(z, Z) CONTRACTS FOR VMI WITH STOCHASTIC DEMAND AND (Q, r) MODEL

by

Chandrachur Paul

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Examination Committee: Prof. Huynh Trung Luong (Chairperson)
Prof. Manukid Parnichkun
Dr. Mongkol Ekpanyapong

Nationality: Indian
Previous Degree: Bachelor of Technology in Mechanical Engineering
Jalpaiguri Government Engineering College
Jalpaiguri, West Bengal, India

Scholarship Donor: AIT Scholarships

Asian Institute of Technology
School of Engineering and Technology
Thailand
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AUTHOR'S DECLARATION

I, Chandrachur Paul, hereby attest that the research for this thesis was carried out per the criteria of 'Asian Institute of Technology'. The works that I have provided in it are original to me and the product of my research. Outside sources have been cited wherever consulted. It is unique and has not been turned in to another institution for credit towards an additional degree. This is the original copy of the thesis, complete with the last edits.

Dated: 3rd May 2024

Name: Chandrachur Paul

Signature:

Chandrachur Paul

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ABSTRACT

Due to the escalating competition and constantly shifting consumer demands, businesses must constantly review their operations. Companies need to collaborate and coordinate with other supply chain participants to extract and provide customers with the most value possible. A Supply Chain Contract (SCC) among the members aims to foster the same. Creating an SCC, which can be beneficial for all the partners (Lummus & Vokurka, n.d.) is necessary, to drive down the operational costs. Driving down the operational costs helps reduce the final product's price. In a competitive market, the lower the price of the product, the higher the chance for the customer to buy it. In turn, the sales of the company increase, and the company generates profit in that way (Agrawal & Yadav, 2020)

In this research, a (z, Z) contract between a manufacturer & retailer under a Vendor Managed Inventory (VMI) system in the case of stochastic demand is addressed and a (Q, r) inventory policy is used. A mathematical model has been developed to formulate a total cost function for the entire supply chain, where the lead time is considered constant and production, delivery, ordering, backorder penalty, and holding costs are fixed. Following that, numerical experiments have been conducted using MATLAB to calculate optimal values of Q & R. Sensitivity analysis with respect to different parameters have subsequently been carried out to verify the robustness of the solution as well.

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LIST OF ABBREVIATIONS

EOQ	= Economic Order Quantity
Q	= Order Quantity
Q*	= Optimal Order Quantity
(Q, r)	= Continuous Review Policy
R	= Reorder Level
R*	= Optimum Reorder Level
SC	= Supply Chain
SCC	= Supply Chain Contracts
SCM	= Supply Chain Management
(s, S)	= Periodic Review Policy
TC	= Total Cost
VMI	= Vendor Managed Inventory
w.r.t.	= with respect to

CHAPTER 1
INTRODUCTION

1.1 Background of the Study

Firms constantly develop new products for sale now and then, to be competitive in the market while consumers are faced with having more choices in selecting suitable products for themselves. If the price is too high, the customer will feel that they are being overcharged. At the same time, if the price is too low, there is a chance that it will be detrimental to the company's profits. Thus, product pricing is a strategy (Faith & Edwin, 2014) and it involves both the buyers and sellers. Here is where a Supply Chain Contract (SCC) comes into place, which helps to make sure that the overall Supply Chain (SC) cost is minimal. If the cost of the SC has a minimum cost, the price of the product can be driven down.

The main aim of the vendor will be to sell more stock, so the vendor will want the retailer to place orders in large quantities. Meanwhile, the retailer needs to set the order quantity(Q) in such a way as to prevent shortage and minimize holding costs. The main goal here will be to create a trade-off solution between the two to create an optimal situation for both parties. Using an SCC, the values of optimal quantity and reorder level are determined in a way that the total cost (TC) of the SC is minimized.

Inventory systems can be categorized into two primary types: Continuous Review (Q, r) systems and Periodic Review (s, S) systems. Under the (Q, r) inventory system, a consistent item quantity is typically maintained in every order, with orders placed whenever the inventory reaches a predefined Reorder Level (R). In the (s, S) inventory system, orders are placed at regular intervals, and Q is determined based on the inventory level at R.

(Q, r) inventory systems require continuous monitoring of physical inventory levels, making their implementation costlier compared to periodic review systems. However, the required safety stock level is usually lower because the demand quantity is the only variable that is unpredictable during the delivery time. Additionally, continuous review systems offer benefits such as real-time inventory updates. In contrast to periodic review systems, continuous review systems are better suited for products with high sales volumes and provide greater control over inventory movements.

Supply Chain demand can be categorized into two types: deterministic demand and stochastic demand. Deterministic demand produces output solely based on parameter values and initial conditions, whereas stochastic demand accounts for inherent randomness. Existing literature (Lee & Cho, 2018) predominantly focuses on deterministic demand, which assumes known demand. However, such assumptions may not be suitable for many systems, where demand is often a random variable with a known distribution. Stochastic models are considerably more complex to handle despite their accuracy.

A contract or agreement between the customer and the vendor is necessary for the execution of VMI. Among the most popular VMI contract types is the (z, Z) kind. In addition to the under and overstocking penalties (b and B), a VMI contract details the minimum amount of inventory (z) and the maximum amount of inventory (Z) at the customer. Penalties can take the nature of revised terms for payments as well as shelving locations for the goods being supplied if the vendor violates the minimum/maximum inventory levels more than a predetermined number of times.

1.2 Problem Statement

Every organization's general goal is to lower the SC's overall cost while adding value for the consumer in the end. There are numerous ways to minimize SC costs. However, in the past, it has been proven that Vendor Managed Inventory (VMI) is very effective, as it helps the working between the two seamlessly and efficiently.

VMI is an inventory management policy that integrates the inventory decisions of the vendor and retailer. However, a contract or agreement between the retailer and the supplier is necessary for the implementation of VMI. Costs incurred by the supplier and the retailer need to be analyzed to make sure that the contract with the best outcome is selected.

Lee & Cho (2018) has devised a model to demonstrate that the implementation of a VMI system can reduce the TC of the entire system in the past. This study typically focuses on periodic review policies. Furthermore, the research predominantly dealt with deterministic demand. Consequently, there remains a gap in developing models for stochastic demand. Hence, this study aims to address this gap by developing a model to minimize supply chain costs through a (Q, r) policy for items with stochastic demand under a VMI system.

1.3 Research Objective

The objective of this research will be to calculate the Economic Order Quantity (EOQ) & reorder level (R) in a VMI inventory system to ensure that the net cost of the SC (which includes the holding cost, order cost, overstocking penalty, understocking penalty, and shortage cost) is minimum under (Q, r) policy and stochastic demand in a (z, Z) supply chain contract.

1.4 Scope and Limitations

The scope is to identify the EOQ & R* of the entire inventory system for a single supplier & single retailer SC system. The following assumptions will be considered in this research:

1. Lead time is constant.
2. The demand is stochastic.
3. The production and delivery costs are fixed.
4. The ordering and holding costs are fixed.
5. The shortage cost is fixed.

CHAPTER 2
REVIEW OF LITERATURE

2.1 Supply Chain Contracts

A SC combines a series of operations with linked supplier-user businesses that go from the initial usage of components in their raw form to the final utilization of the finished product. Many value chains, which comprise all the operations inside an organization that add up to the product's value, adopt SC (Cox et al., 1995). The domain of SC research continues to grow quickly, offering businesses new tools & methods for adapting to shifting market circumstances. The latest developments include resilience, digitalization, sustainability & artificial intelligence (Saberi et al., 2019). There are five categories for supply chain management (SCM) research questions: Inventory Issues (IP), Information Flow (IF), Contracts & Coordination (CC), Network Architecture (ND) and Performance Assessment (PE) (Tiwari, 2013).

