DEVELOPMENT OF TWO VENDORS-SINGLE RETAILER INVENTORY MODEL UNDER VENDOR-MANAGED INVENTORY FOR TWO COMPETING PRODUCTS

by

Jutamart Jomratchakom

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Examination Committee:	Prof. Huynh Trung Luong (Chairperson)
	Prof. Manukid Parnichkun
	Dr. Mongkol Ekpanyapong

Nationality:	Thai
Previous Degree:	Bachelor of Engineering in Industrial Engineering Chiang Mai University Chiang Mai, Thailand
Scholarship Donor:	Mr. Boonkiat and Ms. Jintana Kovavisaruch Scholarship

Asian Institute of Technology
School of Engineering and Technology
Thailand
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AUTHOR'S DECLARATION

I, Jutamart Jomratchakom, declare that the research work carried out for this thesis was in accordance with the regulations of the Asian Institute of Technology. The work presented in it are my own and has been generated by me as the result of my own original research, and if external sources were used, such sources have been cited. It is original and has not been submitted to any other institution to obtain another degree or qualification. This is a true copy of the thesis, including final revisions.

Date: 12/07/2024

Name: Jutamart Jomratchakom

Signature:

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ABSTRACT

Vendor Managed Inventory (VMI) is a modern alternative inventory management strategy that allows vendors to oversee the entire replenishment process. Through information sharing between the members of the supply chain, vendors are able to determine appropriate order quantities for delivery to the retailer in an inventory replenishment cycle. This approach enables vendors to better meet customer demand. On the other hand, the retailer is able to reduce inventory costs and increase long-term trustworthy relationships with the customers, leading to greater customer loyalty and secured sales. It is obvious that VMI contributes to a win-win situation for all supply chain partners. This research advances this field by studying the development of an inventory model involving two vendors and one retailer under Vendor Managed Inventory (VMI) for two competing products. The objective of this research is to determine the optimal replenishment quantities for the vendors so that the expected total profit for the retailer is maximized. To make this research more realistic, an order-up to level inventory policy and lost sales policy when the customers' demands cannot be fulfilled are taken into account. The demand of both vendors will follow a Normal distribution. A mathematical model is developed to determine the optimal order quantities of two vendors with the aim to maximize the expected total profit for the retailer. Subsequently, numerical experiments and sensitivity analyses are conducted to observe the impacts of input parameters on order quantities of both vendors and the expected total profit per time unit. The conclusion identifies potential avenues for future research.

Keywords: Vendor-managed inventory, Inventory management, Competing products, Order-up-to level inventory policy, Lost sales.

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LIST OF ABBREVIATIONS

VMI	= Vendor Managed Inventory
RMI	= Retailer Managed Inventory
IS	= Information Sharing
CRP	= Continuous Replenishment Programs
PSO	= Particle Swarm Optimization

CHAPTER 1
INTRODUCTION

1.1 Background of the Study

To increase competitiveness in the market nowadays, organizations participating in upstream and downstream processes and activities, contributing to the generation of value through the production of goods and services delivered to end consumers, work cooperatively as a network called a supply chain (Christopher, 2016). A supply chain involves every step, from sourcing raw materials, transporting them to production facilities, and delivering the finished products to retail stores or distribution centers where they probably hold inventory and deliver to the end consumers. To ensure the efficient flow of all these processes, supply chain management emerges as a necessary process that every member of a network needs to deal with. An optimized supply chain leads to lower costs, faster response to customer demands, and also improves the overall effectiveness of the entire supply chain. With the aim at lower costs, sometimes this strategy encourages retailers to buy a large batch of products and store a higher volume of inventory than necessary leading to a rise in holding costs and the risk of obsolescence. It is inevitable that the total investment in inventory is massive, and the management of capital allocated to raw materials, work in process, and final products presents significant potential for improvement (Sven, 2015). If the available products kept on hand are not sufficient to meet the customers' demands, it will definitely affect customer service level and incur shortage costs. Hence, the objective of supply chain management aims at balancing the conflicting objectives of achieving high customer service levels, maintaining low inventory levels, and minimizing costs. (Stevens, 1989).

