & Ghahroodi, 2018)	Managing Inventory and formulating costing strategies for deterioration items considering age and price-dependent		Stochastic demand	✓		Quantity-based and age-based policies	Dynamic programming
(Kuksov & Wang, 2014)	Exploring the impact of loss aversion in dynamic and competitive market environments.		Determinants of demand		✓	A consumer-choice model	
(Li et al., 2010)	Reviewing Studies on Deteriorating Inventory	Allowing shortages	Deterministic and stochastic demand	✓		The deteriorating inventory studies	
(Lin & Hung, 2015)	Examining a dual sourcing policy within a two-sided disruptive supply chain		Stochastic demand			a dual-sourcing policy	

Authors	Research topic	Shortages	Demand	Deteriorating product	Consider customer purchasing	Model /Policy	Software
(Ozbay & Ozguven, 2007)	Developing a stochastic humanitarian inventory control model to enhance disaster planning strategies.	Allowing shortages	Stochastic demand	✓		Replenishment policy	
(Shen & Su, 2007)	Reviewing Customer Behavior Modeling in Revenue Management and Auctions: Identifying New Research Opportunities	Allowing shortages and backordering	Stochastic demand	✓	✓	Consumer strategic behavior and reference price effect	MATLAB program
Sivakumar, 2009)	A perishable inventory system with retrial demands and a finite population	Allowing shortages and backordering	Stochastic demand	✓		Continuous review (s, S) Policy	MATLAB program
(Talluri & van Ryzin, 2004)	Optimizing Revenue Management in the Presence of a comprehensive discrete choice model capturing various aspects of consumer behavior	Allowing shortages	Determinants of demand		✓	Customer choice behavior	Dynamic program

Authors	Research topic	Shortages	Demand	Deteriorating product	Consider customer purchasing	Model /Policy	Software
(Tekin et al., 2001)	Contrasting age-based and stock-level control policies within perishable inventory systems	Allowing shortages	Stochastic demand	✓		a Poisson process, (Q,r,T) policy	MATLAB program
(Tunc et al., 2014)).	Redefining the stochastic lot sizing problem by incorporating service-level constraints	Allowing shortages	Stochastic demand		✓	The stochastic lot-sizing	a MIP solver
(Yadavalli et al., 2011)	Developing a model for a perishable inventory system with multiple servers, considering negative customer feedback.	Allowing shortages and backordering	Stochastic demand	✓	✓	a Markovian arrival process (MAP)	

CHAPTER 3
MODEL FORMULATION

The main aims of inventory management are to specify (1) when the items should be ordered and (2) how large the order should be. Practical inventory replenishment ensures that order fill rates can be achieved target while simultaneously managing and minimizing inventory carrying costs. Moreover, the deterioration of products also has an impact the customer's decision on whether to choose quality products at high price or lower price products which are near expiry date. The decline in the freshness of the product is an important determinant of demand. This chapter, we built a mathematical model to assist optimize the total cost of the system. The mathematical inventory model is established within the following assumptions and notation.

3.1 Assumptions

The following assumptions are used for developing the mathematical model in this research:

1. The inventory system is reviewed at fixed interval and appropriate order quantity will be placed in each cycle to raise the inventory level to a fixed amount.
2. Customers' demands of the two groups are random variables that does not depend on price. It is also assumed that the demands of the two customer groups follow Poisson processes.
3. The product has fixed expiry date.
4. Lead time is ignored.
5. Shortage is lost.

3.2 Notations

The purpose of this work is to meet the most favorable replenishment policy for the inventory management model of the deteriorating product. These products have a random lifetime, and the replenishment order is received immediately. The inventory position is checked and replenished at each T units of time and a replenishment quantity is ordered to bring the inventory position to a given level S. Let λ_i $(i = 1,2)$ represents the demand rates of customers in two types. The first group will always buy the product regardless of whether the remaining lifetime is very short while the second group will buy only when the remaining lifetime of the product is larger than a predefined value. The following notations are used in this research.

Summary of notations

λ_T	The total customer demand rate
λ_1	Type I customer's demand rate
λ_2	Type II customer's demand rate
S	Base stock level
T	Cycle length which is also the lifetime of the product
τ	The time at which the lifetime of the product is considered to be short
h	Holding cost per unit per time unit
π	Unit shortage cost per unit
TC	Total cost per unit of time
OH	On hand inventory
$E[I]$	The expected amount of time weighted holding inventory per cycle
$E[B]$	The expected amount of demand lost per cycle
$E[EH]$	The expected ending inventory level at time T
p^p	The probability of positive ending inventory at time T

3.3 Structure of Inventory

The characteristic behavior of the inventory function during a given cycle is separated into two intervals. The first interval is the period $[\ 0, \tau\ ]$when the product is considered to be "new". The second interval is the period $[\ \tau, T\ ]$ where the product is considered to be "old". In the first interval the total demand rate is $\lambda_T = \lambda_1 + \lambda_2$ while in the second interval it is λ_1 because the customer who cares about the lifetime of products will not buy in this interval. The on-hand inventory levels have two possible cases at the time τ, one of these cases is the case that the stock level is greater than or equal to zero, and another one is the stock level is less than zero. The first case is also separated into two subcases including positive expected ending inventory and negative expected ending inventory at time T denoted by cases 1.1 and 1.2 respectively. The mathematical derivations of the above discussed cases/subcases are presented below.