Supply Chain Coordination and Contracts are crucial for fostering cooperation and integration among nodes within the SC (Terzi & Cavalieri, 2004). This is because, in SCM, attempts are made to offer a win-win strategy for each chain link, from suppliers to customers. One example that illustrates the huge number of nodes in a SC is that of a classic SC for manufacturing clothes, where, raw materials are procured from numerous suppliers, followed by producing goods in one or more of the manufacturing facilities (Şen, 2008). After that, the final goods are transported to intermediate storage locations, like distribution centres or warehouses, where they are packaged, loaded, and delivered to retailers/ final customers (Boysen et al., 2021).

To coordinate the decentralized supply chain, Pasternack (2008) originally suggested the use of a supply chain contract. Up until recently, several agreements, including buy-back, rebate, and revenue-sharing contracts, were put forth to further enhance SC performance in the rise of unpredictable customer demand (Saha et al., 2012; Sang, 2013; Zhao et al., 2014).

For VMI to be implemented, a contract or agreement between the manufacturer & retailer is required. In (z, Z) contracts under VMI, the contract specifies the customer's minimum & maximum inventory levels (z & Z respectively) as well as the associated under and overstocking fines (b & B respectively). If the supplier violates the minimum/maximum inventory levels for a specified number of times, the terms of payment may be modified, or the products may be exhibited on different shelves (Lee & Cho, 2018). In the model by Fry et al. (2001), (b, B) & (z, Z) are decided upon by the supplier and the retailer in agreement to reduce the overall cost of the SC, while Z is selected through the retailer to reduce their costs.

2.2 Vendor Managed Inventory

The emergence of VMI as a novel approach to inventory management can be attributed to the advancements in manufacturing technology and information science. The supplier assumes all inventory risks and oversees inventory management under VMI. Procter & Gamble (P&G) and Wal-Mart deployed VMI for the first time in 1985. Another common VMI mode is used by the oil business Petrol-soft Corporation to control gasoline supplies at the service stations they supply to. Supply chain performance can be enhanced, and inventory expenses can be decreased by using the VMI mode. Under the VMI system, the supplier oversees inventory and customers can provide more precise information about market demand. When the selling season is just getting started, the provider can find out about the actual demand. It is normal for the supplier to restock the products by means of a second production run if there is unfulfilled demand (Cai et al., 2016).

Reduced overhead expenses and, if consignment stock is used, inventory transfer costs—where inventory costs are passed on to the supplier—are two advantages of VMI for the retailer. However, the advantages of VMI for the supplier are not entirely clear-cut (Lee & Ren, 2011). Here, the Apple Computer business case study was taken into consideration where it was observed that the company's Irish manufacturing site, Apple Cork, demanded that foreign suppliers establish a supply center close to their plant. Material handling, storage, and freight were covered by the suppliers. Until the materials were dispatched from the hub and delivered to Apple Cork's production line, Apple did not take ownership of the goods. (Philippe-Pierre Dornier, Ricardo Ernst, 1998)

Verma & Chatterjee (2017) examined a situation with multiple retailers and a single supplier, in which the supplier oversees deciding how much inventory to replenish at each retailer so that the total amount stays within a mutually agreed-upon upper limit mentioned in the VMI contract. It turns out that, in comparison to the current alternative replenishment models, the suggested model with the integer ratio policy structure is comparatively more stable. Bertazzi et al. (2021) examined a long-haul transportation issue involving the integration of air freight or less-than-container load (LCL) shipments on one side and full container load (FCL) shipping on the other, including the delivery of a batch of products from a producer to a client. It has been noted that, both on average and in the worst situation, combining LCL and ECL/air freight shipments can result in significant cost savings.

Cai et al. (2016) examined the issue of SC coordination with an option contract under the VMI system and compared it with a subsidy contract. It is noticeable that option contracts can be used with greater flexibility across most supply chains, particularly those involving innovative products. Zammori et al. (2009) examined the general structure of the VMI. It is observed that to bring on flexibility in a VMI agreement, it is important to arrange the VMI agreement into sections that deal with the general and legal components of the agreement; the annexes should handle the technical details and themes specific to the relationship. This makes the deal more flexible.

2.3 (Q, r) Inventory System

A (Q, r) model was created by Handfield et al. (2009) utilizing fuzzy-set representations of the numerous sources of supply chain uncertainty. The lead times, demand, supplier output, penalty fees, and lead times are the sources of risk and uncertainty in the model. One important advantage of the model is its applicability to nearly any empirical demand or lead time distribution; on the other hand, non-normal demand or lead time distributions could complicate the computation of policy parameters in a traditional, stochastic model of a (Q, r) system. A (Q, r) inventory system with fast-moving products was studied by (Chung et al., 2009) on the presumption that the lead time is under uniform distribution.

Hill (2007) considers a (Q, r) system with a lost-sales inventory model that has no fixed order cost, a fixed lead time, and Poisson demand. Controlling slow-moving but crucial and perhaps costly spare components with a lengthy lead time for restocking is one potential use case for such a concept. This study looks at policies that make use of the discovery that by placing orders consecutively, base stock policies for lost sales models can be improved. These policies' efficacy is compared to that of the ordering strategy at predefined, regular intervals and the matching base stock policy.

CHAPTER 3
MATHEMATICAL MODEL

This study is centered on finding the most profitable (z, Z) contract where the whole system is profitable. The inventory system that will be used is VMI, where all inventory is kept at the retailer by the vendor.

3.1 Development of the Mathematical Model

This chapter the mathematical model for dealing with this situation will be formulated, where stochastic demand & lost sales are considered. The demand will be stochastic, following a normal distribution N (μ, σ^2). The review policy that will be used is the (Q, r) policy, where an order of Q units will be placed whenever the inventory level falls to r units, which is the reorder point. Here, the inventory level is the stock on hand. All ordering decisions are made based on the inventory level. Inventory position is defined as the sum of the inventory on hand and the in-transit inventory (i.e., the amount that has been ordered but that has not arrived yet). Holding costs are calculated based on the inventory level.

Notations:

μ: Mean demand per unit of time

μ_L: Mean demand during lead time

SS: Safety Stock

S: Expected Shortage Amount

T: Expected Cycle Time

L: Lead Time

$f_L(x)$: Probability Density Function of the demand during lead time.

h: Holding Cost of the items per unit per time unit

K: Ordering Cost per order

π: Shortage Cost per unit

Z: Maximum Inventory level

z: Minimum Inventory level

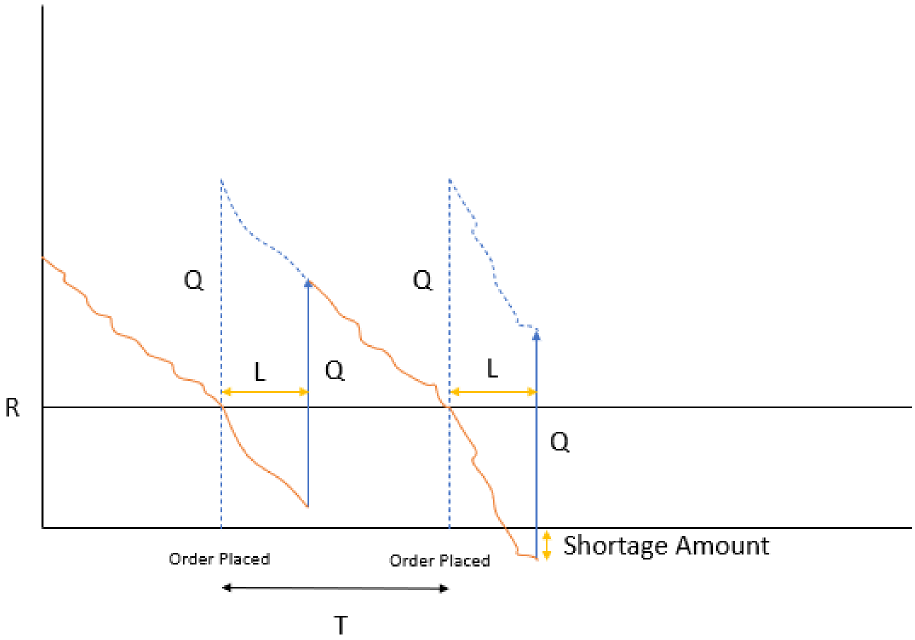
B: Overstocking Penalty per unit per unit time

b: Understocking penalty per unit per unit time

When the (Q, r) inventory policy is applied, the system's overall inventory distribution can be represented as follows:

Figure 3.1

Inventory Level under (Q, r) Policy



Demand will be met during time L+T by the amount Q+R, and demand during the lead time L by the amount R.