Inventory management is recognized as one of the most concerning problems in supply chain management since inventories have an impact in every sector of the supply chain. For example, the purchasing manager may desire to order a large batch of products to obtain benefits from discounts, while the production manager favors holding a great deal of raw material inventory to prevent manufacturing disruptions. Simultaneously, the marketing manager anticipates preserving a large amount of merchandise inventory in order to ensure high service levels of customer service. Hence, getting the right products and services to the right location is essential through a suitable inventory level at a reasonable cost. Also, ensuring customer satisfaction is a quite challenging issue in inventory management (Simchi-Levi et al., 2008). The effective management of this process requires overseeing customer-supplier relationships, managing inventory levels, predicting demand, and continuously monitoring operations at every stage of the supply chain. There are a lot of companies and academics have studied and proposed various strategies and policies to deal with inventory management problems so far. One of the inventory management policies has been successfully implemented in practice is Vendor Managed Inventory (VMI).

Vendor Managed Inventory (VMI) stands as a modern approach for the order-delivery process. The primary change over a traditional order-delivery process is the elimination of the ordering phase. Rather than increasing the pressure on vendors to achieve faster and more accurate deliveries, VMI allows the vendor both authority and responsibility to manage the whole replenishment process (Kaipia et al., 2002). In VMI, the customer or retailer provides the vendor the right to access to information about expected demand, product-related costs, promotional activities and inventory levels (Barratt, 2004). Thereafter, the vendor decides when and how many product quantities will be delivered to the retailer during an inventory replenishment cycle. It is obvious that information sharing for both vendors and retailers contributes to a win-win situation. The advantage for vendors is their enhanced capability to adjust production processes to match customer demand due to early access to information on both actual and forecasted demand (Claassen et al., 2008). Concurrently, suppliers additionally benefit from lower inventory costs and the establishment of long-term trustworthy relationships with the customers, leading to enhanced customer loyalty and secured sales (Vergin and Barr, 1999; Xu et al., 2001). Furthermore, lead time and the risk of demand amplification in the supply chain which is called the bullwhip effect can be alleviated through collaboration among supply chain partners (Dejonckheere et al., 2004; Reiner and Trcka, 2004). With the enormous benefits to both parties, this is obvious that the shift to VMI is necessary. There are some well-known companies, for example, Walmart, Procter and Gamble, Amazon, Bosch, Campbell Soup, and Intel implementing VMI to deal with their inventory system. However, there is a remaining question of why VMI still has not yet been widely adopted as a standard practice across various companies.

Among the research studies conducted on VMI policy to make it realistic so far, most of the research has proposed and developed various mathematical models for VMI system with one vendor and one retailer, one vendor and multiple retailers, and even multiple vendors and multiple retailers. However, considerations for the situation where multiple vendors and one retailer for two products in which vendors act as competitors still have not been addressed properly. In addition, the research studying customer behavior highlighted that the majority of unfulfilled demand is lost without giving a chance for customers to search for an alternative item. Inventory systems with this characteristic tend to be more complicated to analyze and solve. Consequently, the integration of VMI system with two vendors and one retailer in such an inventory system is an interesting topic to be addressed and be the direction of this research.

1.2 Statement of the Problem

To overcome and survive in the competitive market, several companies tried to reduce system wide costs of the supply chain while satisfying service level requirements. VMI policy has emerged to alleviate issues between these conflicting goals. More than decades ago, a lot of research studies highlighted and presented VMI policy in the inventory system. Dong and Xu (2002) have investigated vendor-managed inventory model addresses supply chain relationships, emphasizing inventory systems, purchase prices, and order quantities for short and long terms. The model involves two entities within a supply chain: a buyer and a supplier, with deterministic demand. The findings suggest that implementing VMI can effectively reduce inventory-related costs within the buyer-supplier channel system. Additionally, Razmi et al., (2010) have proposed a two-echelon mathematical model for VMI system with a single vendor and single buyer with just one item. The report pointed out that the VMI system enhances system coordination and consistently reduces costs across all conditions compared to traditional supply chain modes. Zhang et al., (2007) introduced an integrated vendor-managed inventory model for a two-echelon system with a focus on the reduction of order costs. A joint relevant cost model is developed by assuming constant production and demand rates, while considering variations in buyers' ordering cycles and allowing each buyer to replenish more than once per production cycle. This model also incorporates considerations for investment decisions. Numerical examples demonstrate that both the vendor and all buyers can realize significant cost savings through the reduction in ordering costs. Mateen et al., (2015) have presented an approximate expression for the expected total cost of VMI system involving a single vendor and multiple retailers under stochastic demand. The analysis demonstrated that the result obtained using the aforementioned approximation procedure were closely aligned with the actual optimal values. Furthermore, for VMI system with multiple vendors and multiple retailers, Hong et al., (2016) studied a two-echelon distribution network comprising multiple vendors and retailers within traditional and vendor-managed inventory systems under stochastic demand conditions, where unsatisfied demands are lost. After mathematical models were developed, the numerical experiments have been conducted and the results illustrated that the total system inventory cost is lower than that of a traditional system where shortage is permitted.