***Case 1*: when $OH \geq 0$ at time τ**

For ***case 1***, the occurrence probability for this case is given by

$$p_1 = P\{D(\tau)\ \leq\ S\}$$

In which $D(\tau)$ is the total demand during time period τ which follows Poisson distribution

with rate $\lambda_T = \lambda_1 + \lambda_2$

Therefore:

$$p_1 = P\{D(\tau)\ \leq\ S\}$$

$$= \sum_{i=0}^{S} P\{D(\tau) = i\}$$

$$= \sum_{i=0}^{S} e^{-\lambda_T \tau} * \ \frac{(\lambda_T \tau)^i}{i!}$$

***Case 2*: when $OH < 0$ at time τ**

For ***case 2***, the occurrence probability for this case is given by

$$p_2 = P\{D(\tau) > S\}$$
$$= 1 - p_1$$

Let consider ***case 1***; In this case, there are 2 subcases that can occur:

Case 1.1: when $EH \geq 0$ at time T

and ***Case 1.2***: when $EH < 0$ at time T

It is also assumed that the time t_1 be the time to consume the available on hand inventory at time τ.

Then, there will be two scenarios

$$t_1 \geq T - \tau \quad \text{(\textbf{\textit{Case 1.1}})}$$

$$t_1 < T - \tau \quad \text{(\textbf{\textit{Case 1.2}})}$$

Or equivalently:

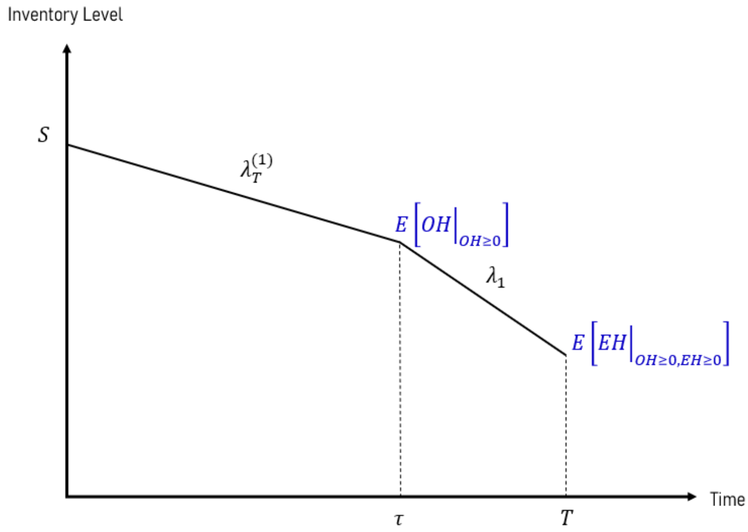
$$E[OH|_{OH\geq 0}] \geq \lambda_1(T - \tau) \quad \text{(\textbf{\textit{Case 1.1}})}$$

$$E[OH|_{OH\geq 0}] < \lambda_1(T - \tau) \quad \text{(\textbf{\textit{Case 1.2}})}$$

Case 1.1*: when $EH \geq 0$ at time *T

Figure 3.1

Inventory Level for Case 1 with Positive Ending Inventory



For ***case 1.1***, the expected inventory level at τ is

$$E[OH|_{OH\geq 0}] = \sum_{i=0}^{S}\left(S - D(\tau)\right) * \frac{P\{D(\tau) = i\}}{p_1}$$

$$= \frac{\sum_{i=0}^{S}(S - i) * \frac{e^{-\lambda_T \tau}(\lambda_T \tau)^i}{i!}}{p_1}$$

The conditional total demand rate can be determined as:

$$\lambda_T^{(1)} = \frac{S - E[OH|_{OH\geq 0}]}{\tau}$$

The expected ending inventory level at T can be expressed as:

$$E\left[EH|_{OH\geq 0,EH\geq 0}\right] = E[OH|_{OH\geq 0}] - \lambda_1(T - \tau)$$

The inventory holding cost per cycle will be

$$E\left[HC|_{OH\geq0,EH\geq0}\right] = h\left\{\begin{array}{c}\frac{S+E[OH|_{OH\geq0}]}{2} * \tau + \\ \frac{E[OH|_{OH\geq0}]+E[OH|_{OH\geq0}]-\lambda_1(T-\tau)}{2} * (T-\tau)\end{array}\right\} \quad \textbf{(1)}$$

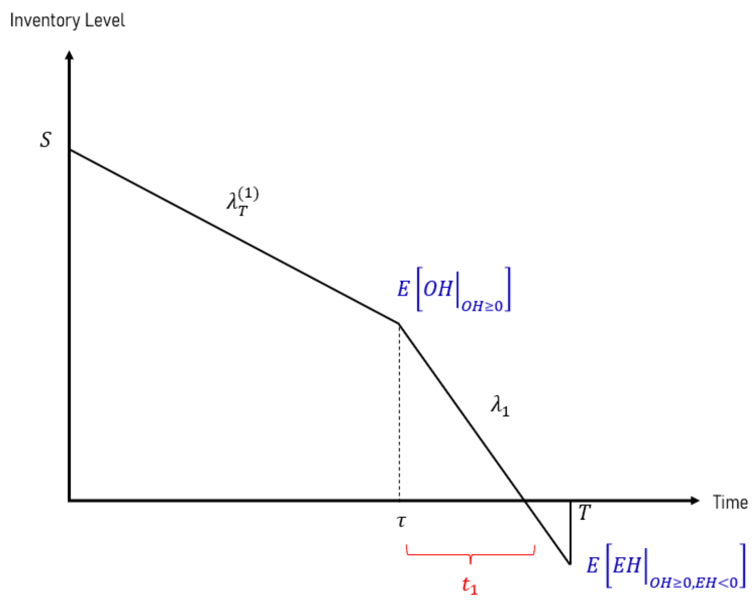
The shortage cost per cycle can be determined as:

$$E\left[SC|_{OH\geq0,EH\geq0}\right] = 0$$

<u>Case 1.2</u>: when EH < 0 at time T

Figure 3.2

Inventory Level for Case 1 with Negative Ending Inventory



For case 1.2, the time duration to consume $E[OH|_{OH\geq0}]$ is

$$t_1 = \frac{E[OH|_{OH\geq0}]}{\lambda_1}$$

The expected ending inventory at T can be expressed as:

$$E\left[EH|_{OH\geq0,EH<0}\right] = E[OH|_{OH\geq0}] - \lambda_1(T-\tau) < 0$$

The inventory holding per cycle will be

$$E\left[HC|_{OH\geq 0,EH<0}\right] = h\left\{\frac{S+E[OH|_{OH\geq 0}]}{2} * \tau + \frac{E[OH|_{OH\geq 0}+0]}{2} * t_1\right\} \quad \textbf{(2)}$$

It is noted that

The component $E[OH|_{OH\geq 0}] - \lambda_1(T-\tau)$ in $(\mathbf{1})$ and 0 in $(\mathbf{2})$ can be written as:

$$Max\{E[OH|_{OH\geq 0}] - \lambda_1(T-\tau), 0\}$$

Furthermore, the component $(T-\tau)$ in **(1)** and t_1 in **(2)** can be written as:

$$Min\,\{T-\tau, t_1\} \;=\; Min\,\{T-\tau, \frac{E[OH|_{OH\geq 0}]}{\lambda_1}\}$$

The shortage cost per cycle can also be determined as:

$$E\left[SC|_{OH\geq 0,EH<0}\right] = \pi * Max[\lambda_1(T-\tau) - E[OH|_{OH\geq 0}], 0]$$

Thus, the expected holding cost and shortage cost per cycle for ***case 1*** can be derived as:

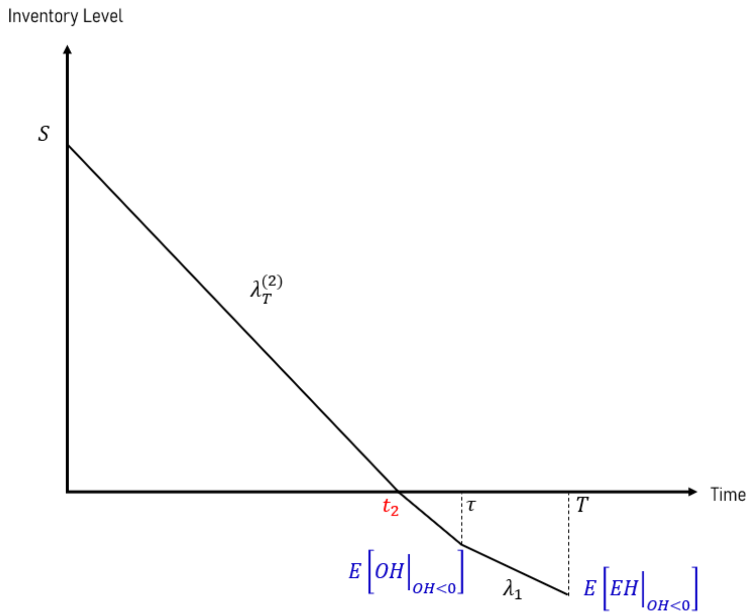
$$E[HC|_{OH\geq 0}] = h\left\{\frac{S+E[OH|_{OH\geq 0}]}{2} * \tau + \frac{E[OH|_{OH\geq 0}] + Max\{E[OH|_{OH\geq 0}] - \lambda_1(T-\tau), 0\}}{2}\right\} * Min\left(T-\tau, \frac{E[OH|_{OH\geq 0}]}{\lambda_1}\right)$$

and $$E[SC|_{OH\geq 0}] = \pi * Max\{\lambda_1(T-\tau) - E[OH|_{OH\geq 0}], 0\}$$

***<u>Case 2</u>*: when $OH < 0$ at time τ**

Figure 3.3

Inventory Level for Case 2 with Negative Ending Inventory



For case 2, the expected inventory level at τ can be defined as follows:

$$E[OH|_{OH<0}] = \sum_{i=s+1}^{\infty} \left(S - D(\tau)\right) \frac{P\{D(\tau) = i\}}{p_2}$$

$$= \sum_{i=s+1}^{\infty} (S - i) \frac{e^{-\lambda_T \tau} * \frac{(\lambda_T \tau)^i}{i!}}{p_2} \quad (< 0)$$

