DEVELOPMENT OF SINGLE VENDOR-TWO RETAILERS INVENTORY MODELS UNDER ORDER-UP-TO LEVEL POLICY WITH VMI

by

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AUTHOR'S DECLARATION

I, Shune Lae Sandar, declare that the research work carried out for this thesis was in accordance with the regulations of the Asian Institute of Technology. The work presented in it are my own and has been generated by me as the result of my own original research, and if external sources were used, such sources have been cited. It is original and has not been submitted to any other institution to obtain another degree or qualification. This is a true copy of the thesis, including final revisions.

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ABSTRACT

In the competitive business era, it is vital to save the system wide cost of SC and improving service level by utilizing appropriate inventory policies. In this research, VMI models with one vendor and two retailers under order-up-to level policy were focused. Two mathematical models were developed to deal with the case when demand is deterministic and no backorder is allowed, and the case when demand is stochastic and backorder and lost sales are considered. Those models were developed under periodic review policy to determine optimal order quantities and replenishment cycle time that can decrease the total cost. Then, numerical experiments and sensitivity analyses were conducted to analyze the applicability of the proposed models and to show the effects on optimal solution when changing of model's input parameters. Finally, conclusions and recommendations are presented.

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LIST OF ABBREVIATIONS

ASM	Adjusted Silver-Meal
CRP	Continuous Replenishment Program
GA	Genetic Algorithm
IIM	Integrated Inventory Management
IS	Information Sharing
JIT	Just In Time
MV-MR-SW	Multi-Vendor Multi-Retailer Single-Warehouse
OUTL	Order-Up-To-Level
PSO	Particle Swam Optimization
RMI	Retailer-Managed Inventory
SC	Supply Chain
VMI	Vendor Managed Inventory
VMI-CS	Vendor Managed Inventory with Consignment Stock

CHAPTER 1
INTRODUCTION

1.1 Background of the Study

A supply chain is a network including suppliers, manufactures, warehouses, retailers, and customers that distributes the products to the end buyers. The entire process is operated based on the market demand. Many companies often have a complicated supply chain network. In managing a supply chain, managers face challenges in controlling the inventory effectively.

In order to cope with these challenges, many researchers have concluded research in order to find appropriate solutions. Then, decentralized control and centralized control were developed in SC system (Petrovic et al. (1999)). All individual entities have the responsibility to determine the replenishment and pricing in decentralized control system (Taleizadeh et al. (2020)). This leads to the unbalanced profit between firms and can increase the cost for vendors. In order to share the total profit between vendors and retailers equally, to increase the total revenue of the whole SC, the decentralization members identify the decisions in such a way that the optimal solution in the centralized system can be achieved. In reality it should be noted that both systems have their own advantages and disadvantages depending on the specific environment.

Many corporations have recognized that there is lack of coordination between firms. As a consequence, they realized that the collaboration among different entities can lead to greater efficiency for the partners in SC. VMI model with one supplier and one buyer that positively affected the total cost reduction was firstly introduced by Dong & Xu (2002). VMI is an organized system that can benefit in the whole supply chain cost. In VMI system, supplier has the responsibility for the inventory decisions at buyers or retailers and takes full control of inventory (Rad et al. (2014)). Information sharing, direct contact with the customers and centralized decisions like replenishment decisions in VMI help for maximizing the revenue and minimizing the system wide cost of the SC. As a result, VMI will help to reduce cost and increase profit for all supply chain members.

Many studies have illustrated the benefits of VMI. Kuk (2004) found that implementing collaborated VMI in the electronic industry can minimize supply chain cost, thus leads to optimization of the production and planning capacity. Yao et al. (2007) highlighted that the obvious benefits are disproportionately spread between firms even though it can save costs. Qinglong et al. (2008) launched replenishment and scheduling strategies for VMI system. They showed that the modified inventory policy can reduce 5 to 20 percent cost than the standard one (Qinglong et al. (2008)).

A two-echelon mathematical model using total inventory cost as performance measure was conducted (Razmi et al. (2010)). The result indicated that VMI system is more advantageous in delivering lower costs in all conditions including back order rather than the traditional system. Mateen et al. (2015) studied a VMI system with multiple retailers reducing the expected cost of the system, and the demand is stochastic and shortage is permitted. Maio & Laganà (2020) showed that VMI in last-mile deliveries outcomes in potential saving costs.

There are many studies related to VMI models including single-vendor single-retailer system, single-vendor multi-retailer system and multi-vendor multi-retailer system based on various replenishment policies. Continuous review policy and periodic review policy are the most common policies. Under periodic review *(R, T)* policy, the inventory is measured every time interval of length T (Taleizadeh et al. (2020)). When the inventory level is at y, an order quantity $R - y$ is placed to reach the inventory level up to R. Under a continuous-review *(r, Q)* policy, a quantity Q is ordered if the inventory position is below the predetermined level i.e., the reorder point r (Taleizadeh et al. (2020)).

Related to development of mathematical models for VMI system, Mateen & Chatterjee (2015) developed various models such as retailer managed replenishment model and four models of VMI with one supplier and multiple retailers. They found that VMI system can handle increasing in setup cost by producing larger amount and providing deliveries faster. However, finding the solution for VMI models with multiple retailers becomes complex and hard. Moreover, determining the inventory level is difficult when back-orders are allowed. As a consequence, there are only a few studies allowing back-orders in VMI models. This issue will be tackled in this research.

1.2 Statement of the Problem

In the competitive business world, companies are trying to provide the value product to the customer with reasonable price while reducing the total supply chain costs. In other words, providing the service levels to end buyers by managing the activities in the supply chain internally as well as externally should be considered.

In managing the supply chain process, many challenges like customer services, cost control and how much to produce and when to deliver the finished products to the customers have been faced. As a consequence, how to effectively manage these challenges is critical for companies to gain more profit, to reduce system-wide cost and to be competitive in a market. Ouyang et al. (2004) presented a single-vendor single-retailer inventory model with stochastic demand and the allowable of shortage, thus can reduce the system expected cost. Although many studies of one-vendor single-retailer VMI systems have been conducted, there was still a research gap in one-vendor and multi-retailers models that is the most practical situation in inventory.

Therefore, many researches have been focused on multiple retailer's scenarios in the first decade of the twenty century. Rad et al. (2014) examined a two-echelon model with finite production of a vendor and two retailers. The traditional RMI system and the integrated VMI policy were compared, and the results highlight that VMI can save more supply chain cost in comparison to RMI. Zhang et al. (2007) developed a joint VMI model for one supplier and multiple retailers system under fixed production rate, fixed demand rate and unequal ordering cycles of the buyers.

For the problems of stochastic demand and uncertain replenishment lead time, Song & Dinwoodie (2008) developed IIM policy that is more efficient than RMI and VMI. Moreover, Mateen et al. (2015) examined the VMI model of a vendor and multiple retailers with common cycle for delivery and allowing the shortage. The achieved outcome utilizing the approximate expressions were somewhat similar to the actual optimal values.

On the one hand, the vendor in VMI system has to create more deliveries of product when the buyers prefer to maintain stock at a quite low level, and this can be problematic. Besides, it is necessary for the vendor to take full control since they are responsible for determining how much to produce, when to replenish to the retailers and when customers require more items. Last but not least, VMI also relies on the vendor and coordination with the buyers to precisely decide forecasts for demands in long term as well as short term. In the absence of accurate forecasts, VMI system can be unable to overcome peak demands and thus causing the Bull whip effect and the supply chain turns into more unstable.

The more retailers include in SC system, the more it is difficult to optimize the SC inventory not only for increasing profit but also for decreasing system cost when back orders are allowed. Considering backorder is more realistic in corporations when considering the replenishment systems. Therefore, it is necessary to tackle the condition of analyzing VMI taking into consideration the practice of partial backordering in VMI.

According to literature, there exists only one study developing a VMI system for one vendor and two buyers under periodic review *(R, T)* and continuous review *(r, Q)* replenishment policies where partial backordering is allowed (Taleizadeh et al. (2020)). They used the common delivery cycle to deliver the items to two retailers. However, there is still lack of a predefined level of back-ordering/backlogging, and the problem of how to allocate the production quantity in one production cycle to multiple retailers has not been dealt with. In this research, this problem will be addressed.

1.3 Objectives of the Study

The purpose of this research is to develop single vendor-two retailers inventory models under order-up-to level policy with VMI. In this research, VMI systems with one vendor and two retailers were considered under a periodic review, aiming to improve the cooperation of the vendor and retailers when partial backordering is allowed. There will be a predetermined level of backorder, and exceeding this level the demand will be lost. The production rate will be fixed that is higher than the total demand of both retailers. The retailers will receive the products if the whole amount of production is finished. And they share their demand information to the vendor.

1.4 Scope and Limitation

The system with the following characteristics will be examined throughout the research:

1. VMI system with one vendor and two retailers.
2. Back orders are permitted.

3. Retailer's demand information is shared with the vendor.
4. Order-up-to level inventory policy is considered.
5. The replenishment cycle for retailers may not be identical.

CHAPTER 2
LITERATURE REVIEW

There are centralized and decentralized systems in supply chain inventory system (Petrovic et al. (1999)). While the central authority brings the entire decision making process in centralized control system, all the individual entities have the responsibility to determine the replenishment and pricing in decentralized control system (Taleizadeh et al. (2020)). In traditional inventory system, all members of the supply chain make their own optimal decision separately, thus increasing the cost for other members. In order to cope with these problems, vendor managed inventory system (VMI) is utilized. In VMI system, the supplier has the responsibility for the inventory decisions at buyers or retailers and takes full control of inventory (Taleizadeh et al. (2020)). (Mateen & Chatterjee (2015)) show that control transfer is advantageous for VMI with regard to the order quantity and delivery cycle from the vendor to the retailers in the matter of the optimal replenishment policy.

Many research works have been developed in dynamic and statistic control policies based on various parameters which are considered as fixed over time or change over time. In managing VMI system, planning the inventory to be optimal is important, including decision making for inventory position and replenishment system (Han et al. (2017)). Inventory models including RMI, consignment inventory, and VMI-CS have been studied (Gümüş et al. (2008)) and the results highlight that VMI-CS is significance coordination approach for not only buyers but also suppliers. Tarhini et al. (2020) investigated a single vendor with multiple buyers cooperating under VMI-CS policy. Cooperation between buyers allows transshipments. Transshipment is the collaboration between buyers permitting shipments of goods between buyers and leads to system wide cost saving rather than direct shipments (Tarhini et al. (2020)).

2.1 Supply Chain Incorporation in VMI

There are two main features for VMI system: integration/coordination and information sharing between the retailers and the vendor for the effectiveness of the VMI (Mateen & Chatterjee (2015)). The process of sharing required amount and inventory level data from the buyer to the vendor is called information sharing (Savaşaneril & Erkip (2010)).

As a result of this process, the vendor can take full control of the inventory and buying function from the retailers. This is called integration (Yao et al. (2007)).

2.1.1 Information Sharing

The previous studies pointed out that the vendor can benefit from a partnership that provides inventory status and demand level when information sharing is allowed between vendor and retailers. Cachon & Fisher (2000) developed a model with the information sharing for a single-vendor multiple-retailer with stochastic demand, then compared with the traditional policy that does not use information exchange process. The results expressed that 2.2% of supply chain cost can be saved with the information sharing policy. Another research proved that information sharing can enhance the various benefits in savings of whole supply chain cost between firms in the VMI system Yao & Dresner (2008).

2.1.2 Supply Chain Integration

In today's technology world, most of the companies find out the technologies and strategies to decrease the system costs, to increase profit and to be competent in the global markets. In order to deal with these problems, integration strategies like vendor managed inventory (VMI), just in time manufacturing (JIT), CRP and quick response have been pointed out. Among those strategies, VMI can take more benefits to the supply chain than others. This is because the supplier can monitor the retailer demand directly and can use this information to plan production runs, schedule deliveries, and manage order volumes and inventory levels at the buyer's stock-keeping facilities (Yao et al. (2007)).