Although deterministic demand has been widely studied, it is evident that stochastic demand is more reflective of real-world conditions. Also, in the existing literature, there is no research on VMI system with a single retailer and two vendors who provide different products. Hence, in this research we consider two vendors who sell two products to a retailer. The two products are competing products and can replace each other. Due to it is possible that sometimes customers may be unwilling to wait until the desired product is replenished, this opens the opportunity for customers to search for an alternative product which comes from the other vendor. For this reason, it is challenging to help the two vendors to make decisions leading to a win-win situation for both when they have VMI contracts with the retailer.

1.3 Objectives of the Study

The aim of this study is to determine the appropriate replenishment quantities for two vendors who deliver alternative products to the retailer in an inventory replenishment cycle. Two vendors and one retailer VMI system will be investigated in which the expected total profit for the retailer is maximized.

1.4 Scope and Limitations

This study’s framework will be carried out according to the underlying assumptions.

1. A VMI system with two vendors and one retailer is considered.
2. The retailer is willing to share demand information with both vendors.
3. The customers’ demands that cannot be fulfilled are completely lost.
4. Order-up-to level inventory policy is considered.
5. The replenishment cycles for both vendors are identical.

6. The products of the two vendors are alternative products, i.e., they can replace each other.

CHAPTER 2
LITERATURE REVIEW

This study attempts to advance a VMI-based inventory model including two vendors and one retailer for two competing products so that the expected total profit for the retailer is maximized. Order-up-to level policy and complete lost sales when shortages occur are taken into account in this study. In the past literature, various studies reported that inventory management can help alleviate the system wide cost as well as enhance the efficiency of the whole supply chain. Although inventory management can help retailers to analyze the number of products stocked in the right location at a reasonable cost and help maintain customer satisfaction, there are still some research gaps that have not been addressed in previous research works. Therefore, to push the VMI system forward more realistic, inventory models should be developed in different scenarios. According to research works in the past, researchers mostly focused on developing VMI system with single vendor and single retailer, single vendor and multiple retailers, or even multiple vendors and multiple retailers. Furthermore, consideration of shortage with full backlogging or partial backlogging is mainly highlighted when the inventory on hand is not enough to fulfill the demand of customers on time. From the foregoing, it can be found that there is no research work conducted on developing inventory models for VMI system with multiple vendors and single retailer at which each vendor is a competitor to the other. To fill this gap, this study focuses on VMI system involving two vendors and one retailer for two competing products. Meanwhile, all shortages in this case are considered to be fully lost sales.

2.1 Review of Vendor Managed Inventory (VMI)

Vendor Managed Inventory (VMI) is a collaborative supply chain strategy which offers mutual benefits for both buyers and suppliers. Supplier is authorized and responsible to manage the entire replenishment process of the buyer’s inventory while the buyer shares inventory and demand information with the supplier. Thereafter, the supplier simply determines both the timing and quantity of product deliveries. This collaboration synchronizes supplier and buyer operations through information sharing and reengineered business process (Yao and Dresner, 2008; Disney and Towill, 2003). By using information technologies, the buyer can share real-time sales and inventory information with supplier and the supplier can utilize this information for planning manufacturing operations, scheduling deliveries, managing order quantities, and monitoring inventory echelons within the buyer's storage facility. The manufacturer now directly assesses consumer demand from end consumers rather than relying on the retailer’s order quantities, while continuing to receive orders from the retailer (Yao and Dresner, 2008). It cannot be denied the shortage of demand visibility has been recognized as a significant challenge in supply chain management, leading to inefficient capacity utilization, inadequate product availability, and elevated inventory levels across all parties (Smaros et al., 2003). Accordingly, the strategic implementation of supply chain collaboration is indispensable in today’s dynamic business environment to enable opportunities for enhancing revenue, strengthening customer loyalty, and improving efficiencies (Ireland and Crum 2005).