The conditional total demand rate can be determined as:

$$\lambda_T^{(2)} = \frac{S - E[OH|_{OH<0}]}{\tau}$$

The expected ending inventory at T can be expressed as:

$$E[EH|_{OH<0}] = E[OH|_{OH<0}] - \lambda_1(T - \tau) \quad (< 0)$$

The expected time to consume S is

$$t_2 = \frac{S}{\lambda_T^{(2)}} = \frac{S\tau}{S - E[OH|_{OH<0}]}$$

The expected holding cost and shortage cost per cycle can be defined as follow:

$$E[HC|_{OH<0}] = h * \frac{S * t_2}{2} = h\left(\frac{S * \left(\frac{S\tau}{S - E[OH|_{OH<0}]}\right)}{2}\right)$$

and

$$E[SC|_{OH<0}] = -\pi * E[EH|_{OH<0}]$$

$$= \pi * \left[\lambda_1(T - \tau) - E[OH|_{OH<0}]\right]$$

Hence; the total inventory cost per time unit will be

$$E[TC] = \frac{1}{T}\left\{p_1 * \left[E[HC|_{OH\geq 0}] + E[SC|_{OH\geq 0}] + p_2 * \left[E[HC|_{OH<0}] + E[SC|_{OH<0}]\right]\right]\right\}$$

CHAPTER 4
NUMERICAL EXPERIMENTS AND SENSITIVITY ANALYSES

Numerical experiments are conducted to present the applicability of the developed mathematical models in Chapter 3. furthermore, sensitivity analyses are conducted to investigate the results of input parameters on the solution. MATLAB software is created to determine the values that optimize of the decision variables, such as base stock level (S) and cycle length (T) to reduce the expected total cost per unit of time.

4.1 Numerical Experiments

The input parameters are used for the base case are shown in the following Table 4.1

Table 4.1

Input Parameters in the Base Case

Parameters	*Value*
Type $\boldsymbol{I}$ customer's demand rate ($\boldsymbol{\lambda_1}$) (units)	150
Type $\boldsymbol{II}$ customer's demand rate ($\boldsymbol{\lambda_2}$) (units)	170
The time at which the lifetime of the product is considered short ($\boldsymbol{\tau}$)	1.5
Holding cost ($\boldsymbol{h}$) (per unit per time unit)	0.02
Unit shortage cost ($\boldsymbol{\pi}$) (฿ per units)	15

To find the solution, the function fmincon in MATLAB is utilized for addressing this nonlinear optimization problem. The outcomes are presented in the following Table 4.2.

Table 4.2

Optimal Solution

S	T	TC
981 units	3 days	8565.4 ฿

4.2 Sensitivity Analysis

Sensitivity analyses are conducted to consider the behavior of the model. All input parameters are diverse one by one at a time while other input parameters remain unvaried. The detailed resultants are proposed in the following subsections.

4.2.1 Effect of Customer's Demand Rate

In this part investigates the value of $Type\ I$ Customer's demand rate (λ_1) and $Type\ II$ Customer's demand rate (λ_2) which are modified from 140 to 180, while other input parameters from the base case are kept unvaried. The base stock level (S), cycle length (T) and total cost (TC) are represented in the following table.

Table 4.3

Effect of Customer's Demand Rate

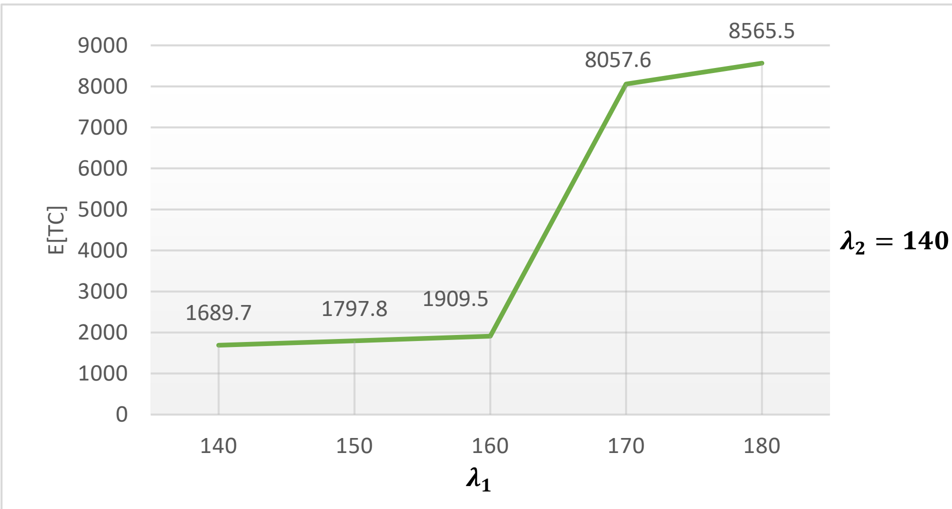
λ_1	λ_2	S	T	TC
140	140	607	2	1689.7
	150	627	2	1797.8
	160	647	2	1909.5
	170	952	3	8057.6
	180	981	3	8565.5
150	140	627	2	1797.8
	150	922	3	7565.2
	160	952	3	8057.5
	170	981	3	8565.4
	180	1335	4	19779
160	140	923	3	7565.1
	150	952	3	8057.4
	160	981	3	8565.4
	170	1011	3	9088.9
	180	1375	4	20965