2.2 Continuous and Periodic Review

There are two major parts of review in controlling the inventory system, i.e., continuous review and periodic review policies (Axsäter (2015)). The inventory level is checked continuously in continuous review policy. When the inventory position is sufficiently low, the buyer will order. After a certain lead time, the buyer will get the order (Vandeput (2020)). The waiting time between triggering the order quantity and receiving it is called lead time. In periodic review policy, the inventory position is monitored every constant single time and the buyers will order when the inventory level is below or at the target level position (Axsäter (2015)). Both reviews have benefits and drawbacks. Continuous review is more benefit for the high demand items while period review is benefit for the lower demand items and for coordination of orders for different items (Axsäter (2015)).

2.3 Ordering Policies

2.3.1 (R, Q) Policy

This policy is a fixed order quantity policy. When the inventory level decreases under the reorder point R, it is necessary to restock for a certain amount of quantity Q. This policy can sometimes be denoted as *(R, nQ)* where there is a need to replenish more than one quantity of Q. Although the inventory level gets the reorder point in continuous review, it is below the reorder point in periodic review (Axsäter (2015)).

2.3.2 (s, S) Policy

The (s, S) policy is close to the (R, Q) policy (Axsäter (2015)). When the inventory level goes under the reorder point s, the maximum amount S is ordered for restoring the inventory level until a target. In this policy, the reorder point is minimum and the number of restored inventory level is maximum. The separation between (s, S) and (R, Q) is that (s, S) policy thinks of the amount of the inventory falling from the reorder level. This policy is known as minimum/maximum inventory policy (Axsäter (2015)).

2.3.3 Order-up-to Level Policy

There is a target level for inventory to help determine the order quantity in this policy. The required amount of order quantity is put to reach that target level. Therefore, the size of the order can be varied depending on the amount of on-hand inventory. this policy is the same as the *(s, S)* policy, i.e., the inventory level will be reviewed at the end of each period. However, the required quantity amount will always be ordered regardless of the observed inventory level.

2.4 Review of Previous Studies Conducted on VMI Systems

Dong & Xu (2002) investigated how VMI model with a vendor and a buyer affects supply chain. They found the improvements through switching to VMI from the conventional system and the essence of VMI impacts on purchase price, purchase quantity and profit. Ouyang et al. (2004) conducted a single vendor single retailer model with the permitting of shortage during the lead time. Two models were developed to consider the case with normal distribution demand during lead time following and the case without distribution. Both models can save a significant amount of system cost.

Zhang et al. (2007) developed a joint model of single vendor and multiple buyers with constant production and demand rates under different ordering as well as multiple replenishment cycles. The results showed that firms can significantly save cost by revising the production cycle of the vendor. Razmi et al. (2010) developed two-echelon mathematical model with one vendor and one buyer. The comparison of tradition and VMI system performances were conducted. They concluded that VMI system is better than the tradition method for reducing the cost in all conditions including back order.

Sadeghi et al. (2013) proposed the multi-vendor multi-retailer single-warehouse (MV-MR-SW) model with the limitation of order quantity per year and area of warehouse. Two algorithms of PSO and GA were developed by Sadeghi et al. (2013). The PSO enhance more for the restrictions on central warehouse according to the results (Sadeghi et al. (2013)). Mateen & Chatterjee (2015) developed single vendor-multiple retailer models with various methods in detailed. They concluded that selecting the correct replenishment policy, operation methods and adjusting the cost of transport can enhance benefits to VMI system and benefits are different between firms (Mateen & Chatterjee (2015)).

Kannan (2015) considered a model of two-echelon supply chains for stochastic demand. He emphasized on matching the total cost by using ASM and Least Unit Cost heuristics, and his approach provided that the vendor has more chances to control the inventory when retailers obtain the same goods (Kannan (2015)). Kaasgari et al. (2016) presented a model for perishable products with single supplier and multiple retailers. GA and PSO algorithms were used for defining order quantity, production and replenishment rates, and the finding highlighted that PSO algorithm is well performed (Kaasgari et al. (2016)).

Han et al. (2017) considered a VMI model where the vendor and retailer are manufacturers. They aimed to analyze the cooperation of the members in decentralized three-echelon system including third-party logistics, and their model was a new manufacturer-manufacturer VMI system (Han et al., 2017). Tarhini et al. (2020) established a model of a vendor with several buyers under VMI with CS policy. The finding showed that the transshipment model between retailers can decrease the total cost and it can also reduce transportation and holding costs (Tarhini et al. (2020)).

Table 2.1

Recent Studies Based in VMI Systems

Paper	System	Key finding
Dong & Xu (2002)	single vendor-single retailer	The VMI can positively affect the profit, purchase price and purchase quantity.
Ouyang et al. (2004)	single vendor-single retailer	By optimizing ordering quantity, lead time, delivery cycles and reorder point, the total cost is minimized (Ouyang et al. (2004)).
Zhang et al. (2007)	single vendor-multiple retailers	The proposed model can significantly save total cost by revising the production cycle of the vendor (Zhang et al. (2007)).
Razmi et al. (2010)	single vendor-single retailer	The VMI system performs better for lowering the cost from all around sides (Razmi et al. (2010)).
Sadeghi et al. (2013)	multiple vendors-multiple retailers	The hybrid PSO algorithm performs better for the limitation of order quantity per year and area of warehouse (Sadeghi et al. (2013)).
Mateen & Chatterjee (2015)	single vendor–multiple retailer system	Selecting the correct replenishment policy, operation methods and adjusting the cost of transport can enhance benefits to VMI system and benefits between firms are different (Mateen & Chatterjee (2015))
Kannan (2015)	single vendor-multiple retailers	The vendor gets more chances to consolidate the demand and to determine the replenishment cycle times when there are many retailers obtaining the same goods (Kannan (2015)).

Paper	System	Key finding
Kaasgari et al. (2017)	single vendor-multiple retailers	The increase of deterioration rate, discount and demand can increase the total cost (Kaasgari et al. (2017)).
Han et al. (2017)	single vendor-multiple retailers	The proposed model was new manufacturer-manufacturer VMI for three-echelon system to equalize the total cost splitting (Han et al. (2017)).
Tarhini et al. (2020)	single vendor-multiple retailers	The transshipment model between retailers can reduce the total cost encountered by both supplier and retailers (Tarhini et al. (2020)).

2.5 Review of Previous Studies Conducted on VMI Replenishment Policy under Stochastic Demand

Song & Dinwoodie (2008) considered the problem of SC with uncertain demands and lead times. They developed IIM policy using the stochastic dynamic that is more efficient than RMI and two pulled type VMI systems. Yao & Dresner (2008) considered a two-level supply chain that consists of one manufacturer and one retailer by implementing IS, CRP, and VMI. It is shown that using IS, CRP, and VMI save inventory cost between organizations but that are not proportionally shared to corporations.

Qinglong et al. (2008) introduced joint replenishment and scheduling strategies for VMI system. They showed that the modified inventory policy can reduce 5 to 20 percent cost than the standard *(s, S)* policy (Qinglong et al. (2008)). Rad et al. (2014) examined a two-echelon model with finite production of a vendor and two retailers. The traditional RMI system and the integrated VMI policy were compared, and the results highlight that VMI can save more supply chain cost in comparison to RMI.

Choudhary & Shankar (2015) observed the switching model of single vendor and multiple retailers from the Information sharing to VMI system. The expected cost, inventory level and shipment volumes were considered under stochastic demand. The results revealed that considerable performance can be get while the setup cost of supplier is minimal and issue of order is maximum (Choudhary & Shankar (2015)). Mateen & Chatterjee (2015) developed single vendor-multiple retailer models with various methods in detailed. They concluded that selecting the correct replenishment policy, operation methods and adjusting the cost of transport can enhance benefits to VMI system and benefits are different between firms (Mateen & Chatterjee (2015)).

Pacheco et al. (2017) proposed a new OUTL policies by changing reorder points and lot size under continuous review. That resulted in lower lot sizes in terms of bullwhip effect reduction, decreased stockout and improved service level (Pacheco et al., 2017). Taleizadeh et al. (2020) developed the VMI system of one vendor and two buyers with backordering and lost sales. *(r, Q)* and *(R, T)* policies are applied to match the performances. He concluded that when shortage is allowed, both policies under VMI have advantages and disadvantages in separate situations.

Table 2.2

Recent Studies of Replenishment Policy Based on Stochastic Demand of VMI Systems

Paper	**Demand**	**Replenishment policy**	**Controlled factors**	**Supply chain level**
Song & Dinwoodie (2008)	Stochastic	Continuous review	Order quantity Stock level	One vendor-One retailer
Qinglong et al. (2008)	Stochastic	Continuous review	Order quantity Can-order level	One vendor-Multiple retailers
Yao & Dresner (2008)	Stochastic	Periodic review	Replenishment frequency Order quantity Safety stock	One vendor-One retailer
Rad et al. (2014)	Stochastic	Continuous review	Demand Ordering cost Holding cost	One vendor-Two retailers

Paper	**Demand**	**Replenishment policy**	**Controlled factors**	**Supply chain level**
Choudhary & Shankar (2015)	Stochastic	Continuous review	Expected cost Lot sizes Inventory level	One vendor-Multiple retailers
Mateen & Chatterjee (2015)	Stochastic	Periodic review	Safety factor Replenishment cycle	One vendor-Multiple retailers
Pacheco et al. (2017)	Stochastic	Continuous review	Reorder point Lot sizes	One vendor-Multiple retailers
Taleizadeh et al. (2020)	Deterministic Stochastic	Periodic review Continuous review	Order quantity Back-order level	One vendor-Two retailers

CHAPTER 3
MATHEMATICAL MODEL DEVELOPMENT

This research focus on deriving a VMI system with one vendor and two retailers under order-up-to level policy. Two mathematical models will be derived in this chapter to deal with the case when demand is deterministic and no backorder is allowed, and the case when demand is stochastic and backorder and lost sales are considered.

3.1 Mathematical Model for One Vendor and Two Retailers Considering No Backorder under Deterministic Demand

In this section, a VMI system with one vendor and two retailers with no backorder was considered. It is assumed that the replenishment cycles for the two retailers are different and the demand rate is deterministic. The following notations will be used in this section.

i	=	Index for retailers
j	=	Time index
T	=	Basic cycle time
k_i	=	An integer that represents how many basic cycles in one replenishment cycle of retailer i
k_v	=	An integer that represents how many basic cycles in one replenishment cycle of vendor, $k_v = LCM(k_1, k_2)$; LCM = The least common multiple
t_i	=	$k_i T$: Replenishment cycle for retailer i
t_v	=	$k_v T$: Replenishment cycle for vendor
q_i	=	Order quantity of retailer i
Q	=	Order quantity of vendor
d_i	=	Average demand rate of retailer i
D	=	Average demand rate of vendor ($D = \sum d_i$)
h_{ri}	=	Holding cost of retailer i per unit per time unit
h_v	=	Holding cost of vendor per unit per time unit
H_{ri}	=	Holding cost of retailer i per replenishment cycle time
H_v	=	Holding cost of vendor per replenishment cycle time
s_{ri}	=	Transportation cost to retailer i per unit