Expected Cycle Length:

The expected elapsed time between two successive orders is defined as the Expected Cycle Length.

T=Q/μ

Shortage:

The demand quantity that cannot be fulfilled by the current stock on hand is known as a shortage. When demand exceeds the reorder point during the lead time, there is a shortage.

Denote x to be the demand during lead time.

The demand during lead time can be,

- $x \geq R$: It is the situation where a shortage occurs. Hence the shortage amount for this situation is (x-R)
- $x < R$: It is the situation where there is no shortage. So, the shortage amount in this situation is zero.

Expected shortage amount during one cycle (S);

$=E\ [\max\ (D_L - R, 0)]$

$=\int_0^\infty Max(x - R, 0) f_L(x) dx$

$=\int_0^R\ (0) f_L(x) dx + \int_R^\infty (x - R) f_L(x) dx$

$=\int_R^\infty (x - R) f_L(x) dx$

Safety Stock:

The expected minimum on-hand inventory just before the order gets replenished is known as safety stock.

We have:

SS=E [Minimum Inventory level before the order arrives]

$= \int_0^\infty (R - x) f_L(x) dx$

$= \int_0^\infty R f_L(x) dx - \int_0^\infty x\, f_L(x) dx$

We know,

$\int_0^\infty f_L(x) dx = 1$

And, $\int_0^\infty x\, f_L(x) dx = \mu_L$

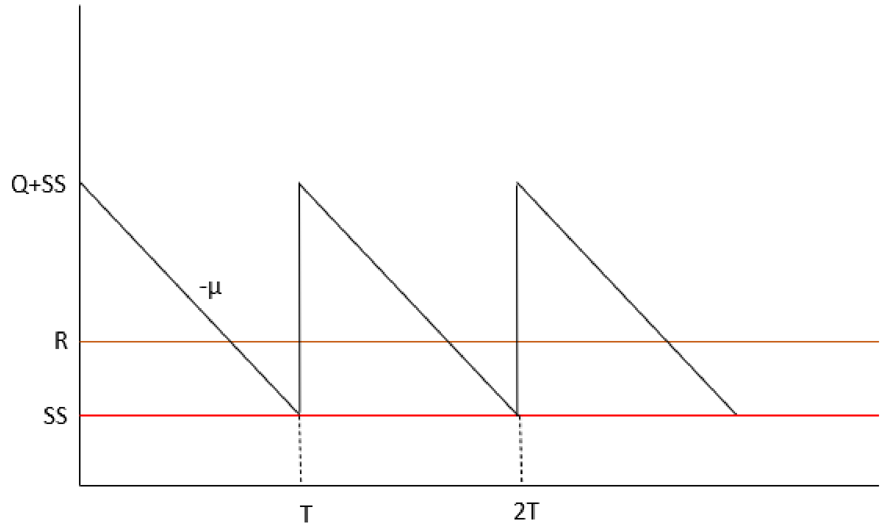
Also, $\mu_L = \mu L$

So,

$SS = R - \mu L$

Figure 3.2

Inventory Distribution of the Retailer Stock



The above diagram will be used for calculation purposes. It helps to approximate the cost components involved in the supply chain.

Now, we will calculate each of the costs one by one.

Order Cost:

The order cost is defined as the cost to place an order.

We know,

Expected Cycle Length (T)=Q/µ

So, the average ordering cost per unit time K/T=Kµ/Q

Holding Cost:

The cost incurred while holding inventory is called the holding cost. It includes all the components like warehouse employee costs, warehouse maintenance and all other forms of costs that are dependent on the amount of inventory/stock held. It is directly proportional to the inventory amount held in the warehouse.

Holding Cost per unit of time= $\frac{h*\int_0^T I dt}{T}$

As per the second diagram,

The average inventory during a cycle will be,

$I=\frac{Q}{2}+SS$

Hence,

Holding Cost per unit time

$=h*(\frac{Q}{2}+SS)$

Shortage Cost:

Shortage Cost is defined as the cost that is incurred when there is a shortage amount, i.e., when the demand exceeds the on-hand inventory or safety stock during lead time. Since we are using a lost sales policy, it will be calculated per unit.

Shortage Cost=π*S

Shortage Cost per unit time= πS/T= πμS/Q

Where,

S is the Shortage amount, that has been proved before.

$S=\int_R^\infty (x-R)f_L(x)dx$

Understocking Penalty Costs:

Understocking penalty is defined as the penalty that will be charged if the minimum inventory level (z) is violated.

It is noted that the value R must be higher than z, otherwise, the requirement of minimum inventory z will be violated in every cycle, and hence, the penalty cost will be too high.

The ending inventory in every cycle will be,

$EI=R-D_L$

So, if $R-D_L<z$, then the penalty cost will be charged for the amount: $z-R+D_L$; otherwise, there will be no penalty charge.

The expected amount for penalty charge in each cycle, therefore can be determined as:

$\int_0^\infty Max\{z-R+x,0\}f_{D_L}(x)dx$

$=\int_0^{R-z} Max\{z-R+x,0\}\,f_{D_L}(x)dx+\int_{R-z}^\infty Max\{z-R+x,0\}\,f_{D_L}(x)dx$

$=0+\int_{R-z}^\infty (z-R+x)\,f_{D_L}(x)dx$

$=\int_{R-z}^\infty (z-R+x)\,f_{D_L}(x)dx$

The expected charge in one cycle due to the violation of minimum inventory is,

$b\int_{R-z}^{\infty}(z - R + x)\, f_{D_L}(x)dx$

The expected charge per unit of time due to the violation of minimum inventory is,

$\frac{b\mu \int_{R-z}^{\infty}(z-R+x) f_{D_L}(x)dx}{Q}$

Overstocking Penalty:

The magnitude of the overstocking penalty is what happens when the inventory level goes above the maximum inventory level (Z).

The maximum inventory level is at the start of each cycle, after which it starts depleting due to the fulfillment in demand.

The starting inventory in a cycle is:

SI=Q+ EI

We know,

EI=R-D_L

So,

SI=Q+ R-D_L

It is noted that Z must be less than the maximum inventory Q+R, otherwise, the constraint related to Z is redundant (i.e., will never be violated). So, if Q+ R-D_L>Z, the overstocking penalty will be charged on the amount Q+ R-D_L-Z. Else there will be no penalty charged.

Denote x as the demand during lead time then.