λ_1	λ_2	S	T	TC
170	140	952	3	8057.4
	150	981	3	8565.3
	160	1011	3	9088.8
	170	1375	4	20965
	180	1415	4	22186
180	140	981	3	8565.2
	150	1011	3	9088.8
	160	1375	4	20965
	170	1415	4	22186
	180	1454	4	23442

Total inventory cost with the horizontal axis will present the value of $\lambda_1 = 140$ to180 and the line will show the values of total cost function when $\lambda_2 = 140$ is shown in the following Figure 4.1

Figure 4.1

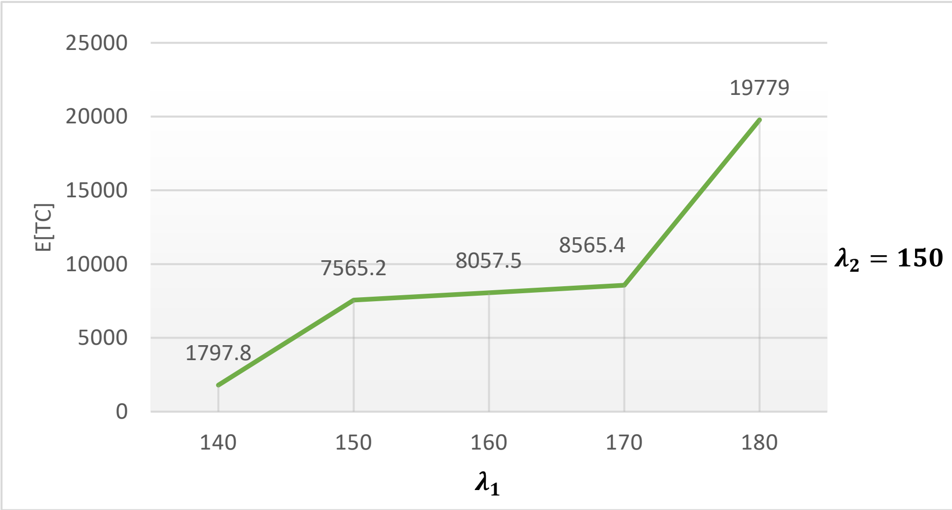
The Total Cost for the Value of λ_1=140 to 180 and the Values λ_2=140



Total inventory cost with the horizontal axis will present the value of $\lambda_1 = 140$ to180 and the line will show the values of total cost function when $\lambda_2 = 150$ is shown in the following Figure 4.2

Figure 4.2

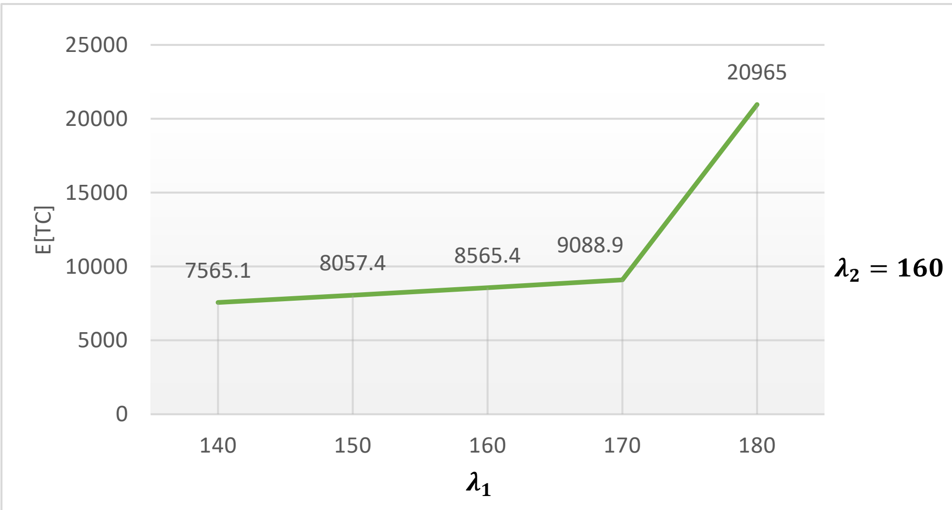
The Total Cost for the Value of λ_1*=140 to 180 and the Values* λ_2*=150*



Total inventory cost with the horizontal axis will present the value of $\lambda_1 = 140$ to180 and the line will show the values of total cost function when $\lambda_2 = 160$ is shown in the following Figure 4.3

Figure 4.3

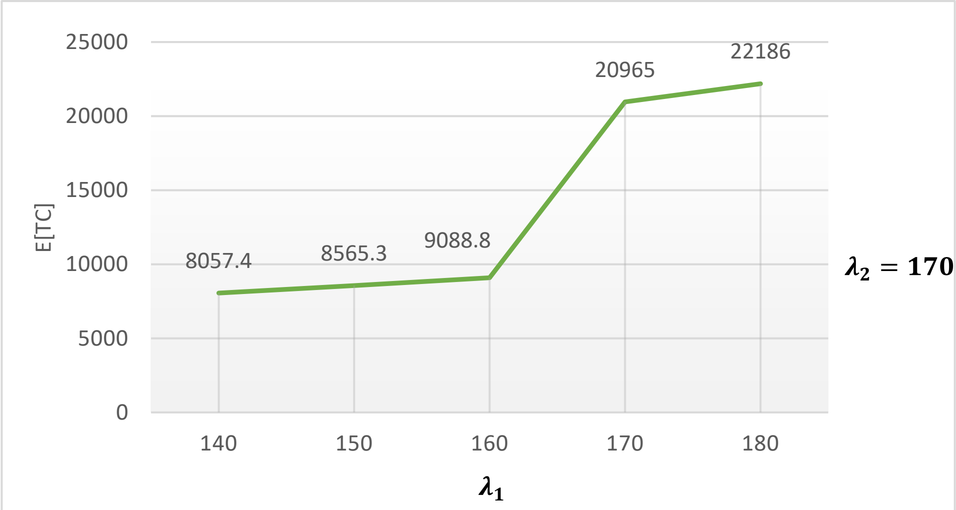
The Total Cost for the Value of λ_1*=140 to 180 and the Values* λ_2*=160*