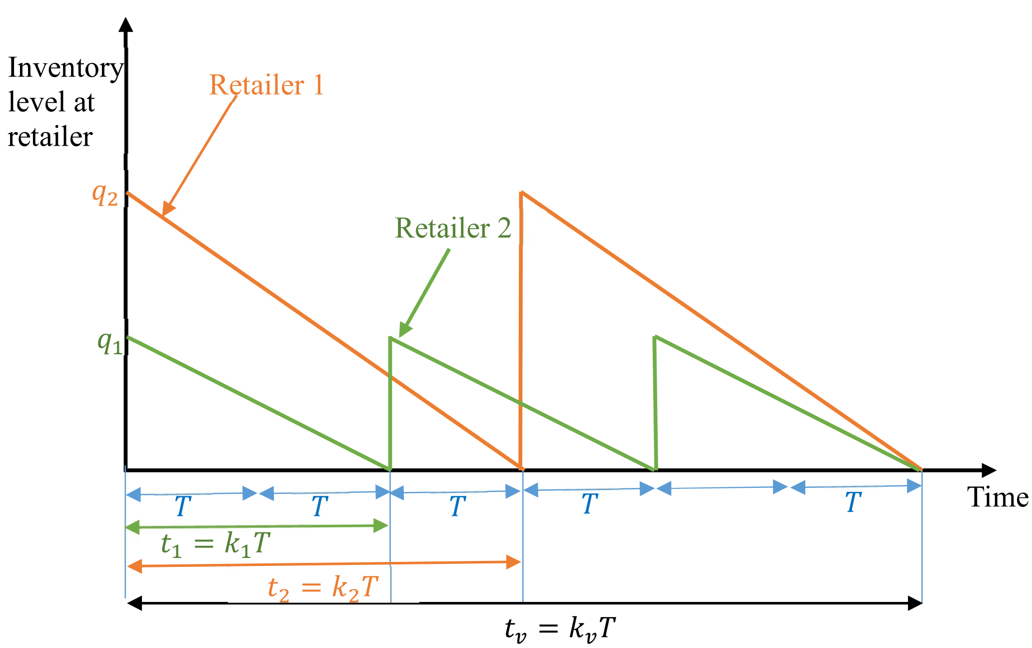
s_v	=	Transportation cost to vendor per unit
f_{ri}	=	Replenishment cost per shipment to retailer i
f_v	=	Replenishment cost per shipment to vendor
S_{ri}	=	Ordering cost of retailer i per replenishment cycle time
S_v	=	Ordering cost of vendor per replenishment cycle time
TC_r	=	Total cost of both retailers
TC_v	=	Total cost of vendor
TC	=	Total cost of supply chain

3.1.1 Total Cost Function for Two Retailers

Consider a two-retailers system with different replenishment cycles $t_1 = k_1T$ for retailer 1 and $t_2 = k_2T$ for retailer 2, respectively. The order quantities for retailer 1 and retailer 2 are q_1 and q_2 respectively. Figure 3.1 shows the replenishment cycles of the two retailers for a basic delivery cycle of the vendor. Holding cost of each retailer is the multiplication of the holding cost per unit per unit time and the time-accumulated inventory holding which is the area under the triangle. Ordering cost for each retailer in a replenishment cycle is the summation of transportation cost and replenishment cost.

Figure 3.1

Replenishment Cycles of the Retailers for a Basic Delivery Cycle of the Vendor



Equation 3.1 shows the holding cost and Equation 3.2 presents the ordering cost at the retailer.

It is noted that:

Replenishment cycle time for retailer 1, $t_1 = k_1 T$

Replenishment cycle time for retailer 2, $t_2 = k_2 T$

$$\text{Holding cost/replenishment cycle time}$$
$$= (\frac{\text{replenishment cycle time} * \text{order quantity}}{2}) * \text{holding cost/unit}$$

$$H_{ri} = \frac{t_i q_i}{2} \times h_{ri}$$

$$H_{ri} = \frac{k_i T q_i}{2} \times h_{ri} \tag{3.1}$$

$$\text{Ordering cost/replenishment cycle time}$$
$$= (\text{order quantity} \times \text{transportation cost per unit})$$
$$+ \text{replenishment cost per shipment}$$

$$S_{ri} = q_i s_{ri} + f_{ri} \tag{3.2}$$

in which $q_i = d_i t_i = d_i k_i T$.

The total cost of the retailers is the summation of total holding cost and total ordering cost of the two retailers.

$$\text{Total cost of retailers} = \text{total holding cost} + \text{total ordering cost}$$

$$TC_r = \sum_{i=1}^{2} (H_{ri} + S_{ri}) \tag{3.3}$$

$$TC_r = \sum_{i=1}^{2} (\frac{k_i T q_i}{2} h_{ri}) + (q_i s_{ri} + f_{ri}) \tag{3.4}$$

Hence, the total cost of two retailers per unit time can be computed as in Equation 3.5.

$$Total\ cost\ of\ two\ retailers\ /\ unit\ time = \frac{TC_r}{t_i}$$

$$Total\ cost\ of\ two\ retailers\ /\ unit\ time$$
$$= \sum_{i=1}^{2} \frac{(\frac{k_i T q_i}{2}) h_{ri} + (q_i s_{ri} + f_{ri})}{k_i T} \tag{3.5}$$

3.1.2 Total Cost Function for Vendor

The total cost function of the vendor includes the total ordering cost and the total holding cost. Figure 3.2 shows the two retailers with different replenishment patterns and the vendor's replenishment pattern of $t_v = k_v T$. The order quantity of vendor is $Q = \sum_{i=1}^{2} q_i \frac{t_v}{t_i}$. In order to determine whether a retailer will order at the end of a period j or not, decision variables (X_i^j) are defined as follow:

$$\text{For retailer i: } X_i^j = \begin{Bmatrix} 1; if\ an\ order\ is\ placed\ at\ the\ end\ of\ period\ j \\ 0; if\ no\ order\ is\ placed\ at\ the\ end\ of\ period\ j \end{Bmatrix} \tag{3.6}$$

where: $j = 0,1,2, \ldots, k_v - 1$

It is noted that,

$$X_i^j = \begin{Bmatrix} 1; if\ (j\ mod\ k_i) = 0 \\ 0; if\ (j\ mod\ k_i)\ > 0 \end{Bmatrix} \tag{3.7}$$

When the retailer orders at the end of period j, ($j\ mod\ k_i$) will return 0 and $X_i^j = 1$. When the retailer does not order, ($j\ mod\ k_i$) will return a non-zero reminder and $X_i^j = 0$. For example, at the end of period 2 in one replenish cycle of the vendor in Figure 3.3, $X_1^2 = 1$ and $X_2^2 = 0$, respectively for the two retailers. It means that at the end of period 2 retailer 1 will order and retailer 2 will not order. The general equation for calculating the vendor's remaining inventory level at specific points in time can be developed as follows.

Inventory level at vendor at the beginning of each period is as follows:

Inventory level at the beginning of period 1: $INV_1 = Q - (q_1 X_1^0 + q_2 X_2^0)$

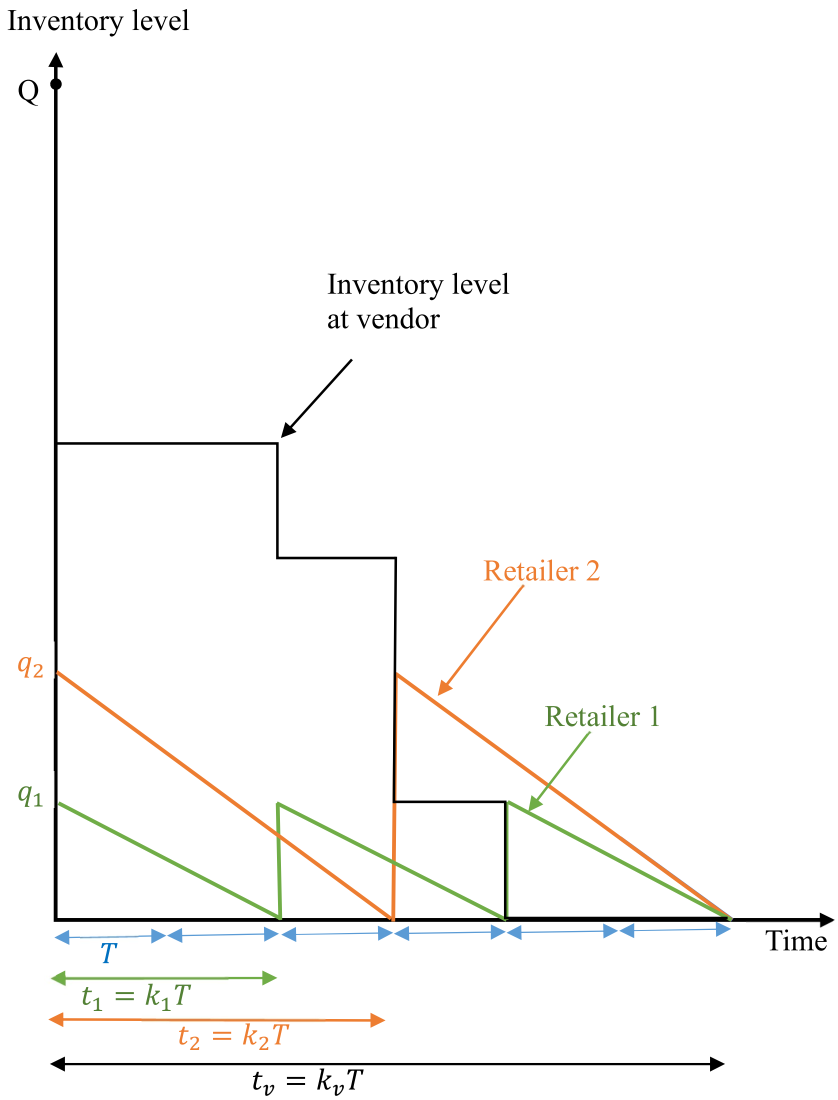
Inventory level at the beginning of period 2: $INV_2 = INV_1 - (q_1 X_1^1 + q_2 X_2^1)$

⋮

Inventory level at the beginning of period k_v: $INV_{k_v} = INV_{k_{v-1}} - \left(q_1 X_1^{k_{v-1}} + q_2 X_2^{k_{v-1}}\right)$

Figure 3.2

Inventory Replenishment Pattern at Vendor



By summing up the above terms together, the remaining inventory at the beginning of each period can be developed as below:

$$INV_1 = Q - \sum_{i=1}^{2}\sum_{j=0}^{0} q_i X_i^j$$

$$INV_2 = Q - \sum_{i=1}^{2}\sum_{j=0}^{1} q_i X_i^j$$

$$INV_3 = Q - \sum_{i=1}^{2}\sum_{j=0}^{2} q_i X_i^j$$

$$\vdots$$

$$INV_{k_v} = Q - \sum_{i=1}^{2}\sum_{j=0}^{k_{v-1}} q_i X_i^j = 0$$

The holding cost of the vendor can be computed by using the above equations.

Holding cost for vendor, $H_v\ for\ period\ 1 :\ INV_1.T.h_v$

Holding cost for vendor, $H_v\ for\ period\ 2 :\ INV_2.T.h_v$

Holding cost for vendor, $H_v\ for\ period\ 3 :\ INV_3.T.h_v$

$$\vdots$$

Holding cost for vendor, $H_v\ for\ period\ k_v :\ INV_{k_v}.T.h_v$

$$Total\ holding\ cost\ for\ vendor = T.h_v \sum_{n=1}^{k_v} INV_n \tag{3.8}$$

Ordering cost for vendor in one replenishment cycle is the summation of transportation cost and replenishment cost. This can be stated as follow:

$$\begin{aligned}\text{Ordering cost of vendor}\\ &= (\text{order quantity} \times \text{transportation cost per unit})\\ &+ \text{replenishment cost per shipment}\end{aligned}$$

$$S_v = (Q.s_v) + f_v \tag{3.9}$$

The total cost of the vendor for one replenishment cycle can be obtained by adding the total holding cost and the total ordering cost.

$$\text{Total cost of vendor} = \text{total holding cost} + \text{total ordering cost}$$

$$TC_v = \left(T.h_v \sum_{n=1}^{k_v} INV_n \right) + (Q.s_v + f_v) \tag{3.10}$$

$$Total\ cost\ of\ vendor\ per\ unit\ time = \frac{TC_v}{t_v} \tag{3.11}$$

$$Total\ cost\ of\ vendor/unit\ time = \frac{\left(T.h_v \sum_{n=1}^{k_v} INV_n\right) + (Q.s_v + f_v)}{k_v * T} \quad (3.12)$$