The expected amount for overstocking penalty for each cycle will be determined as:

$\int_0^{\infty} Max[Q + R - Z - x, 0]\, f_{D_L}(x)dx$

$= \int_0^{Q+R-Z} Max[Q + R - Z - x, 0] \quad f_{D_L}(x)dx + \int_{Q+R-Z}^{\infty} Max[Q + R - Z - x, 0]$

$f_{D_L}(x)dx$

$=\int_0^{Q+R-Z}(Q + R - Z - x)\, f_{D_L}(x)dx + 0$

$=\int_0^{Q+R-Z}(Q + R - Z - x)\, f_{D_L}(x)dx$

Overstocking penalty in one cycle =$B\int_0^{Q+R-Z}(Q + R - Z - x)\, f_{D_L}(x)dx$

Overstocking Penalty per unit time = $\frac{B\mu \int_0^{Q+R-Z}(Q+R-Z-x)\, f_{D_L}(x)dx}{Q}$

3.2 Formulation of Total Cost Function

The supply chain contract will have two levels namely z and Z. Z represents the maximum inventory level that can be held by the retailer, and in case there is overstocking, the penalty that will be levied upon the vendor is B per unit. Similarly, z is the minimum inventory level that the vendor should keep at the retailer, otherwise an understocking penalty b per unit will be levied upon the vendor.

TC Function:

The TC of the supply chain consists of:

- Understocking Penalty
- Overstocking Penalty
- Shortage Cost
- Holding Cost
- Ordering Cost

Hence,

TC of the SC per unit of time

=Understocking Penalty+ Overstocking Penalty+ Shortage Cost+ Holding Cost+ Ordering Cost

$$= \frac{b\mu\int_{R-z}^{\infty}(z-R+x)f_{D_L}(x)dx}{Q} + \frac{B\mu\int_{0}^{Q+R-Z}(Q+R-Z-x)\, f_{D_L}(x)dx}{Q} + \frac{\pi\mu\int_{R}^{\infty}(x-R)f_{D_L}(x)dx}{Q} + h\left(\frac{Q}{2} + R - \mu_L\right) + \frac{K\mu}{Q}$$

CHAPTER 4
NUMERICAL EXPERIMENT & SENSITIVITY ANALYSIS

4.1 Numerical Experiment with Base Case

Numerical experiments are conducted with the goal in mind to evaluate incentives for both the distributor and supplier to apply the (z, Z) contract in this chapter. The following assumptions are considered:

- It must be noted that R should be higher than z, otherwise, the requirement of minimum inventory z will be violated in every cycle, and hence the penalty cost will be too high. So, R>z.
- Also, Z should be less than the maximum inventory Q+R, otherwise there will be no overstocking penalty charged. So, Q+R>Z.
- The shortage cost should be slightly higher than the profit generated from the item sale per unit. This is because lost sales signify the loss of opportunity to earn profits.
- Overstocking and understocking penalties are charged in such a way that the penalties do not exceed the shortage cost.

We shall consider that the demand follows the normal distribution.
The numerical experiments will be conducted using MATLAB

Input parameters used here are as follows:

- Price of one item=100 Baht
- Holding cost per unit per week= (0.3*100)/52 Baht
- Lead time= 1 Week
- Shortage cost per unit= 12 Baht
- Overstocking penalty per unit = 9 Baht
- Understocking penalty per unit = 7 Baht
- Upper Z limit= 400
- Lower z limit= 320

- Standard deviation of Demand per week = 20 Units
- Ordering Cost per order= 80 Baht
- Mean demand per week= 120 Units

Output consisting of optimal results of Q, R and minimum TC are: -

Q*: 80

R*: 472

Minimum value of TC: 782.44

4.2 Sensitivity Analyses

This section will assess how the input parameters have an impact on the EOQ, optimal reorder level, and the TC of the SC. Parameters like price of an item, holding cost per unit per week, lead time, shortage cost per unit, overstocking penalty per unit, understocking penalty per unit, upper inventory limit (Z), lower inventory limit (z), standard deviation of demand per week, ordering cost per order and the mean demand per week will be investigated.

Effect of the Holding Cost Per Unit Per Week (h):

Here, the holding cost as a percentage of unit price will be varied between 20% - 45% of the product price, with other parameters constant. The results are in Table 4.1. The variations can also be summarized as a line chart showing the trend of Q*, R* & TC in Figure 4.1.

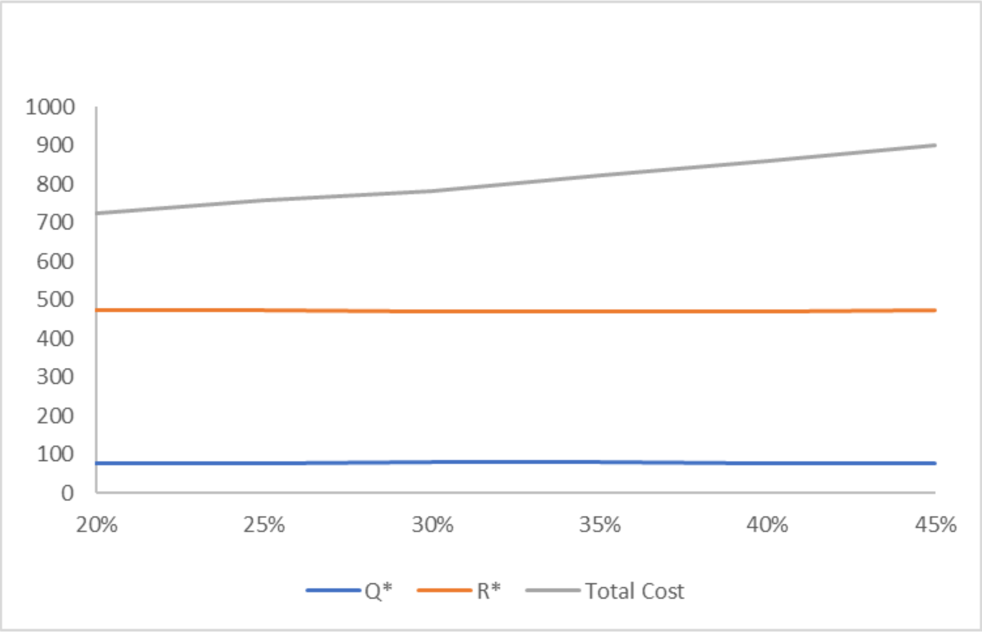
Table 4.1

Variation in Q, R*, TC w.r.t. Holding Cost Per unit Per Week*

h	Q*	R*	TC
20%	77	474	723.55
25%	78	473	757.34
30%	**80**	**472**	**782.442**
35%	80	472	821.56
40%	79	472	861.225
45%	79	473	901.914

Figure 4.1

Variation in Q, R*, TC w.r.t. Holding Cost per Unit per Week*



According to the results, it shows that when the holding cost as a percentage of unit product price is increased, then the value of Q* and R* remains mostly the same with slight variations. The value of TC however, increases. These trends are reasonable.

Effect of the Unit Item Price (p):

In this part, the price of an item will be varied between 80 - 130, with other parameters constant. The results are in Table 4.2. The variations can also be summarized as a line chart, which shows the trend of Q*, R* & TC in Figure 4.2.