Total inventory cost with the horizontal axis will present the value of $\lambda_1 = 140$ to180 and the line will show the values of total cost function when $\lambda_2 = 170$ is shown in the following Figure 4.4

Figure 4.4

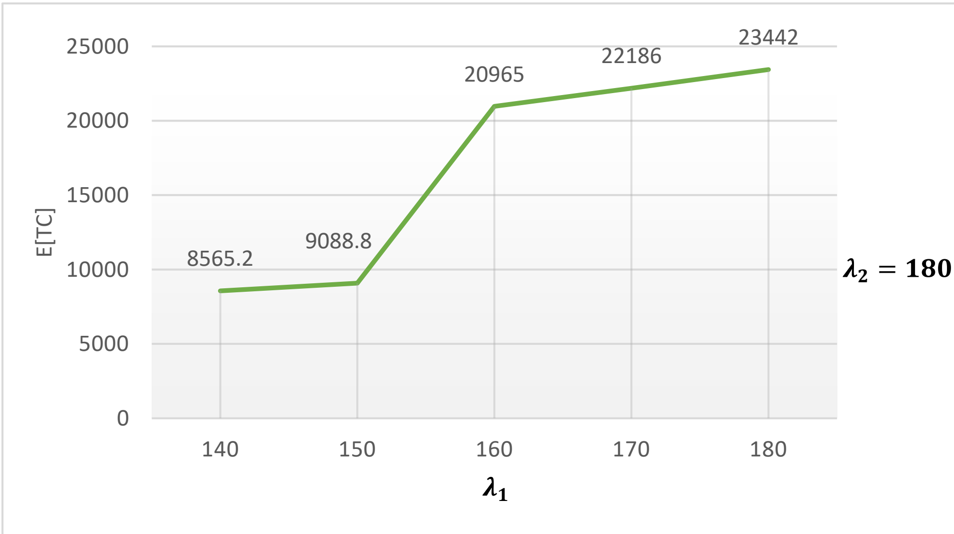
The Total Cost for the Value of λ_1=140 to 180 and the Values λ_2=170



Total inventory cost with the horizontal axis will present the value of $\lambda_1 = 140$ to180 and the line will show the values of total cost function when $\lambda_2 = 180$ is shown in the following Figure 4.5

Figure 4.5

The Total Cost for the Value of λ_1=140 to 180 and the Values λ_2=180



Following the result demonstrated in Table 4.1 and Figure 4.1 to Figure 4.5, when the demand rate of *Type I* customer (λ_1) or and *Type II* customer (λ_2) increase the base stock level of the cycle will increase. This happens since of the increasing movement in the cycle length generates an increase in the base stock level of the inventory, and hence, the total cost will increase. The above trends are due to the truth to escape too high a shortage cost, we should increase the base stock level to cope with the higher demand.

4.2.2 Effect of the Time at which the Remaining Lifetime of the Product is Considered to be Short

In this part, the value of the time at which the lifetime of the product is considered to be short and changed from 1 to 2, while keeping other input parameters identical to the base case. The base stock level (S) ,cycle length (T), and total cost (TC) are proposed in the following table.

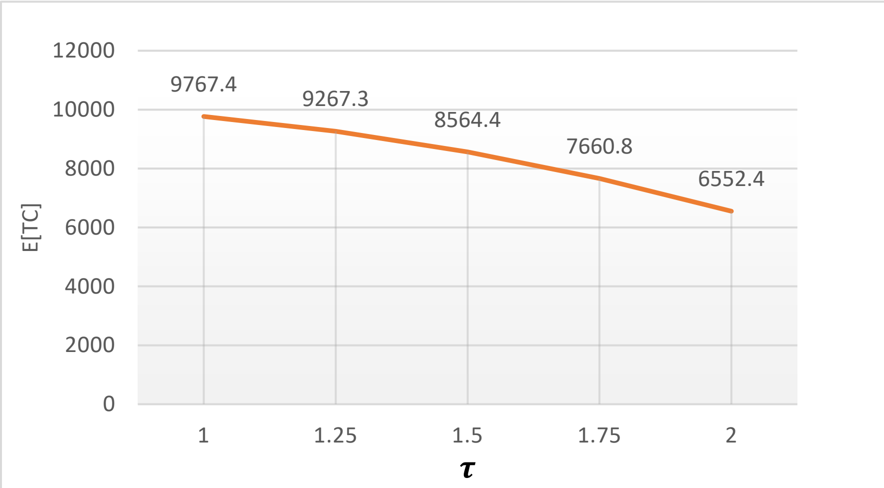
Table 4.4

Effect of the Time at which the Remaining Lifetime of the Product is Considered to be Short

τ	S	T	TC
1	981	3	9767.4
1.25	981	3	9267.3
1.5	981	3	8564.4
1.75	982	3	7660.8
2	983	3	6552.4

Figure 4.6

The Total Inventory Cost with the Time at which the Lifetime of the Product



Following the result demonstrated in Table 4.4 and Figure 4.6, it is shown that when τ increases, the cycle length is not affected but the base stock level (S) can slightly increase to prevent shortages. The total cost function will decrease. This trend is reasonable the increase in τ means the higher demand in the first interval before the end of the cycle length, and the total cost will show a decreased movement.