3.1.3 Total Cost Function for Supply Chain

The total cost of supply chain per unit time can be derived as follows:

$$\mathrm{TC} = \text{Total cost of vendor/unit time} + \text{Total cost of two retailers /unit time}$$

$$TC = \frac{\left(T.h_v \sum_{n=1}^{k_v} INV_n\right) + (Q.s_v + f_v)}{k_vT} + \sum_{i=1}^{2} \frac{(\frac{k_iTq_i}{2})h_{ri} + (q_is_{ri} + f_{ri})}{k_iT} \quad (3.13)$$

3.2 Mathematical Model for One Vendor and Two Retailers Considering Backorder and Lost Sales under Stochastic Demand

In this section, a VMI system with one vendor and two retailers with backorder and lost sales is considered under a periodic review (R, T) policy. It is assumed that the two retailers have a common cycle time, and the demand rate is stochastic. The order will be placed at the end of every replenishment cycle. Then, the order will arrive after a fixed lead time. The order quantities will not be the same for every period depending on the on-hand inventory amount. The following notations are used in this section:

i	=	Index for retailers
T	=	Common cycle
T_i°	=	The point in time that occurs the backorder and lost sales
t_v	=	kT : Replenishment cycle for the vendor
k	=	An integer that represents the number of cycles in one replenishment cycle of vendor
d_i	=	Average demand rate of retailer i (unit per time unit)
d_i'	=	Average fulfilled demand rate of retailers i
D	=	Average demand rate of vendor ($D = \sum d_i'$) unit per time unit
q_i	=	Order quantity of retailer i
Q	=	Order quantity of vendor
π_{bi}	=	The unit cost of back-ordering given by retailer i
π_{li}	=	The unit cost of lost sale given by retailer i
SS_i	=	Safety stock of retailer i

Z_α	=	Safety factor with stock-out probability α
σ	=	Standard deviation of demand through lead time
R_i	=	Maximum inventory level of retailer i under periodic review (R, T) system
L_i	=	Length of lead time of retailer i (unit time)
β	=	Back-ordering rate
$1-\beta$	=	Lost-sale rate
X_i	=	Demand during the interval $(T+L_i)$ of retailer i, which has a probability density function f_X with finite mean $d_i.(T+L_i)$ and standard deviation $\sigma_i\sqrt{T+L_i}$
h_{ri}	=	Holding cost of retailer i per time unit
h_v	=	Holding cost of vendor per time unit
H_{ri}	=	Holding cost of retailer i per replenishment cycle time
H_v	=	Holding cost of vendor per replenishment cycle time
s_{ri}	=	Transportation cost to retailer i per unit
s_v	=	Transportation cost to vendor per unit
f_{ri}	=	Replenishment cost per shipment to retailer i
f_v	=	Replenishment cost per shipment to vendor
S_{ri}	=	Ordering cost of retailer i per replenishment cycle time
S_v	=	Ordering cost of vendor per replenishment cycle time
TC_r	=	Total cost of both retailers
TC_v	=	Total cost of vendor
TC	=	Total cost of supply chain

3.2.1 Total Cost Function for Two Retailers

In this section, the total cost function for two retailers will be developed. Figure 3.3 shows the inventory level with stock out under periodic review policy. The order quantities for each period will be different based on the amount of on-hand inventory. And the target stock level R_i is the sum of expected demand during the protection interval $(T+L_i)$ and the safety stock (SS).

$$R_i = SS_i + d_i.(T+L_i)$$

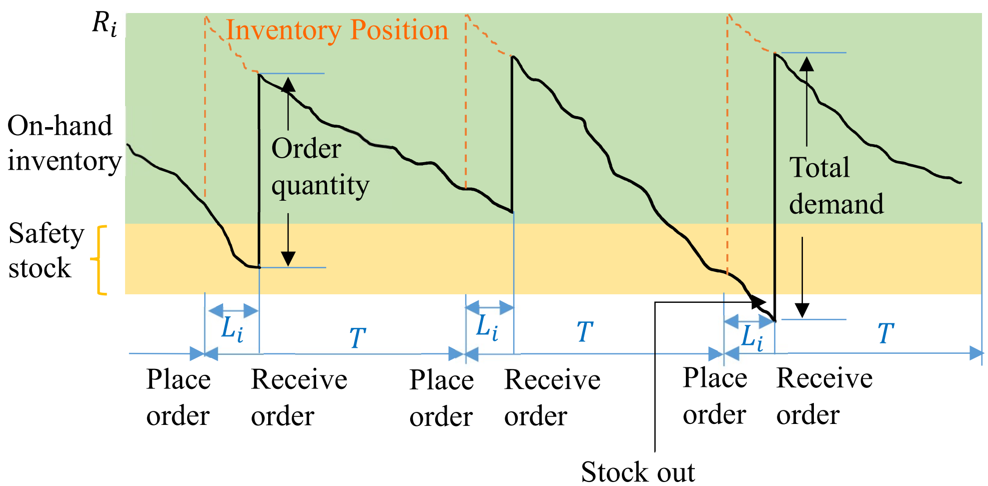
Safety stock can be defined as the product of standard deviation of demand during a review cycle plus lead time with safety factor Z_α in which with α is the stock-out probability.

$$\text{Safety stock} = Z_\alpha \times \textit{standard deviation of demand during protection interval } (T + L_i)$$

$$SS_i = Z_\alpha\, \sigma\sqrt{T + L_i}$$

Figure 3.3

Inventory Level with Stock Out under Periodic Review



Assume that the demand during a review cycle and lead time follows a normal distribution with mean $d_i.(T + L_i)$ and standard deviation $\sigma_i\sqrt{T + L_i}$,then $R_i = SS_i + d_i.(T + L_i)$. The expected shortage quantity $E(X - R_i)^+$ can be determined as followed:

$$E(X - R_i)^+ = \int_{R_i}^{\infty} (x - R_i) f_i(x)\, dx$$

$$E(X - R_i)^+ = \int_{R_i}^{\infty} (x - R_i) \frac{1}{\sigma\sqrt{2\pi(T + L_i)}} e^{-\frac{1}{2}\left(\frac{x - d_i.(T+L_i)}{\sigma\sqrt{T+L_i}}\right)^2} dx \qquad (3.14)$$

Then, the backlogged amount and lost sales amount in one cycle can be determined as followed:

$$Backlog\ amount = \beta.E(X - R_i)^+ \tag{3.15}$$

$$Lost\ sale\ amount = (1 - \beta).E(X - R_i)^+ \tag{3.16}$$

Figure 3.4

A Replenishment Cycle with Backorder and Lost Sales for Retailer i

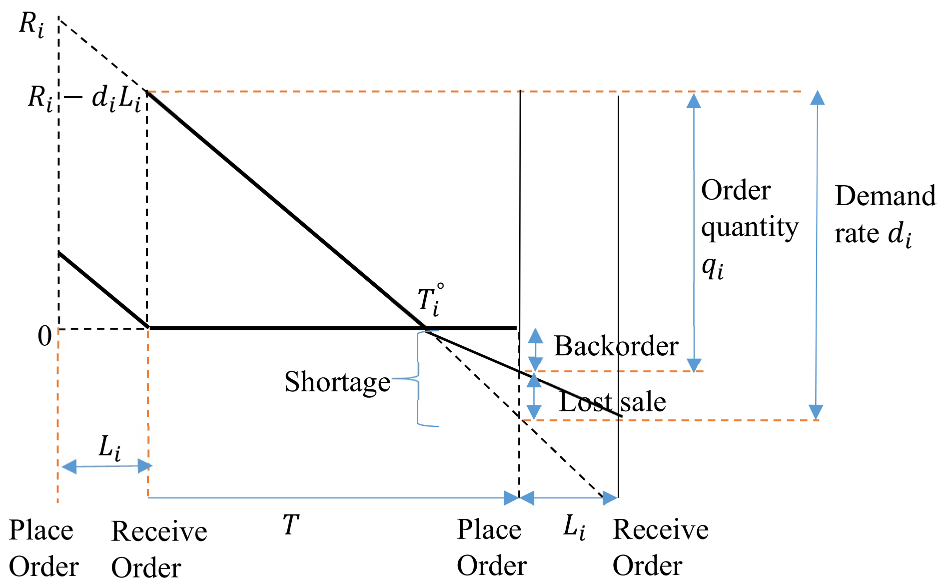


Figure 3.4 shows the replenishment cycle with back order and lost sales for retailer i. The point in time T_i° is derived as $\frac{R_i}{d_i}$ where ($T \geq T_i^{\circ}$). The expected holding cost of each retailer is the multiplication of the holding cost per unit per unit time and the time-accumulated inventory holding.

$$Holding\ cost\ of\ retailer\ i = \left(\frac{R_i * T_i^{\circ}}{2} - (R_i - d_iL_i)L_i\right)h_{ri} \tag{3.17}$$

Ordering cost for each retailer is the summation of transportation cost and replenishment cost. The order amount in each cycle is $q_i = d_iT_i^{\circ} + \beta.E(X - R_i)^+ = R_i + \beta.E(X - R_i)^+$.

$$Ordering\ cost\ of\ retailer\ i = q_is_{ri} + f_{ri} \tag{3.18}$$

The backordering cost is the multiplication of the unit cost of backordering and the expected backorder quantity. The lost sales cost is the multiplication of unit cost of lost sales and the expected lost sales quantity.

$$Back\ ordering\ cost = \pi_{bi}.\beta.E(X - R_i)^+ \tag{3.19}$$

$$Lost\ sales\ cost = \pi_{li}.(1 - \beta).E(X - R_i)^+ \tag{3.20}$$

Therefore, the total cost of two retailers is the summation of total ordering cost, total holding cost and total backordering cost and total lost sales cost for each retailer.

$$\begin{aligned}\text{Total cost of retailers} &= \text{total ordering cost} + \text{total holding cost}\\ &+ \text{total back ordering cost} + \text{total lost sales cost}\end{aligned}$$

$$\begin{aligned}TC_r &= \sum_{i=1}^{2}\left(\frac{R_i * T_i^{\circ}}{2} - (R_i - d_iL_i)L_i\right)h_{ri} + (q_is_{ri} + f_{ri}) + \pi_{bi}.\beta.E(X - R_i)^+\\ &+\pi_{li}.(1-\beta).E(X - R_i)^+\end{aligned} \tag{3.21}$$

$$Total\ cost\ of\ retailer\ i\ per\ unit\ time = \frac{TC_r}{T}$$

$$= \sum_{i=1}^{2}\frac{\left(\frac{R_i * T_i^{\circ}}{2} - (R_i - d_iL_i)L_i\right)h_{ri} + (q_is_{ri} + f_{ri}) + \pi_{bi}.\beta.E(X - R_i)^+ + \pi_{li}.(1-\beta).E(X - R_i)^+}{T} \tag{3.22}$$

3.2.2 Total Cost Function for Vendor

In this section, the total cost function for vendor will be developed. Figure 3.5 shows the inventory cycle for VMI system during period $t_v = kT$. The average fulfilled demand rate of retailers i, d_i' is as followed.

$$d_i' = \frac{d_i * T_i^{\circ} + \beta.E(X - R_i)^+}{T}$$

Then, the average demand rate of the supplier is $D = \sum_{i=1}^{2} d_i'$. If the supplier does not deliver the product to the retailers and the demand is fulfilled directly from the inventory of the supplier, then the time weighted accumulated inventory in a cycle at the supplier is:

$$D.t_v * \frac{t_v}{2} = \frac{D * t_v^2}{2}$$

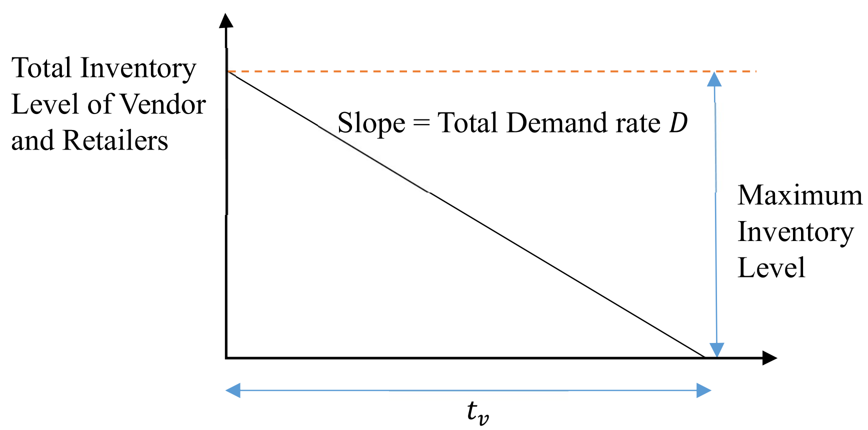
Therefore, the actual time-weighted accumulated inventory in a cycle at the supplier is:

$$\frac{D * t_v^2}{2} - \sum_{i=1}^{2} \frac{T * q_i}{2} * k$$

in which $k = \frac{t_v}{T}$.