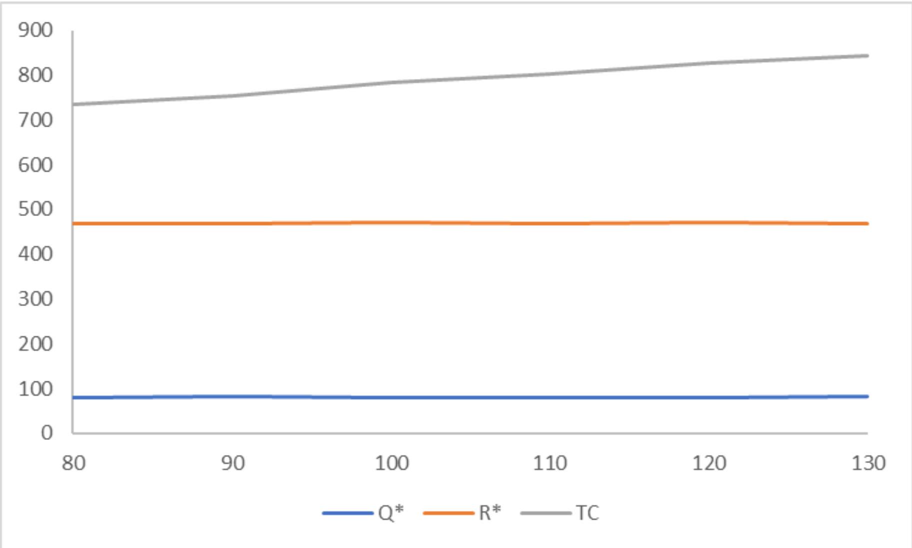
Table 4.2

Variation in Q, R*, TC w.r.t. unit item price*

p	Q*	R*	TC
80	80	471	735.6
90	81	471	754.59
100	**80**	**472**	**782.44**
110	80	471	803.797
120	79	472	827.986
130	81	471	843.328

Figure 4.2

Variation in Q, R*, TC w.r.t. Unit Item Price*



According to the results, it shows that when the unit product price is increased, then the values of Q* and R* remain mostly the same with slight variations. But, the TC increases with the increase in the unit product price. These trends are reasonable.

Effect of the Lead Time (L):

In this part, the lead time will be varied between 0.8-1.3, with other parameters constant. The results are in Table 4.3. The variations can also be summarized as a line chart, which shows the trend of Q*, R* & TC in Figure 4.3.

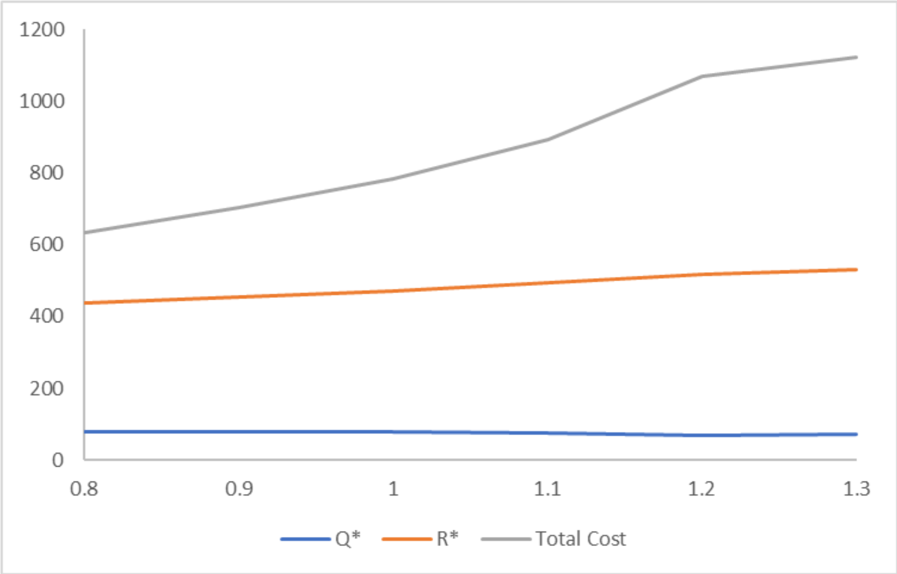
Table 4.3

Variation in Q, R*, TC w.r.t. Lead Time*

L	**Q***	**R***	**TC**
0.8	80	438	635.403
0.9	80	454	702.64
1	**80**	**472**	**782.44**
1.1	76	493	893.747
1.2	68	518	1069.714
1.3	72	532	1123.23

Figure 4.3

Variation in Q, R*, TC w.r.t. Lead Time*



According to the results, it shows that as the lead time increases, the value of Q* decreases, and the value of R* increases. The TC increases with the increase in the lead time. These results are reasonable.

Effect of the Ordering Cost Per Order (K):

In this part, the ordering cost per order will be varied between 60-110, with other parameters constant. The results are in the table 4.4. The variations can also be summarized as a line chart showing the trend of Q*, R* & TC in Figure 4.4

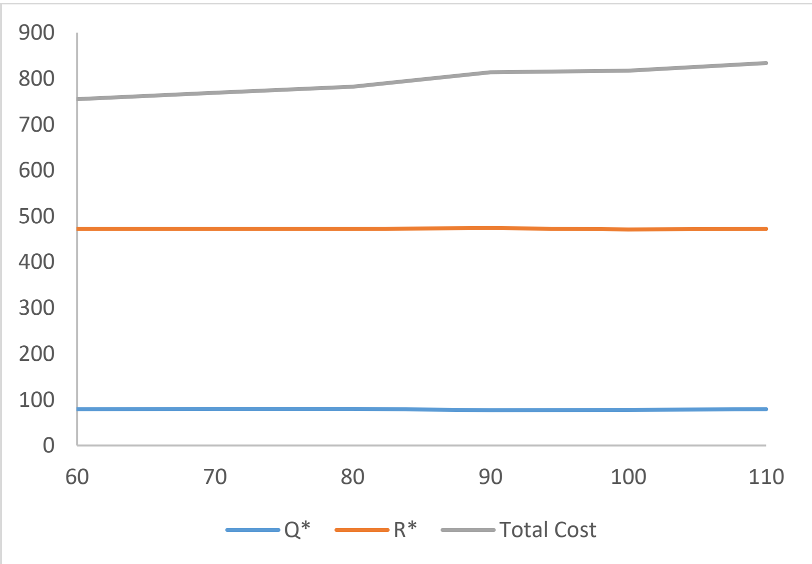
Table 4.4

Variation in Q, R*, TC w.r.t. Ordering Cost per Order*

K	Q*	R*	TC
60	79	472	755.134
70	80	472	768.793
80	**80**	**472**	**782.44**
90	77	474	814.01
100	81	471	817.415
110	79	472	833.773

Figure 4.4

Variation in Q, R*, TC w.r.t. ordering cost per order*



According to the results, the values of Q*, R* do not have much changes. This is because the ordering cost per order is just in one part of the TC expression, and is adjusted accordingly to reduce the TC. The change in the value of R* can do little to influence the total cost, as the other components are fixed. However, the value of Q* can influence the ordering cost inversely. So, the Q* & R* values are adjusted to minimise the TC of the SC. However, the TC increases with the increase in the amount of holding cost per order.

The effect of the Shortage Cost Per Unit (s):

In this part, the shortage cost per unit will be varied between 0.8-1.3, with other parameters constant. The results are in Table 4.5. The variations can also be summarized as a line chart showing the Q*, R*, & TC trend in Figure 4.5.

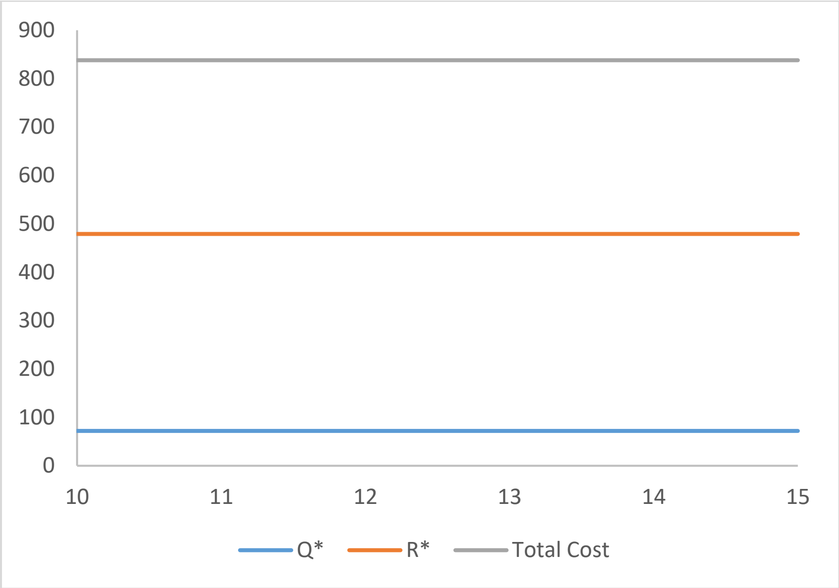
Table 4.5

Variation in Q, R*, TC w.r.t. Shortage Cost per Unit*

s	**Q***	**R***	**TC**
10	80	472	782.44
11	80	472	782.44
12	**80**	**472**	**782.44**
13	80	472	782.44
14	80	472	782.44
15	80	472	782.44

Figure 4.5

Variation in Q, R*, TC w.r.t. Shortage Cost per Unit*



The observation is that the variation of shortage cost per unit does not affect the TC, nor the Q* or R*. This can be because the solution is derived in such a way that there is no shortage amount.