4.2.3 Effect of Holding Cost per Unit per Time Unit

In this part, the value of the holding cost is changed from 0.01 to 0.03 per unit per time unit, while other input parameters are identical to the base case. The base stock level (S), cycle length (T), and total cost (TC) are represented in the following table.

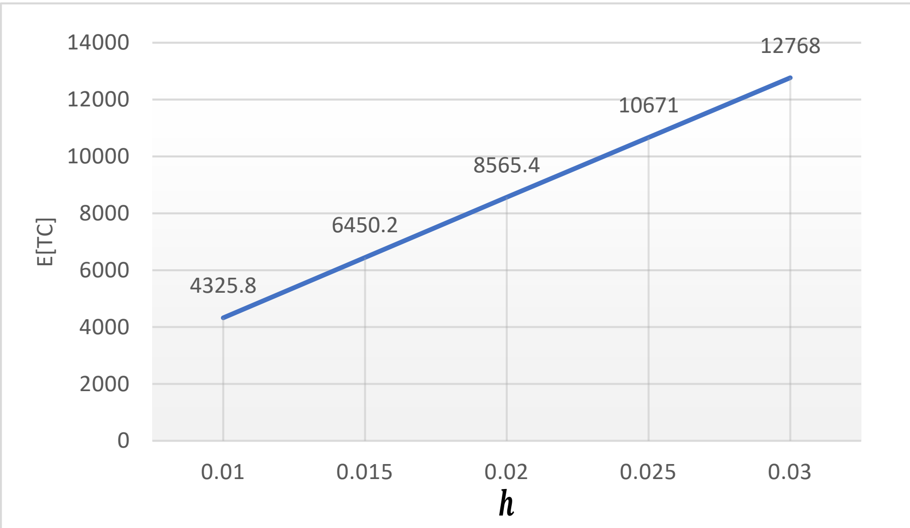
Table 4.5

Effect of Holding Cost per Unit per Time Unit

h	S	T	TC
0.01	986	3	4325.8
0.015	984	3	6450.2
0.02	981	3	8565.4
0.025	979	3	10671
0.03	977	3	12768

Figure 4.7

Total Inventory Cost with the Holding Cost



Following the result given in Table 4.5 and Figure 4.7, it is noted that when the holding cost increases, the total inventory cost of the cycle increases likewise. In addition, it can be remarked that the increase in holding costs will discourage holding inventory from decreasing trend of the base stock level. This is to escape too high inventory holding costs. However, the cycle length will not be affected.

4.2.4 Effect of Unit Shortage Cost per Unit

In this part, the value of unit shortage is changed from 10 to 19 per unit, while keeping other input parameters identical to the base case. The base stock level (S),cycle length (T), and total cost (TC) are represented in the following table.

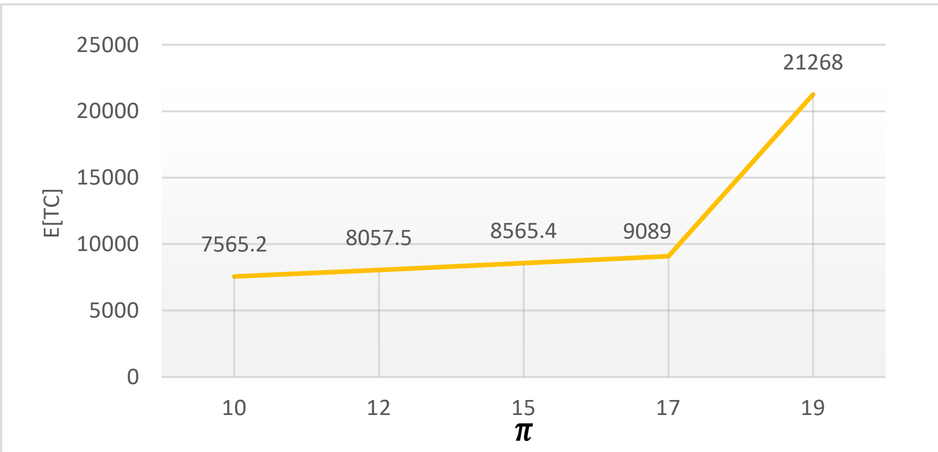
Table 4.6

Effect of Unit Shortage Cost per Unit

π	S	T	TC
10	922	3	7565.2
12	952	3	8057.5
15	981	3	8565.4
17	1011	3	9089.0
19	1385	4	21268

Figure 4.8

The Total Inventory Cost with the Shortage Cost



Following the result given in Table 4.6 and Figure 4.8, it is noted that when the shortage cost increases, the base stock level will increase. The truth is that we should increase the base stock level to escape too high shortage cost. Related to the total cost, it can be remarked that the total cost will increase due to the increase in the shortage cost. The cycle length can be slightly increased also when the shortage cost is too high.