Figure 3.5

An Inventory Cycle for VMI System



Then, the inventory holding cost per time unit of the vendor is:

$$Holding\ cost\ of\ vendor/unit\ time = \frac{1}{t_v}\left(\frac{D * t_v^2}{2} - \sum_{i=1}^{2} \frac{T * q_i}{2} * k\right) * h_v \quad (3.23)$$

Ordering cost for the vendor is the summation of transportation cost and replenishment cost. The amount of order in each cycle is $Q = k.\sum_{i=1}^{2} q_i$.

$$Ordering\ cost\ of\ vendor = Qs_v + f_v \quad (3.24)$$

Therefore, the total cost of the vendor is the summation of total ordering cost and total holding cost.

$$\text{Total cost of the vendor } = \text{total ordering cost} + \text{total holding cost}$$

$$Total\ cost\ of\ vendor\ per\ unit\ time = \frac{TC_v}{t_v}$$

$$= \frac{\left(\frac{D * t_v^2}{2} - \sum_{i=1}^{2} \frac{T * q_i}{2} * k\right) * h_v + (Qs_v + f_v)}{t_v} \quad (3.25)$$

3.2.3 Total Cost Function for Supply Chain

The total cost of supply chain per unit time can be derived as follows:

$$\mathrm{TC} = \text{Total cost of vendor/unit time} + \text{Total cost of retailer i/unit time}$$

$$TC = \frac{\left(\frac{D * t_v^2}{2} - \sum_{i=1}^{2} \frac{T * q_i}{2} * k\right) * h_v + (Qs_v + f_v)}{t_v} + \sum_{i=1}^{2} \frac{\left(\frac{R_i * T_i^{\circ}}{2} - (R_i - d_i L_i) L_i\right) h_{ri} + (q_i s_{ri} + f_{ri}) + \pi_{bi}.\beta.E(X - R_i)^+ + \pi_{li}.(1-\beta).E(X-R_i)^+}{T} \tag{3.26}$$

In which

$R_i = SS_i + d_i.(T + L_i)$

So,

$$TC = \frac{\left(\frac{D * k^2 T^2}{2} - \sum_{i=1}^{2} \frac{T * q_i}{2} * k\right) * h_v + (Qs_v + f_v)}{kT} + \sum_{i=1}^{2} \frac{\left(\frac{SS_i + d_i.(T + L_i) * T_i^{\circ}}{2} - (SS_i + d_i.(T + L_i) - d_i L_i) L_i\right) h_{ri} + (q_i s_{ri} + f_{ri}) + \pi_{bi}.\beta.E(X - R_i)^+ + \pi_{li}.(1-\beta).E(X-R_i)^+}{T}$$

$$TC = \frac{\left(\frac{D * k^2 T^2}{2} - \sum_{i=1}^{2} \frac{T * q_i}{2} * k\right) * h_v + (Qs_v + f_v)}{kT} + \sum_{i=1}^{2} \frac{\left(\frac{SS_i + d_i.(T + L_i) * T_i^{\circ}}{2} - (SS_i + d_i.T) L_i\right) h_{ri} + (q_i s_{ri} + f_{ri}) + \pi_{bi}.\beta.E(X - R_i)^+ + \pi_{li}.(1-\beta).E(X-R_i)^+}{T}$$

CHAPTER 4
NUMERICAL EXPERIMENTS

In this chapter, numerical experiments were conducted using fminsearchbnd function in MATLAB in order to illustrate the applicability of the developed mathematical model. By using the fminsearchbnd function, the optimal solution can be determined.

4.1 Numerical Experiment for One Vendor and Two Retailers VMI System with No Backorder and Lost Sales

In this section, numerical experiments of a VMI system with two retailers and one vendor was conducted using MATLAB. By using the fminsearchbnd function in MATLAB, the optimal values for the two decision variables related to basic cycles in one replenishment cycle of the two retailers, i.e., (k_1, k_2) and the optimal total cost can be determined. The input parameters in the base case for the two retailers and vendor are given in table 4.1.

Table 4.1

Input Data for Total Cost Function of No Backorder System

Parameter	Retailer 1	Retailer 2	Vendor
d_i	60	80	-
h_{ri}	2	3	
h_v	-	-	4
S_{ri}	1800	1600	-
S_v	-	-	3200
f_{ri}	2000	2200	-
f_v			3500
T	1	1	1

For the above input parameters, using lower bound = [1,1] and upper bound = [100,100], the optimal values which result from the fminsearchbnd solver in Matlab are: $k_1 = 5, k_2 = 5$ and the value of the total cost function is 6255.

4.2 Sensitivity Analysis for One Vendor and Two Retailers VMI System with No Backorders and Lost Sales

In this section, the effects of input parameters are examined. The input parameters such as the demand rates of the two retailers, the holding costs of the two retailers and the vendor, the ordering costs of the two retailers and the vendor, the replenishment costs of the two retailers and the vendor, and the basic cycle time are investigated.

4.2.1 The Impacts of Changing the Demand Rate of Retailer

The effects of demand rates of the two retailers are examined by changing those one by one (the value of the demand rate is changed from 50 to 70 for retailer 1 and from 70 to 90 for retailer 2) while the other parameters are maintained at initial value. The optimal results are illustrated in the following tables.

Table 4.2

Effects of Demand Rate of Retailer 1

d_1	d_2	k_1	k_2	k_v	q_1	q_2	Q	Total cost per unit time
50	80	5	5	5	250	400	650	6245.0
55	80	5	5	5	275	400	675	6250.0
60	80	5	5	5	300	400	700	6255.0
65	80	5	5	5	325	400	725	6260.0
70	80	2	6	6	140	480	900	7486.7

From the results in Table 4.2, changes of retailer 1's demand rate influence the total cost for one unit time. When increasing the retailer 1's demand rate, the order quantity of vendor and the total cost rise up. It is noted that the optimal values of (k_1, k_2) are somewhat stable with the exception of high demand rate, i.e., $d_1 = 70$.

Table 4.3 shows the influence of changing demand rate of retailer 2. The same trends are observed as seen in the case of changing the demand rate of retailer 1. These trends are reasonable since when the total demand of item in the system increases, the holding cost of the system will increase.

Table 4.3

Effects of Demand Rate of Retailer 2

d_1	d_2	k_1	k_2	k_v	q_1	q_2	Q	Total cost per unit time
60	70	5	5	5	300	350	650	6220.0
60	75	5	5	5	300	375	675	6237.5
60	80	5	5	5	300	400	700	6255.0
60	85	5	5	5	300	425	725	6272.5
60	90	5	5	5	300	450	750	6290.0

4.2.2 The Impacts of Changing the Holding Cost

In order to find out the effect of retailers' holding cost, sensitivity analysis is conducted for two retailers and one vendor, the results are shown in Table 4.4 for retailer 1, Table 4.5 for retailer 2 and Table 4.6 for vendor, respectively. When holding cost is changed, the other parameters in the base case are maintained constant.

Table 4.4

Effects of Holding Cost of Retailer 1

h_{r1}	h_{r2}	k_1	k_2	k_v	q_1	q_2	Q	Total cost per unit time
1	3	5	5	5	300	400	700	6105.0
2	3	5	5	5	300	400	700	6255.0
3	3	5	5	5	300	400	700	6405.0
4	3	5	5	5	300	400	700	6555.0
5	3	5	5	5	300	400	700	6705.2

When changing the holding cost of retailer 1, there is no changes in the delivery cycles of vendor and retailers according to Table 4.4. Similarly, order quantities of both the vendor and the two retailers are unchanged. Anyway, the total cost per unit time will increase as a consequence.

Table 4.5

Effects of Holding Cost of Retailer 2

h_{r1}	h_{r2}	k_1	k_2	k_v	q_1	q_2	Q	Total cost per unit time
2	1	5	5	5	300	400	700	5855.0
2	3	5	5	5	300	400	700	6255.0
2	5	3	5	15	180	400	2100	7281.7
2	7	3	5	15	180	400	2100	7681.7
2	9	5	3	15	300	240	2100	7668.5

According to Table 4.5, the total cost for one unit time and the order quantity of the vendor increase with respect to the increasing of retailer 2's holding cost. Then, the delivery cycles and the order quantities of the two retailers are also affected as a consequence of changing the holding cost of retailer 2. The upward trend of the total cost function in Table 4.4 and 4.5 is understandable.

Table 4.6

Effects of Holding Cost of Vendor

h_v	k_1	k_2	k_v	q_1	q_2	Q	Total cost per unit time
1	3	4	12	180	320	1680	6733.3
3	5	5	5	300	400	700	6395.0
4	5	5	5	300	400	700	6255.0
7	2	4	4	120	320	560	7643.3

Table 4.6 presents the effect of changing the vendor's holding cost while other parameters are kept intact. Unlike the results of changing the retailers' holding costs,

the change of vendor's holding cost is quite strange. It shows an unstable effect on the total cost per unit time and the delivery cycles and the order quantities of the whole system. In details, there exist the maximum solutions for delivery cycles and order quantities while the total cost can reach a minimum solution. These trends may happen because when the vendor's holding cost increases, the vendor will firstly increase the delivery frequency to the retailers for reducing the holding cost to alleviate the increase in total cost. The increase in total cost at the beginning is also due to the increase in replenishment cost. However, the increasing trend of the total cost will stop when the delivery frequency is high enough because regardless of the increase trend in vendor's holding cost, the holding cost component in the total cost function will be reduced.

4.2.3 The Impacts of Changing the Ordering Cost

In this part, the change of ordering costs will be analyzed when other input parameters are maintained at initial values. The obtained results of delivery cycles, order quantities and the inventory cost are presented in Tables 4.7, 4.8 and 4.9 for retailer 1, retailer 2 and vendor, respectively.

Table 4.7

Effects of Ordering Cost of Retailer 1

S_{r1}	S_{r2}	k_1	k_2	k_v	q_1	q_2	Q	Total cost per unit time
1100	1600	5	5	5	300	400	700	5555.0
1400	1600	5	5	5	300	400	700	5855.0
1800	1600	5	5	5	300	400	700	6255.0
2200	1600	5	5	5	300	400	700	6655.0
2500	1600	5	5	5	300	400	700	6955.0

When changing the ordering cost of retailers and vendor, it has no effect on the delivery cycles of vendor and retailers as shown in Tables 4.7, 4.8 and 4.9. The order quantities of both the vendor and two retailers are stable. However, there is an upward trend for the total inventory wide cost when increasing the retailers' and vendor' ordering costs. These trends are reasonable.

Table 4.8

Effects of Ordering Cost of Retailer 2

S_{r1}	S_{r2}	k_1	k_2	k_v	q_1	q_2	Q	Total cost per unit time
1800	1000	5	5	5	300	400	700	5655.0
1800	1300	5	5	5	300	400	700	5955.0
1800	1600	5	5	5	300	400	700	6255.0
1800	1900	5	5	5	300	400	700	6555.0
1800	2100	5	5	5	300	400	700	6755.0

Table 4.9

Effects of Ordering Cost of Vendor

S_v	k_1	k_2	k_v	q_1	q_2	Q	Total cost per unit time
2600	5	5	5	300	400	700	6105.0
2800	5	5	5	300	400	700	6155.0
3200	5	5	5	300	400	700	6255.0
3600	5	5	5	300	400	700	6355.0
4000	5	5	5	300	400	700	6455.0

4.2.4 The Impacts of Changing the Replenishment Cost

In this section, the replenishment costs of retailers are changed for investigating the results of it when other input parameters are maintained at initial values. The optimal results are presented in Tables 4.10, 4.11 and 4.12 for retailer 1, retailer 2 and vendor, respectively.