The Effect of the Overstocking Penalty Per Unit (B):

In this part, the overstocking penalty per unit will be varied between 7.5-10, with other parameters constant. The results are in Table 4.6. The variations can also be summarized as a line chart, which shows the trend of Q*, R* & TC in Figure 4.6.

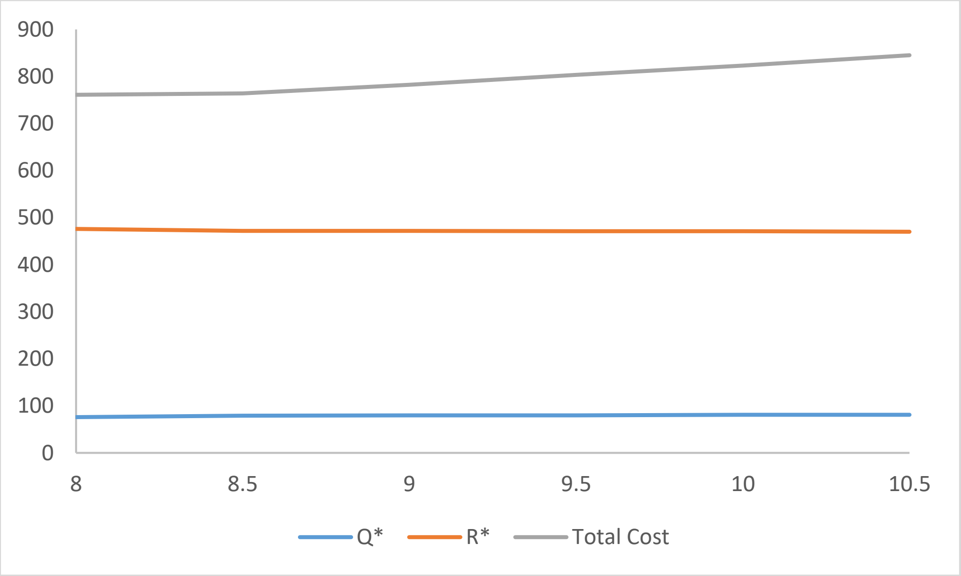
Table 4.6

Variation in Q, R*, TC w.r.t. Overstocking Penalty per Unit*

B	Q*	R*	TC
8	76	476	761.106
8.5	79	472	764.01
9	**80**	**472**	**782.44**
9.5	80	471	803.18
10	81	471	822.98
10.5	81	470	845.204

Figure 4.6

Variation in Q, R*, TC w.r.t. Overstocking Penalty per Unit*



In this case, the increase in overstocking penalty leads to the increase in total cost. This is reasonable because the overstocking penalty per unit is a multiplication factor to one of the parts of the total cost expression, and that part is added linearly. The Q* and R* values are adjusted to reduce the total cost accordingly, although they are not very much varied in number.

Effect of the Understocking Penalty per Unit (b):

In this part, the understocking penalty per unit will be varied between 6-8.5, with other parameters constant. The results are in Table 4.7. The variations can also be summarized as a line chart showing the Q*, R* & TC trend in Figure 4.7.

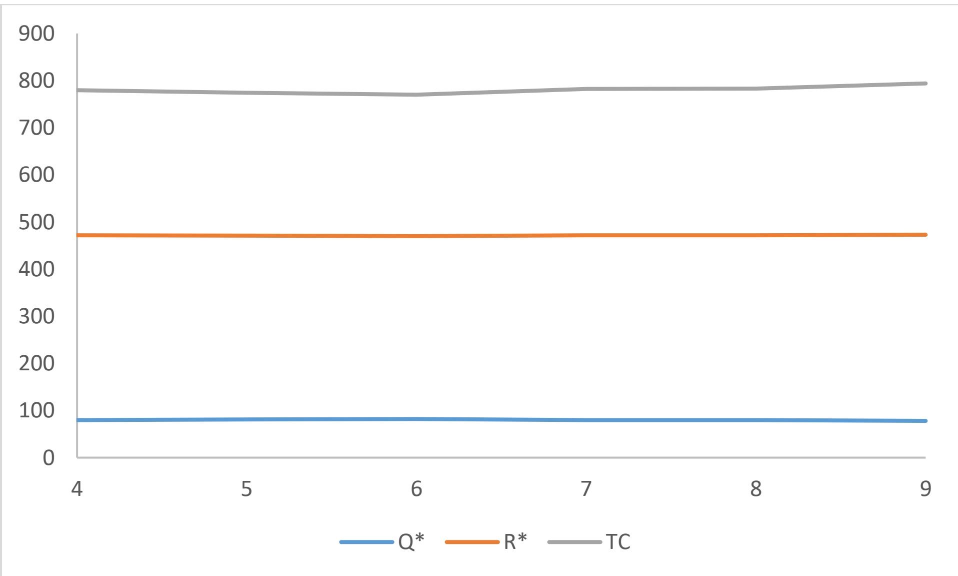
Table 4.7

Variation in Q, R*, TC w.r.t. Understocking Penalty per Unit*

b	Q*	R*	TC
4	80	472	779.885
5	81	471	774.465
6	82	470	770.102
7	**80**	**472**	**782.44**
8	80	472	782.87
9	78	473	793.962

Figure 4.7

Variation in Q, R*, TC w.r.t. Understocking Penalty per Unit*



In this case, the Q* and R* values remain mostly the same as the value of b increases. The TC of the supply chain also increases gradually. This trend is reasonable.

The Effect of the Upper Inventory Limit (Z):

In this part, the upper inventory limit will be varied between 380-430, with other parameters constant. The results are in Table 4.8. The variations can also be summarized as a line chart showing the trend of Q*, R* & TC in Figure 4.8.