CHAPTER 5
CONCLUSIONS AND RECOMMENDATIONS

The major goals of many companies have aimed to minimize inventory investment, maximize customer service, and ensure efficient plant operation. The deteriorating products become a significant factor in controlling inventory costs. The decline in the freshness of the product is an important determinant of demand because it also impacts the customer's decision. Therefore, dealing with the inventory of deteriorated products and understanding customer purchasing behavior are interesting research directions. The execution of this work is the development of an inventory model for deteriorating products under a replenishment policy considering customer purchasing behavior. Customers' demands are considered for in which the first group choose quality products at high price and the second group can buy lower price products which are near expiry date and it is also assumed that the demands of the two customer groups follow Poisson processes. The developed mathematical model was considered to define the optimal solutions of two decision variables, i.e., Cycle length which is also the lifetime of the product (T), and base stock level (S). According to the results, the following interesting characteristics of the optimal solutions have been noted.

- The optimal base stock level is affected by the customer's demand rate, the time at which the remaining lifetime of the product is considered to be short, the holding cost and the shortage cost. The optimal base stock level tends rise in response to the increase of the above input parameters.

- The optimal cycle length is affected by the customer's demand rate and the shortage cost. The optimal cycle length tends to rise in response to the increase of the above input parameters excluding the time at which the remaining lifetime of the product is considered to be short and the holding cost are not be affected.

5.1 Recommendations

To guide future research, several potential extensions can be explored, including the following.

- This research considers the inventory model with stochastic demand of known distribution. For the future study, this can be continued to consider uncertain demand where the distribution of the demand is unknown.
- Other types of deteriorating products such as bakery, dairy, health and fresh products can be considered. For these products, the deterioration rate will increase with time.
- Additional considerations, such as constraints related to warehouse space, pricing and promotion campaign, etc. can be integrated into next research.

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APPENDIX

MATLAB PROGRAMMING CODES

```
%unknown factor
 % TC "total cost"
 % S "base stock level"
 % T "time at which the lifetime product"
   %define x(1,2,3) to use optimize to find TC,S,T
 % x [1,2]
 % x(1) = S
 % x(2) = T

 function ETC = TestObjFun1(x)

   %define parameters
   tau = 1.5;
   lamda1 = 150;
   lamda2 = 170;
   h = 0.02;
   pii = 15;
   lamdaT = lamda1+lamda2;

   %case1 : when OH >= 0
   %define
       p1=0;

   for i=0:x(1)

       p1=p1+((exp(-lamdaT*tau))*(((lamdaT*tau)^i)/gamma(i+1)));

   end

   p2 = 1-p1;

       %############# Case 1.1 ###############

   % define for (below is used to represent summation of 0 to S
   for EOH_case1)

   sum1 = 0;
   sum2 = 0;

       for k = 0:x(1)

           sum1=sum1+(x(1)-k);
           sum2=sum2+(exp(-
   lamdaT*tau)*(((lamdaT*tau)^k))/gamma(k+1));

       end

       sum3=sum1*sum2;


   EOH_case1 = sum3/p1;
```

```
lamdaT1 = (x(1)-EOH_case1)/tau;

EEH11 = EOH_case1-(lamda1*(x(2)-tau));

EHC11 = h*((((x(1)+EOH_case1)/2)*tau)+ ((EOH_case1+EOH_case1-
(lamda1*(x(2)-tau)))/2)*(x(2)-tau));

ESC11 = 0;

    %############# Case 1.2 ###############

    %define t1 " is the time duration to consume "

t1 = EOH_case1/lamda1;

EEH12 = EOH_case1-(lamda1*(x(2)-tau));

EHC12 = h*((((x(1)+EOH_case1)/2)*tau)+ (((EOH_case1+0)/2)*t1)
);

ESC12 = pii*max(((lamda1*(x(2)-tau))-EOH_case1),0);


    %############# Total Case 1 ############

EHC_case1 = h*((((x(1)+EOH_case1)/2)*tau)+((EOH_case1+ ...
    max((EOH_case1-(lamda1*(x(2)-tau))),0))/2))*min((x(2)-
tau),(EOH_case1/lamda1));

ESC_case1 = pii*(max(((lamda1*(x(2)-tau))-EOH_case1),0));

    %#############  Case 2 ###############

    %define EOH_case2

    sum4 = 0;
    sum5 = 0;

    for j=x(1)+1:5000

    sum4 = sum4+((x(1)-j));
    sum5 = sum5+(((exp(-
lamdaT*tau))*(((lamdaT*tau)^j)/(gamma(j+1))))/p2);
    end

    sum6 = sum4*sum5;

EOH_case2 = sum6/p2;

lamdaT2 = (x(1)-EOH_case2)/tau;

EEH_case2 = EOH_case2-(lamda1*(x(2)-tau));

t2 = (x(1)*tau)/(x(1)-EOH_case2);
```

```
EHC_case2 = h*(x(1)*t2)/2;

ESC_case2 = pii*((lamda1*(x(2)-tau))- EOH_case2);


%########################### Total cost
###############################%

 ETC =
(1/x(2))*((p1*(EHC_case1+ESC_case1))+(p2*(EHC_case2+ESC_case2))
);

end
```