The results from sensitivity analysis show that adjusting the replenishment costs has an impact only on total cost function. The higher the replenishment cost, the more total cost of the inventory system will be resulted.

Table 4.10

Effects of Replenishment Cost of Retailer 1

f_{r1}	f_{r2}	k_1	k_2	k_v	q_1	q_2	Q	Total cost per unit time
1400	2200	5	5	5	300	400	700	6135.0
1600	2200	5	5	5	300	400	700	6175.0
2000	2200	5	5	5	300	400	700	6255.0
2400	2200	5	5	5	300	400	700	6335.0
2800	2200	5	5	5	300	400	700	6415.0

Table 4.11

Effects of Replenishment Cost of Retailer 2

f_{r1}	f_{r2}	k_1	k_2	k_v	q_1	q_2	Q	Total cost per unit time
2000	2000	5	5	5	300	400	700	6215.0
2000	2200	5	5	5	300	400	700	6255.0
2000	2600	5	5	5	300	400	700	6335.0
2000	3000	5	5	5	300	400	700	6415.0
2000	3200	5	5	5	300	400	700	6455.0

Table 4.12

Effects of Replenishment Cost of Vendor

f_v	k_1	k_2	k_v	q_1	q_2	Q	Total cost per unit time
3100	5	5	5	300	400	700	6155.0
3300	5	5	5	300	400	700	6205.0
3500	5	5	5	300	400	700	6255.0
3900	5	5	5	300	400	700	6355.0
4300	5	5	5	300	400	700	6455.0

4.2.5 The Impacts of Changing the Basic Cycle Time

In order to consider the consequence of common cycle time, sensitivity analysis was conducted by changing the common cycle time when other input parameters are remained the same. The optimal findings are shown in Table 4.13.

Table 4.13

Effects of Basic Cycle Time

T	k_1	k_2	k_v	q_1	q_2	Q	Total cost per unit time
1	5	5	5	300	400	700	6255
2	1	1	2	120	160	560	21100
3	1	1	3	180	240	1260	38360
4	1	1	4	240	320	2240	81380
5	1	1	5	300	400	3500	172000

From the above result, it can be observed that increasing of common cycle time will produce the increase in the total cost for the whole system. Besides, it has effects on delivery cycles and order quantities of not only vendor but also retailers. It can be seen that changes of common delivery cycle will affect much on all outputs.

4.3 Numerical Experiment for One Vendor and Two Retailers VMI System with Backorders and Lost Sales

In this section, numerical experiments of a VMI system with two retailers and one vendor in the consideration of backorder and lost sales was conducted using MATLAB. By using the fminsearchbnd function in MATLAB, the optimal values for the decision variables related to basic cycles in one replenishment cycle (T) and the optimal total cost can be determined. The input parameters in the base case for the two retailers and vendor are given in table 4.14.

Table 4.14

Input Parameters for Total Cost Function of Backorder and Lost Sales System

Parameter	Retailer 1	Retailer 2	Vendor	Probability
d_i	60	80	-	-
h_{ri}	2	3	-	-
h_v	-	-	4	-
S_{ri}	1500	1900	-	-
S_v	-	-	2800	-
f_{ri}	1200	1300	-	-
f_v	-	-	1900	-
L_i	0.05	0.04	-	-
π_{bi}	200	180	-	-
π_{li}	220	210	-	-
σ	18	16	-	-
β	-	-	-	0.5
Z_α	-	-	-	1.881(α=3%)

For the above input parameters, using lower bound = [1,1] and upper bound = [50,50], the optimal values of the basic cycle time from the fminsearchbnd solver in Matlab can be determined as follows:

When T is set to be real number: The results are: $T = 6.0186, k = 3$ and the value of the total cost function is 3005.2355.

When T is set to be integer number: The results are: $T = 5$, $k = 3$ and the value of the total cost function is 2975.543.

4.4 Sensitivity Analysis for One Vendor and Two Retailers VMI System with Backorders and Lost Sales

The effects of input parameters are examined in this section. The input parameters such as the demand rates of the two retailers, the holding costs of the two retailers and the vendor, the ordering costs of the two retailers and the vendor, the replenishment costs of the two retailers and the vendor, the lead time of the two retailers, backordering costs paid by the two retailers, lost sale costs paid by the two retailers, standard deviation of demand through lead time, back ordering rate, and the safety factor with stock-out probability α are investigated.

4.4.1 The Impacts of Changing the Demand Rate of Retailer

In this part, how the demand rates of the two retailers affect the output parameters such as the common cycle time, the basic cycle in one replenishment cycle of the vendor, the order quantities of two retailers and one vendor, and the total cost of the entire system will be examined. The value of retailer 1's demand rate is changed from 54 to 66 and that of retailer 2 is from 74 to 86 but other parameters are maintained at initial values. Then the results are presented in the following tables.

Table 4.15

Effects of Demand Rate of Retailer 1

d_1	d_2	k	T	t_v	q_1	q_2	Q	Total cost per unit time
54	80	4	5.9505	23.8018	407	553	3841	2924.8389
		4	5	20	410	557	3869	2895.7975
57	80	3	6.0636	18.1907	432	563	2986	2984.0921
		3	5	15	428	557	2956	2955.4338
60	80	3	6.0186	18.0557	448	559	3020	3005.2355
		3	5	15	446	557	3010	2975.5430
63	80	3	5.9746	17.9238	463	555	3054	3026.3422
		3	5	15	464	557	3065	2995.7547
66	80	3	5.9316	17.7948	478	552	3088	3047.4021
		3	5	15	483	557	3119	3016.0543

The results in Table 4.15 and Table 4.16 shown that increasing the demand rates of retailers has an impact on decision variables and the total cost of the whole VMI system. The parameters such as the cycle time of the retailers and the vendor, and the order quantities changed as the effects of changing the retailers' demand rate. There exists maximum result for the delivery cycles of retailers for setting T as a real number. The delivery cycle of the vendor decreases. The order quantity of the examined retailer increases, moreover, a maximum order quantity for other retailer and a minimum order quantity for vendor have been occurred. Anyway, the total wide cost of VMI increases.

Table 4.16

Effects of Demand Rate of Retailer 2

d_1	d_2	k	T	t_v	q_1	q_2	Q	Total cost per unit time
60	74	4	5.9964	23.9855	446	521	3869	2906.1797
		4	5	20	446	521	3869	2877.0028
60	77	4	5.9280	23.7119	442	533	3900	2936.8199
		3	5	15	446	539	2956	2946.0683
60	80	3	6.0186	18.0557	448	559	3020	3005.2355
		3	5	15	446	557	3010	2975.5430
60	83	3	5.9529	17.8588	443	571	3044	3035.5806
		3	5	15	446	575	3065	3005.0663
60	86	3	5.8895	17.6684	439	583	3068	3065.6786
		3	5	15	446	593	3119	3034.6330

For setting T as an integer, the delivery cycles of the retailers are the same, but that of vendor decreases. It is noted that the order quantity of the other retailer is unchanged. However, the order quantity of the examined retailer increases and there exists a minimum value result for the vendor's order amount. Also, the total cost increases as a result of increasing order quantity.

4.4.2 The Impacts of Changing in the Holding Cost

The altering of holding costs of the entire system will be analyzed when other input parameters are maintained at initial values. The results are illustrated in Table 4.17 for retailer 1, Table 4.18 for retailer 2 and Table 4.19 for vendor, respectively.

Table 4.17

Effects of Holding Cost of Retailer 1

hr_1	hr_2	k	T	t_v	q_1	q_2	Q	Total cost per unit time
1.6	3	4	6.0872	24.3489	452	565	4069	2858.4474
		4	5	20	446	557	4014	2828.1776
1.8	3	4	5.9715	23.8861	445	555	3998	2913.2449
		4	5	20	446	557	4014	2882.7626
2	3	3	6.0186	18.0557	448	559	3020	3005.2355
		3	5	15	446	557	3010	2975.5430
2.2	3	3	5.9119	17.7358	441	550	2971	3059.5608
		3	5	15	446	557	3010	3030.1280
2.3	3	3	5.8606	17.5818	437	545	2948	3086.4332
		3	5	15	443	552	2985	3057.3432

Table 4.18

Effects of Holding Cost of Retailer 2

hr_1	hr_2	k	T	t_v	q_1	q_2	Q	Total cost per unit time
2	2.7	4	6.0726	24.2905	451	564	4060	2871.7876
		4	5	20	446	557	4014	2841.4451
2	2.8	4	6	24	447	557	4016	2903.9214
		4	5	20	446	557	4014	2873.4126
2	3	3	6.0186	18.0557	448	559	3020	3005.2355
		3	5	15	446	557	3010	2975.5430
2	3.2	3	5.8861	17.6583	439	548	2959	3068.7240
		3	5	15	445	556	3002	3039.4692
2	3.3	3	5.823	17.4689	435	542	2930	3100.0192
		3	5	15	439	548	2961	3071.1412

According to the results of altering retailer 1's holding cost, decreasing trends are observed on delivery cycles of the two retailers and vendor as well as the order quantity of the vendor when T is set as a real number. The order quantities of the two retailers are unstable. For the case of setting T as an integer, it is noted that there is a stable trend for T. There are decreasing trends on the delivery cycle of vendor and order quantities of whole system, but the effects are not significant. Anyway, the total cost per unit time increases for both cases due to the increase of retailer's holding cost as shown in Table 4.17. Similarly, the same trends are observed when changing the retailer 2's holding cost as given in Table 4.18.

Table 4.19

Effects of Holding Cost of Vendor

h_v	k	T	t_v	q_1	q_2	Q	Total cost per unit time
3.5	3	6.0127	18.0382	447	558	3018	2977.3547
	3	5	15	446	557	3010	2953.8017
4	3	6.0186	18.0557	448	559	3020	3005.2355
	3	5	15	446	557	3010	2975.5430
4.5	3	6.0244	18.0733	448	559	3023	3033.1139
	3	5	15	446	557	3010	2997.2844
5	3	6.0303	18.0909	449	560	3026	3060.9901
	3	5	15	446	557	3010	3019.0257
5.5	3	6.0361	18.1084	449	561	3028	3088.8639
	3	5	15	446	557	3010	3040.767

The effect of the vendor's holding cost is different from that of retailers' holding costs. When changing the vendor's holding cost, it can be observed that the value of k is not change. For the case when setting T as a real number, the delivery cycles and order quantities of the two retailers as well as the vendor increase a small amount when increasing the vendor's holding cost. Consequently, the total system wide cost increases. When T is set to be integer, the delivery cycles of retailers and the vendor are stable. Therefore, the order quantities of the two retailers and the vendor are also stable. Likewise, the total cost of the whole system rises when vendor's holding cost increases.

4.4.3 The Impacts of Changing the Ordering Cost

The sensitivity analysis is conducted to examine the effects of retailers' ordering cost and vendor's holding cost when other parameters are maintained at initial values. The results of these changes are presented in Tables 4.20, 4.21 and 4.22 for retailer 1, retailer 2 and vendor, respectively.

Table 4.20

Effects of Ordering Cost of Retailer 1

Sr_1	Sr_2	k	T	t_v	q_1	q_2	Q	Total cost per unit time
1200	1900	3	5.8944	17.6833	439	548	2963	2954.8703
		3	5	15	446	557	3010	2925.5430
1300	1900	3	5.9361	17.8083	442	552	2982	2971.7757
		3	5	15	446	557	3010	2942.2097
1400	1900	3	5.9775	17.9324	445	555	3001	2988.5633
		3	5	15	446	557	3010	2958.8764
1500	1900	3	6.0186	18.0557	448	559	3020	3005.2355
		3	5	15	446	557	3010	2975.5430
1600	1900	3	6.0594	18.1783	451	563	3039	3021.7945
		3	5	15	446	557	3010	2992.2097

According to Table 4.20 and Table 4.21, there is a slight increase in the delivery cycle of the retailers when T is set to be a real number when retailer's ordering cost increases. Hence, the ordering quantities and the total cost of the whole system increase but just marginally. When T is set to be integer, there is no effect on delivery cycles of the two retailers as well as the vendor. Hence, the order quantities of the entire system do not change. It is noted that the total cost increases because of the increasing of retailer's ordering cost.