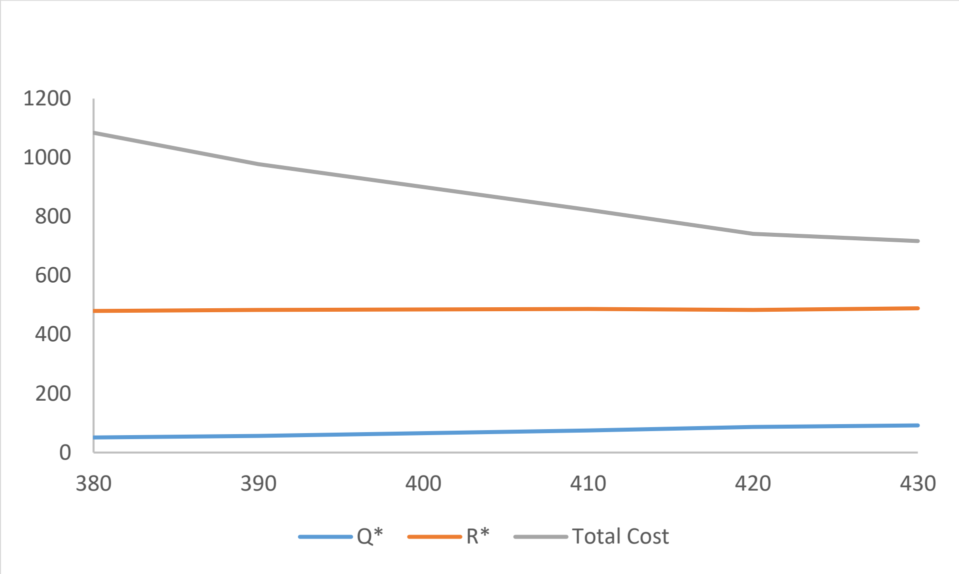
Table 4.8

Variation in Q, R*, TC w.r.t. Upper Inventory Limit*

Z	Q*	R*	TC
380	61	470	948.664
390	68	473	872.58
400	**80**	**472**	**782.44**
410	89	472	724.331
420	96	476	695.78
430	107	475	650.475

Figure 4.8

Variation in Q, R*, TC w.r.t. upper inventory limit*



In this case, it is seen that the increase in upper inventory level Z causes the TC to decrease but, the Q* value increases. The R* value also increases gradually. These trends are reasonable.

The Effect of the Lower Inventory Limit (z):

In this part, the lower inventory limit will be varied between 290-340, with other parameters constant. The results are in Table 4.9. The variations can also be summarized as a line chart, which shows the trend of Q*, R* & TC in Figure 4.9.

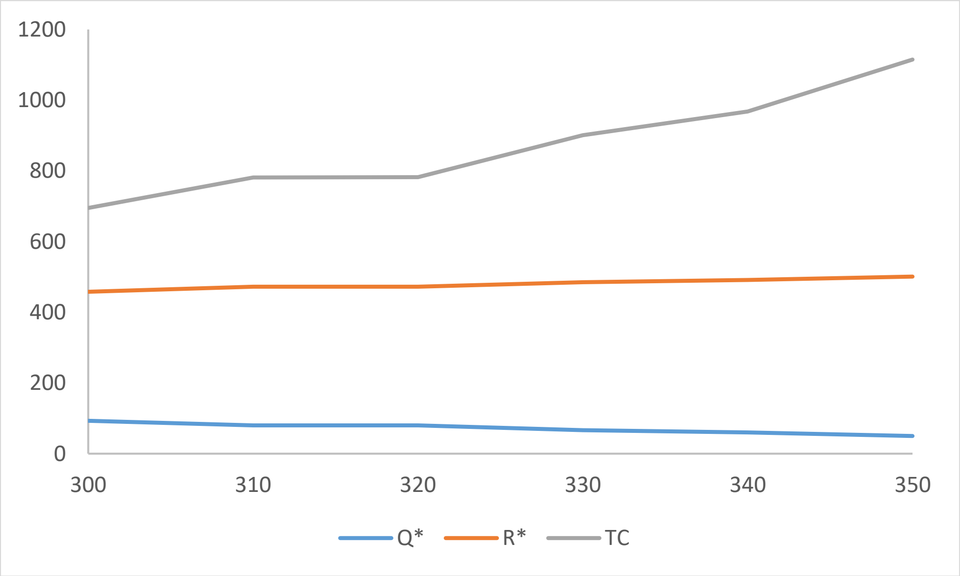
Table 4.9

Variation in Q, R*, TC w.r.t. Lower Inventory Limit*

z	Q*	R*	TC
300	93	458	695.131
310	80	472	780.878
320	**80**	**472**	**782.442**
330	66	485	900.775
340	60	491	968.239
350	50	501	1114.918

Figure 4.9

Variation in Q, R*, TC w.r.t. Lower Inventory Limit*



Here it is observed that as the value of the lower limit z is increased, the value of Q* decreases while value of R* and TC increase. These trends are reasonable.

The Effect of the Mean Demand (mu):

The mean demand will be varied between 100-150, keeping the other parameters constant in this section. The results are in Table 4.10. The variations can also be summarized as a line chart showing the trend of Q*, R* & TC in Figure 4.10.

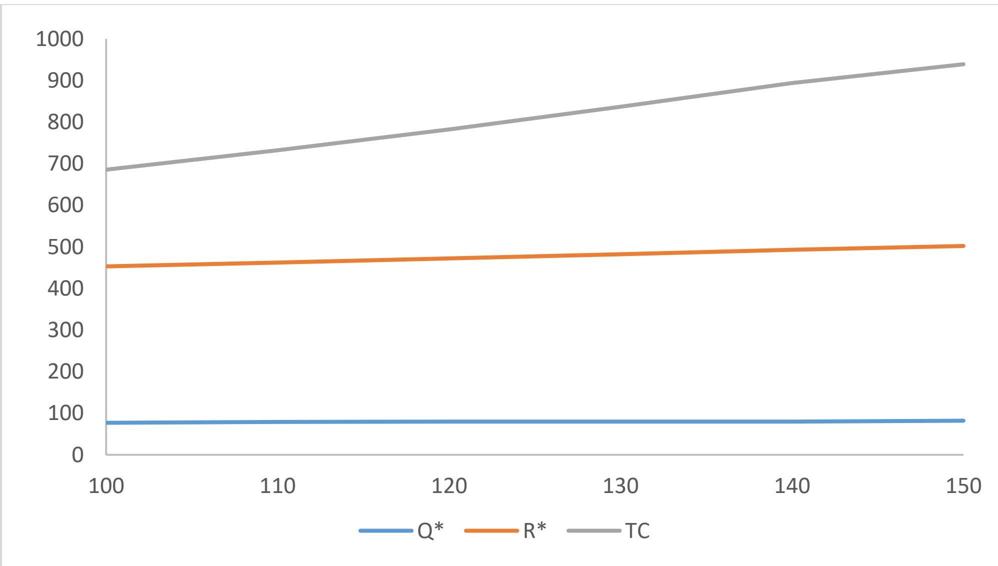
Table 4.10

Variation in Q, R*, TC w.r.t. Mean Demand*

mu	Q*	R*	TC
100	77	453	685.4218
110	79	462	731.905
120	**80**	**472**	**782.442**
130	80	482	836.447
140	80	493	893.347
150	82	502	938.9191

Figure 4.10

Variation in Q, R*, TC w.r.t. Mean Demand*



In this case, it is observed that as the mean demand increases, the value of Q*, R* and TC increases as well. These trends are reasonable.

The Effect of Standard Deviation of Demand (sigma):

The standard deviation of demand will be varied between 17-22, keeping the other parameters constant in this section. The results are in Table 4.11. The variations can also be summarized as a line chart showing the Q*, R* & TC trend in Figure 4.11.