Table 4.21

Effects of Ordering Cost of Retailer 2

Sr_1	Sr_2	k	T	t_v	q_1	q_2	Q	Total cost per unit time
1500	1600	3	5.8944	17.6833	439	548	2963	2954.8703
		3	5	15	446	557	3010	2925.5430
1500	1700	3	5.9361	17.8083	442	552	2982	2971.7757
		3	5	15	446	557	3010	2942.2097
1500	1800	3	5.9775	17.9324	445	555	3001	2988.5633
		3	5	15	446	557	3010	2958.8764
1500	1900	3	6.0186	18.0557	448	559	3020	3005.2355
		3	5	15	446	557	3010	2975.5430
1500	2000	3	6.0594	18.1783	451	563	3039	3021.7945
		3	5	15	446	557	3010	2992.2097

Table 4.22

Effects of Ordering Cost of Vendor

S_v	k	T	t_v	q_1	q_2	Q	Total cost per unit time
2600	3	5.9912	17.9736	446	557	3008	2994.1334
	3	5	15	446	557	3010	2962.2097
2700	3	6.0049	18.0147	447	558	3014	2999.6908
	3	5	15	446	557	3010	2968.8764
2800	3	6.0186	18.0557	448	559	3020	3005.2355
	3	5	15	446	557	3010	2975.5430
2900	3	6.0322	18.0967	449	560	3027	3010.7676
	3	5	15	446	557	3010	2982.2097
3000	3	6.0458	18.1375	450	561	3033	3016.2873
	3	5	15	446	557	3010	2988.8764

From Table 4.22, the delivery frequencies, order quantities and the total cost of the whole system increase when setting T as a real number as the result of increasing the vendor's ordering cost. However, there is no change on delivery cycles and order quantities of the whole system when T is set to be integer. It is noted that there exists an increasing trend for the total cost per unit time caused by increasing the vendor's ordering cost.

4.4.4 The Impacts of Changing the Replenishment Cost

In order to find out the effect of replenishment cost, sensitivity analysis was conducted by changing the replenishment cost while other input parameters are unchanged. The optimal results are presented in the following tables.

Table 4.23

Effects of Replenishment Cost of Retailer 1

fr_1	fr_2	k	T	t_v	q_1	q_2	Q	Total cost per unit time
900	1300	3	5.8944	17.6833	439	548	2963	2954.8703
		3	5	15	446	557	3010	2925.5430
1000	1300	3	5.9361	17.8083	442	552	2982	2971.7757
		3	5	15	446	557	3010	2942.2097
1100	1300	3	5.9775	17.9324	445	555	3001	2988.5633
		3	5	15	446	557	3010	2958.8764
1200	1300	3	6.0186	18.0557	448	559	3020	3005.2355
		3	5	15	446	557	3010	2975.5430
1300	1300	3	6.0594	18.1783	451	563	3039	3021.7945
		3	5	15	446	557	3010	2992.2097

From the result in Tables 4.23 and 4.24, when setting T as a real number, the delivery cycles of the two retailers and the vendor increases when the retailer's replenishment cost increases, thus the order quantities and total cost of the entire system increase. However, the change of the replenishment cost of retailer has no effect on the delivery cycles when it is set to be an integer. So, the other output parameters such as ordering quantities of retailers and vendor are unchanged. But there is a slight increase in the total cost of the whole supply chain.

Table 4.24

Effects of Replenishment Cost of Retailer 2

fr_1	fr_2	k	T	t_v	q_1	q_2	Q	Total cost per unit time
1200	1000	3	5.8944	17.6833	439	548	2963	2954.8703
		3	5	15	446	557	3010	2925.5430
1200	1100	3	5.9361	17.8083	442	552	2982	2971.7757
		3	5	15	446	557	3010	2942.2097
1200	1200	3	5.9775	17.9324	445	555	3001	2988.5633
		3	5	15	446	557	3010	2958.8764
1200	1300	3	6.0186	18.0557	448	559	3020	3005.2355
		3	5	15	446	557	3010	2975.5430
1200	1400	3	6.0594	18.1783	451	563	3039	3021.7945
		3	5	15	446	557	3010	2992.2097

From the results in Table 4.25, all decision variables, i.e., delivery cycles, order quantities and total cost of the whole system is increased when setting T as real number by increasing vendor's replenishment cost. However, the increase of the replenishment cost of vendor does not alter the frequencies of delivery and order quantities of the entire system when T is set as an integer. The total cost per unit time, however, increases a little bit.

Table 4.25

Effects of Replenishment Cost of Vendor

f_v	k	T	t_v	q_1	q_2	Q	Total cost per unit time
1700	3	5.9912	17.9736	446	557	3008	2994.1334
	3	5	15	446	557	3010	2962.2097

f_v	k	T	t_v	q_1	q_2	Q	Total cost per unit time
1800	3	6.0049	18.0147	447	558	3014	2999.6908
	3	5	15	446	557	3010	2968.8764
1900	3	6.0186	18.0557	448	559	3020	3005.2355
	3	5	15	446	557	3010	2975.5430
2000	3	6.0322	18.0967	449	560	3027	3010.7676
	3	5	15	446	557	3010	2982.2097
2100	3	6.0458	18.1375	450	561	3033	3016.2873
	3	5	15	446	557	3010	2988.8764

4.4.5 The Impacts of Changing the Lead Time

In this section, the change of lead times of the two retailers are examined when remaining input parameters are kept at initial values. The obtained results are presented in Table 4.26 for retailer 1 and Table 4.27 for retailer 2.

Table 4.26

Effects of Lead Time of Retailer 1

L_1	L_2	k	T	t_v	q_1	q_2	Q	Total cost per unit time
0.040	0.04	3	6.0179	18.0536	447	559	3018	3004.8943
		3	5	15	446	557	3008	2975.248
0.045	0.04	3	6.0182	18.0547	447	559	3019	3005.0646
		3	5	15	446	557	3009	2975.3953
0.050	0.04	3	6.0186	18.0557	448	559	3020	3005.2355
		3	5	15	446	557	3010	2975.5430
0.055	0.04	3	6.0189	18.0568	448	559	3021	3005.4068
		3	5	15	447	557	3011	2975.6913
0.060	0.04	3	6.0193	18.0579	448	559	3023	3005.5786
		3	5	15	447	557	3012	2975.8400

Table 4.27

Effects of Lead Time of Retailer 2

L_1	L_2	k	T	t_v	q_1	q_2	Q	Total cost per unit time
0.05	0.030	3	6.0177	18.0531	448	558	3017	3004.8068
		3	5	15	446	556	3008	2975.1676
0.05	0.035	3	6.0181	18.0544	448	559	3019	3005.0207
		3	5	15	446	557	3009	2975.3549
0.05	0.040	3	6.0186	18.0557	448	559	3020	3005.2355
		3	5	15	446	557	3010	2975.5430
0.05	0.045	3	6.019	18.057	448	559	3022	3005.4513
		3	5	15	446	558	3012	2975.7322
0.05	0.050	3	6.0194	18.0583	448	560	3023	3005.6680
		3	5	15	446	558	3013	2975.9222

From Tables 4.26 and 4.27, when setting T to be a real number, the delivery cycles of the vendor and the retailers increase because of the increase of a retailer's lead time. It is noted that the order quantity of the other retailer is unchanged. But the order quantities of the examined retailer and the vendor increase slightly, which indicates the increasing of total cost per unit time. When setting the delivery cycles to be integer, the results of sensitivity analysis showed that the delivery cycles are constant even though the retailer's lead time has changed. The order quantity of the other retailer is also constant. But the order quantities of the examined retailer and the vendor as well as the total cost increase due to the effect of increasing lead time.

4.4.6 The Impacts of Changing the Backordering Cost

In this section, the change of lead times of the two retailers are examined when remaining input parameters are kept at initial values. The obtained results are presented in Table 4.26 for retailer 1 and Table 4.27 for retailer 2.

According to the results, the delivery cycles increase due to the increase of backordering cost of a retailer when T is set to be real number. It is noted that the order quantities of retailers are not changed, but that of vendor increases. Thus, the total cost shows the upward trend. For the case of setting T as an integer, the delivery cycles and order quantities of the whole system is remained the same even though the retailer's backordering cost is slightly increased. Nevertheless, the total cost of the supply chain increases as presented in Tables.

Table 4.28

Effects of Backordering Cost of Retailer 1

π_{b1}	π_{b2}	k	T	t_v	q_1	q_2	Q	Total cost per unit time
160	180	3	6.0164	18.0493	448	559	3019	3003.5243
		3	5	15	446	557	3010	2975.5364
180	180	3	6.0175	18.0525	448	559	3020	3004.3799
		3	5	15	446	557	3010	2975.5397
200	180	3	6.0186	18.0557	448	559	3020	3005.2355
		3	5	15	446	557	3010	2975.5430
220	180	3	6.0196	18.0589	448	559	3021	3006.091
		3	5	15	446	557	3010	2975.5464
240	180	3	6.0207	18.0621	448	559	3021	3006.9464
		3	5	15	446	557	3010	2975.5497

Table 4.29

Effects of Backordering Cost of Retailer 2

π_{b1}	π_{b2}	k	T	t_v	q_1	q_2	Q	Total cost per unit time
200	140	3	6.0167	18.0501	448	559	3019	3003.7157
		3	5	15	446	557	3010	2975.5428
200	160	3	6.0176	18.0529	448	559	3020	3004.4756
		3	5	15	446	557	3010	2975.5429
200	180	3	6.0186	18.0557	448	559	3020	3005.2355
		3	5	15	446	557	3010	2975.5430

π_{b1}	π_{b2}	k	T	t_v	q_1	q_2	Q	Total cost per unit time
200	200	3	6.0195	18.0586	448	559	3021	3005.9953
		3	5	15	446	557	3010	2975.5431
200	220	3	6.0205	18.0614	448	559	3021	3006.7550
		3	5	15	446	557	3010	2975.5433

4.4.7 The Impacts of Changing the Lost Sale Cost

In this section, sensitivity analysis is performed by changing lost sale cost from 180 to 260 (for retailer 1) and from170 to 250 (for retailer 2) when other input parameters are remained the same for examining the effect of lost sale cost. The optimal results are presented in Table 4.30 and Table 4.31.

Table 4.30

Effects of Lost Sale Cost of Retailer 1

π_{l1}	π_{l2}	k	T	t_v	q_1	q_2	Q	Total cost per unit time
180	210	3	6.0164	18.0493	448	559	3019	3003.5243
		3	5	15	446	557	3010	2975.5364
200	210	3	6.0175	18.0525	448	559	3020	3004.3799
		3	5	15	446	557	3010	2975.5397
220	210	3	6.0186	18.0557	448	559	3020	3005.2355
		3	5	15	446	557	3010	2975.5430
240	210	3	6.0196	18.0589	448	559	3021	3006.091
		3	5	15	446	557	3010	2975.5464
260	210	3	6.0207	18.0621	448	559	3021	3006.9464
		3	5	15	446	557	3010	2975.5497

From the results, increase in lost sale cost of a retailer will lead to increasing trends for delivery cycles of the whole system, the order quantity of vendor and total cost for the case of setting T as a real number. The trends for order quantities of retailers are stable.

However, when setting integer value for T, the stable trends on the delivery cycles and order quantities of the whole system are observed while there is still an increasing trend on total cost per unit time.

Table 4.31

Effects of Lost Sale Cost of Retailer 2

π_{l1}	π_{l2}	k	T	t_v	q_1	q_2	Q	Total cost per unit time
220	170	3	6.0167	18.0501	448	559	3019	3003.7157
		3	5	15	446	557	3010	2975.5428
220	190	3	6.0176	18.0529	448	559	3020	3004.4756
		3	5	15	446	557	3010	2975.5429
220	210	3	6.0186	18.0557	448	559	3020	3005.2355
		3	5	15	446	557	3010	2975.5430
220	230	3	6.0195	18.0586	448	559	3021	3005.9953
		3	5	15	446	557	3010	2975.5431
220	250	3	6.0205	18.0614	448	559	3021	3006.7550
		3	5	15	446	557	3010	2975.5433

4.4.8 The Impacts of Changing the Standard Deviation of Demand

In this section, the standard deviations of the demand are changed from 16 to 20 and 14 to 18 for the two retailers one by one. Other parameters are maintained at constant values. The optimal outcomes are displayed in the following two tables.

Table 4.32

Effects of Standard Deviation of Demand of Retailer 1

σ_1	σ_2	k	T	t_v	q_1	q_2	Q	Total cost per unit time
16	16	3	6.0362	18.1086	440	561	3000	2978.5913
		3	5	15	437	557	2983	2951.3482

σ_1	σ_2	k	T	t_v	q_1	q_2	Q	Total cost per unit time
17	16	3	6.0274	18.0821	444	560	3010	2991.8565
		3	5	15	442	557	2996	2963.383
18	`16	3	6.0186	18.0557	448	559	3020	3005.2355
		3	5	15	446	557	3010	2975.5430
19	16	3	6.0098	18.0295	452	558	3030	3018.7282
		3	5	15	451	557	3024	2987.8292
20	16	3	6.0011	18.0033	456	557	3040	3032.3347
		3	5	15	456	557	3038	3000.2424

Table 4.33

Effects of Standard Deviation of Demand of Retailer 2

σ_1	σ_2	k	T	t_v	q_1	q_2	Q	Total cost per unit time
18	14	3	6.0479	18.1436	450	552	3006	2969.5106
		3	5	15	446	548	2983	2942.1947
18	15	3	6.0332	18.0995	449	556	3013	2987.3134
		3	5	15	446	553	2996	2958.8018
18	16	3	6.0186	18.0557	448	559	3020	3005.2355
		3	5	15	446	557	3010	2975.5430
18	17	3	6.0041	18.0122	447	562	3028	3023.2768
		3	5	15	446	562	3024	2992.4188
18	18	3	5.9896	17.9689	446	566	3035	3041.4376
		3	5	15	446	566	3038	3009.4294

From the results, when retailer's standard deviation of demand increases the delivery cycles of the retailers and the vendor decrease when setting T to be real number. The order quantity of another retailer also decreases. However, not only the order quantities of examined retailer and vendor but also the total cost of the whole system increase as the result of delivery cycles increase. When setting T as an integer, the delivery cycles of the whole system and order quantity of the other retailer are constant and there exist increasing trends for the examined retailer's order quantity, vendor's order quantity and total inventory cost.

4.4.9 The Impacts of Changing the Backordering Rate

For the effect of backordering rate, the sensitivity analysis is performed by altering its value when remained parameters are remained unchanged. The results are presented in Table 4.34.

Table 4.34

Effects of Backordering Rate

β	k	T	t_v	q_1	q_2	Q	Total cost per unit time
0.3	3	6.0195	18.0585	448	559	3020	3005.9905
	3	5	15	446	557	3010	2975.5444
0.4	3	6.019	18.0571	448	559	3020	3005.6130
	3	5	15	446	557	3010	2975.5437
0.5	3	6.0186	18.0557	448	559	3020	3005.2355
	3	5	15	446	557	3010	2975.5430
0.6	3	6.0181	18.0543	448	559	3020	3004.858
	3	5	15	446	557	3010	2975.5424
0.7	3	6.0176	18.0529	448	559	3020	3004.4804
	3	5	15	446	557	3010	2975.5417

From the above results, increasing the backordering rate will lead to the decreasing trends in delivery cycles of the system when T is set to real number. However, the order quantities are not affected. Besides, the total cost per unit time decreases. In contrast, there exist no effect on delivery cycles and order quantities for the entire system for setting the integer value of T. Anyway, the total cost decreases as shown in Table 4.34.

4.4.10 The Impacts of Changing the Safety Factor Associated with Stock Out Probability α

In this section, the safety factor associated with stock-out probability α is changed examine its effect. The other input parameters are remained unchanged. The optimal results are illustrated in Table 4.35.

Table 4.35

Effects of Safety Factor Associated with Stock-out Probability α

Z_α	k	T	t_v	q_1	q_2	Q	Total cost per unit time
1.645	4	5.9484	23.7938	433	544	3907	2931.8654
	3	6	18	469	591	3181	2923.7357
1.751	3	6.0646	18.1937	445	558	3009	2983.8210
	3	6	18	441	552	2978	2942.015
1.881	3	6.0186	18.0557	448	559	3020	3005.2355
	3	5	15	446	557	3010	2975.5430
2.054	3	5.9643	17.8928	452	561	3038	3039.0163
	3	6	18	454	564	3054	3020.7338
2.326	3	5.8901	17.6702	459	565	3071	3100.9136
	3	6	18	466	575	3122	3093.0667

It can be observed from the results in Table 4.35 that there exists a maximum delivery cycle for retailers and there is a decrease trend for that of vendor because of the increase in safety factor when T is set to be a real number. The order quantities of retailers increase, and a minimum value of vendor's order quantity exists. The upward trend is resulted in total cost. However, there exist minimum values for delivery cycles and order quantities of two retailers as well as vendor when setting T to be integer. Anyway, the total cost per unit time still increases.

CHAPTER 5
CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

In this research, VMI models with one vendor and two retailers under order-up-to level policy were focused to help improve the relationships between firms and reduce the total cost of the whole supply chain. Firstly, two mathematical models were developed to deal with the case when demand is deterministic and no backorder is allowed, and the case when demand is stochastic and backorder and lost sales are considered. Secondly, numerical experiments and sensitivity analysis were conducted to illustrate the applicability of the proposed models and the impact of changes in model's parameters. Optimal values of decision variables and total cost can be determined.

The results from the analyses of a VMI model with no backorder under deterministic demand are as follows:

- Changes of demand rates have impacts on the order quantity of vendor and the total cost per unit time.
- Only the total cost of the supply chain gets affected by changing the holding cost of retailers as well as the ordering cost and replenishment cost of retailers and vendor.
- Changing the holding cost of vendor and the basic cycle time affects all the decision variables and the total cost of the whole system.

The results from the analyses of a VMI model with backorder and lost sales under stochastic demand are as follows:

- Changing of demand rates and standard deviation of demands have impact on all decision variables and total cost when T is set to be real number, and this affects to the order quantity of examined retailer and vendor as well as the total cost when T is set to be integer.
- Total cost and all decision variables are affected by increasing retailer's holding cost for the case of setting T as a real number. When T is set to be integer, only the common cycle of retailer is not changed.

- All output parameters change due to the changing of holding cost of vendor, ordering cost and replenishment cost of the whole system when T is set as real number. But only total cost changes when setting T as an integer.
- While lead time of retailer changes, only order quantity of retailer is constant for the case of setting T as a real number. However, the examined order quantity of retailer, vendor's order quantity and total cost per unit time are affected when T is set to be integer.
- When the backordering cost, lost sale cost and backordering rate of the whole system changes, the order quantities of the two retailers are constant for setting the real number of T value. Other decision variables are affected. But, for setting the integer value of T, only the total cost gets affected.
- When changing the safety factor, all decision variables such as the delivery cycles and order quantities of the entire system as well as the total cost per unit time get affected.

5.2 Recommendations

In this research, a VMI system with one vendor and two retailers are studied under periodic review (R, T) policy. This can be expanded to a VMI system with multiple vendors and multiple retailers as a future research direction. That will be more realistic and relevant for business corporation. Furthermore, continuous review policy can be considered so as to lower cost of the whole system. Last but not least, transshipments between the retailers are also the possible future works because this can increase the service level and prevent shortage when inventory is still available in the system.

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APPENDIC

MATLAB program for VMI system with no backorders and lost sales:

```
function TC = nobl(k)
T=1;
d=[60,80];%demand rate
hr=[2,3];%holding cost
hv=4;
Sr=[1800,1600];%transportation cost
Sv=3200;
fr=[2000,2200];%replenishment cost
fv=3500;

kv=lcm(floor(k(1)),floor(k(2)));
q(1)=d(1)*k(1)*T;%order quantity
q(2)=d(2)*k(2)*T;
Q = q(1)*kv/k(1) + q(2)*kv/k(2);
INV = zeros(kv,2);
  for j=0:(kv-1)
  L =  bsxfun(@rdivide,j(:),k);
  R = mod(L,1);
  Xij = R <= 0;
  B = Xij.*q;

  INV(j+1,:)=Q-cumsum(B,2);
  Q=INV(j+1,2);
  end
INVsum= sum(INV(:,2));

TCretailers=(d(1)*k(1)*T*hr(1)/2)+(Sr(1)+fr(1)/k(1)*T)+(d(2)*k(2)*T*hr(2)/2)+(Sr(
2)+fr(2)/k(2)*T);
TCvendor=((T*hv*INVsum)+Sv+fv)/kv*T;
TC=TCretailers+TCvendor;
```

```
end
options = optimset('Display','iter','PlotFcns',@optimplotfval);
k0 = [1,1];
LB = [1,1];
UB = [100,100];
[k, fval, exitflag, output] = fminsearchbnd(@nobl, k0,LB,UB, options);
 disp("k1 = " +num2str(k(1))+ "k2 = " +num2str(k(2)));
```

MATLAB program for VMI system with backorders and lost sales:

```
function TCbls=kt(T)
di=[60,80];%demand rate
hri=[2,3];%holding cost
hv=4;
Sri=[1500,1900];%transportation cost
Sv=2800;
fri=[1200,1300];%replenishment cost
fv=1900;
Li=[0.05 0.04];
pibi=[200 180];
pili=[220 210];
sigma =[18 16];
beta = 0.5;
Zalpha = 2.326;

t=floor(T(1));
k=floor(T(2));
tv=k*(t);
SSi=Zalpha*sigma.*sqrt(T(1)+Li);
Ri=SSi+di.*(T(1)+Li);
   for i =1:2
  d = di(i);
  L = Li(i);
  R = Ri(i);
  sig = sigma(i);
```

```
    fun=@(x,R,t,d,L,sig) (x-R).*(1/(sig*sqrt(2*pi*(t+L))).*exp(-1/2*((x-d*(t+L))/(s
    ig*sqrt(t+L))).^2));
  Expi(i)=integral(@(x) fun(x,R,t,d,L,sig),R,Inf);
  end
BacklogAmount=beta*Expi;
LostsaleAmount=(1-beta)*Expi;
Tidot=Ri./di;
HCRi=((Ri.*Tidot./2)-(Ri-di.*Li).*Li).*hri;
qi=Ri+beta*Expi;
Q=k*sum(qi);
OCRi=Sri+fri;
BC=pibi.*BacklogAmount;
LSC=pili.*LostsaleAmount;
TCretailers=HCRi+OCRi+BC+LSC;
TCr_per_unittime=sum(TCretailers)./T(1);
didet=(di.*Tidot+beta.*Expi)./T(1);
D=sum(didet);
HCV=hv/tv*(D*tv^2/2-sum(T(1)*qi.*k/2));
OCV=Sv+fv;
TCvendor=HCV+OCV;
TCv_per_unitime=TCvendor/tv;
TCbls=sum(TCr_per_unittime)+TCv_per_unitime;
end
options = optimset('Display','iter','PlotFcns',@optimplotfval);
T0 = [1 1];
LB = [1 1];
UB = [50 50];
[T, fval, exitflag, output] = fminsearchbnd(@kt, T0,LB,UB, options);
 disp("T="+num2str(T));
```