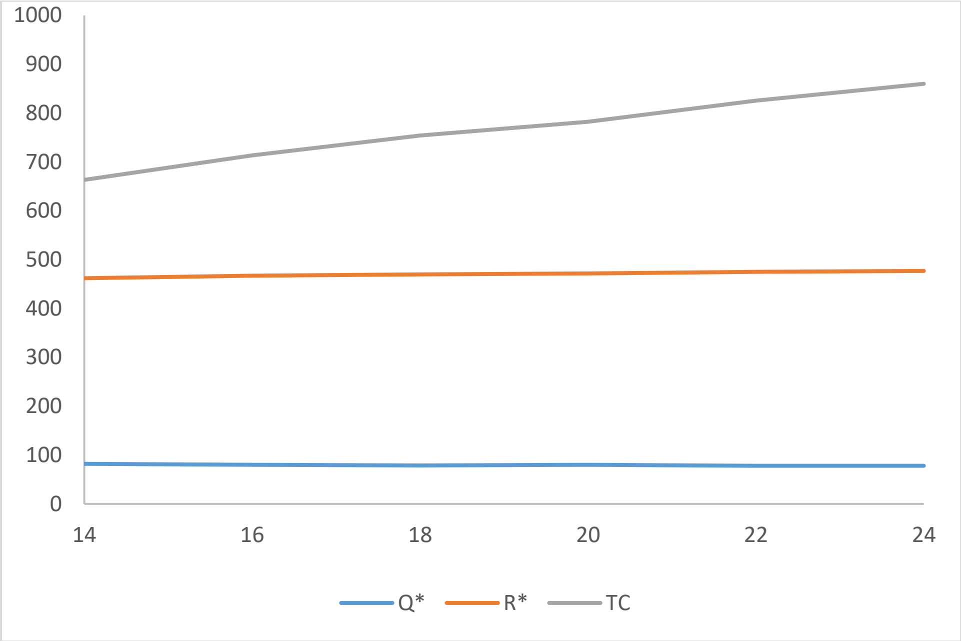
Table 4.11

Variation in Q, R*, TC w.r.t. Standard Deviation of Demand*

sigma	Q*	R*	TC
14	82	462	663.3844
16	80	467	713.649
18	79	470	754.1395
20	**80**	**472**	**782.44**
22	78	475	825.511
24	78	477	860.01

Figure 4.11

Variation in Q, R*, TC w.r.t. Standard Deviation of Demand*



Here, it is observed that as the standard deviation of demand increases, the value of Q* decreases gradually. The value of R* increases gradually as well. Also, the TC value increases. These trends are reasonable.

CHAPTER 5
CONCLUSIONS & RECOMMENDATIONS

5.1 Conclusion

In this research, the EOQ and the R* of the whole inventory system for a (z, Z) supply chain contract are determined, in such a manner that the overall SC cost is minimized. A system under VMI for a single supplier & single retailer supply chain under (Q, r) inventory policy following stochastic demand is taken into consideration. The TC function, which is the sum of the understocking penalty, overstocking penalty, shortage cost, ordering cost & holding cost is derived. After that, the TC function is minimized using MATLAB, with decision variables Q* and R*. Following that, sensitivity analyses are performed to ensure that the MATLAB code is robust.

In summary, the following points can be concluded from the analysis of this research:

- When the holding cost percentage is increased, then the value of Q* and R* remains mostly the same with slight variations. The value of TC, however, increases.
- When the unit product price is increased, then the values of Q* and R* remain mostly the same with slight variations. The TC, however, increases.
- With the increase in lead time, the value of Q* decreases, and the value of R* increases. However, the TC increases with the increase in the lead time.
- When the ordering cost per order increases, values of Q* and R* do not have many changes. However, the TC increases with the increase in the amount of holding cost per order.
- Variation in shortage cost per unit does not affect the TC, nor the Q* or R*.
- The increase in the overstocking penalty leads to an increase in TC. The Q* and R* values are adjusted to reduce the TC accordingly, although they are not very much varied in number.
- The Q* and R* values remain mostly the same as the value of the understocking penalty increases. The TC of the SC also increases gradually.

- An increase in upper inventory level Z causes the TC to decrease. But the Q* value increases. The R* value also increases gradually.
- As the value of the lower limit z is increased, the value of Q* decreases while the value of R* and TC increase.
- As the mean demand increases, the values of Q*, R*, and TC increase as well.
- As the standard deviation of demand increases, the value of Q* decreases gradually. The value of R* increases gradually. Also, the TC value increases.

The proposed model in this research helps to get a more realistic and accurate application of the (z, Z) contract as real-life demand is not deterministic but stochastic. Sensitivity analyses of the parameters in the TC function will help supply chain designers make better decisions while setting the parameters and getting a better deal by negotiating the same.

5.2 Recommendations

The following points can be considered for further extensions of this research:

- Lead time is a constant in this research. Further research can be done where variable lead times are considered.
- In this research, a lost-sales policy is considered. In future research, a backorder policy can be considered instead.
- The (Q, r) inventory policy is considered in this research. In future research, the (s, S) inventory policy can be considered instead.

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APPENDIX

MATLAB CODE

```
syms k
p = 100; %price of one item
h = (0.30*p)/52; %per unit holding cost
L = 1; %lead time
s = 12; %per unit shortage cost
B = 9; %overstocking penalty
b = 7; %understocking penalty
Z = 400; %maximum inventory limit
z = 320; %minimum inventory limit
K = 80; % holding cost per unit per time
mu = 120; % mean demand
sigma = 20; % sigma of demand
DL = mu * L; % demand during lead time
sigmaL = sigma * (L)^2; %sigma during lead time

% Define the normal pdf
m = @(y)normpdf(y, DL, sigmaL);

% Define the integrands of TC function
SC = @(y, q, r) (s*mu)*(1/q)*(y - r) .*m(y)
OP = @(y, q, r) (B*mu)*(1/q)*(q + r - Z - y) .*m(y)
UP = @(y, q, r) (b*mu)*(1/q)*(z - r + y) .*m(y)

% Objective function to minimize
TC = @(y) integral(@(y) SC(y, y(1), y(2)), x(2), Inf)+integral(@(y) OP(y, y(1), y(2)),
0, y(1)+y(2)-Z)+integral(@(y) UP(y, y(1), y(2)), y(2)-z, Inf)+h*(y(1)/2+y(2)-
DL)+(K*mu)/y(1);

% Initial guess for decision variables [q, r]
y0 = [400,350];

% Define linear inequality constraints A*x >= C
A = [-1,-1;0,-1];
C = [-Z,-z];


% Define lower and upper bounds for decision variables
lb = [0, 0];
ub = [Inf, Inf];

%No equality constraints, so these are empty
Aeq=[];
beq=[];
```

```
% Perform optimization
options = optimoptions('fmincon', 'StepTolerance', 2e-26, 'ConstraintTolerance', 1e-25);
[y_opt, fval] = fmincon(TC, y0, A, C, Aeq, beq, lb, ub, [], options);

Qstar = double(y_opt(1));
Rstar = double(y_opt(2));

%calculate probability density function
a = (1 / (sigmaL * sqrt(2 * pi))) * exp(-(k - DL)^2 / (2 * sigmaL^2));

%calculate minimum numeric value of Overstocking penalty
integrand_OP = (B * mu)*(1/Qstar) * (Qstar + Rstar - Z - k) * a;
OP1 = 0;
OP2 = Qstar + Rstar - Z;
OP_Numeric = int(integrand_OP, k, OP1, OP2);

%calculate minimum numeric value of Understocking Penalty
integrand_UP = (b * mu)*(1 / Qstar) * (z- Rstar +k) * a;
UP1 = Rstar-z;
UP2 = inf;
UP_Numeric = int(integrand_UP, k, UP1, UP2);

%calculate minimum numeric value of Shortage Cost
integrand_SC = (s*mu)*(1/Qstar)*(k-Rstar) * a;
SC1 = Rstar;
SC2 = inf;
SC_Numeric = int(integrand_SC, k, SC1, SC2);

%calculate minimum numeric value of Ordering Cost
OC_Numeric= (K*mu)/(Qstar);

%calculate minimum numeric value of Holding Cost
HC_Numeric= h*((Qstar/2)+Rstar-DL);


TC_Numeric=double(OC_Numeric)+double(HC_Numeric)+double(OP_Numeric)+double(UP_Numeric)+double(SC_Numeric);

disp("Optimized value of Q: " + Qstar);
disp("Optimized value of R: " + Rstar);
fprintf("Minimum value of TC: " +TC_Numeric);
```