For several decades, many researchers have reported the benefits of VMI system and provided additional evidence demonstrating that implementing VMI can significantly reduce inventory costs. Waller et al. (1999) conducted a simulation-based study on the effects of VMI. Their findings illustrate how substituting traditional purchase orders with inventory replenishments allows suppliers to enhance service levels and reduce overall supply chain costs. In the following years, Lee et al. (2000) and Xu et al. (2001) confirmed the cost-saving benefits of information sharing in the supply chain. Their studies emphasize how information sharing mitigates forecasted demand variability because of the bullwhip effect and enhances supply chain coordination. In the same way, Kaipia et al. (2002) has addressed the question “What are you losing if you let your customer place orders?” by analyzing the cases focused on time performance when making the switch to VMI instead of the traditional order-delivery process. By introducing VMI, the final result shows that VMI proves notably more efficient compared to frequent purchase orders. The customer experienced a reduction in slow-moving inventory from 10% to 4% of the total inventory. According to Disney and Towill (2003), this study analyzed the comparative performance of a traditional supply chain and VMI, with a particular focus on how each structure influences the bullwhip effect within the supply chain. Evidently, the VMI strategy is notably employed to manage demand fluctuations caused by the Bullwhip effect, thereby preventing excessive inventory holding and ultimately reducing the overall cost of the supply chain. Furthermore, Yao and Dresner (2008) conducted a study on information sharing, vendor-managed inventory and continuous replenishment through an analytical model. Their findings indicate that these supply chain strategies result in significant reduction in inventory for both manufacturer and retailer.

Although several research works contribute to understanding the VMI system by developing analytical models that examine how essential supply chain factors, such as ordering cost, retail price, salvage value, and others, influence potential cost savings, VMI has not achieved widespread adoption as the standard approach to replenishment in supply chains. Implementation is frequently hindered by practical challenges, such as the need for standardized processes and information sharing systems within the supply chain. Therefore, many researchers still put more effort and simulate scenarios to be more complicated and practical in their approaches.

2.2 Review of Existing Studies on VMI Systems

The number of studies examining vendor-managed inventory has grown considerably to make the shift to VMI happen. In the early stages, many studies simplified supply chain structures by considering only a single vendor and a single retailer. In 2002, Dong and Xu (2002) examined VMI challenges using a supply chain relationship model, with particular emphasis on inventory systems, purchase prices, and purchase quantities. The model includes solely a buyer and a supplier. The findings indicate that introducing a VMI program can lower inventory costs across the buyer-supplier channel system, without requiring changes to the cost characteristics of the channel or the end-market demand level. Thereafter, Ouyang et al. (2004) introduced integrated production inventory models for single-vendor single-buyer scenarios, where stochastic demand and shortage during the lead time is permitted. They aimed at minimizing the joint total expected cost through simultaneous optimization of order quantity, reorder point, lead time, and the quantity of lots delivered from vendor to buyer. Finally, the findings suggest that their models can lead to substantial cost reductions. Additionally, Song and Dinwoodie (2008) considered the inventory management challenges in a supply chain environment impacted by uncertain replenishment lead times and fluctuating demand. Computational results showed that VMI policies are better than the best traditional retailer-managed inventory policy (RMI). During the same period, Yao and Dresner (2008) constructed a simple model involving a single manufacturer and retailer before implementation of IS, CRP, and VMI systems. Moreover, orders that the retailer cannot immediately fulfill due to stockouts are backordered with associated penalty costs. According to their research, IS, CRP, and VMI contribute diverse benefits in reducing inventory costs for organizations, and these advantages are not uniformly allocated between retailers and manufacturers. Similarly, Razmi et al. (2010) have also analyzed the performance of the VMI system compared to traditional supply chain operations. Sensitivity analyses have been implemented to assess how changes in the relevant parameters affect the overall cost comparison between the two systems. The result has been shown that the VMI system improves system coordination and consistently achieves cost savings across all conditions, even with backorders. Furthermore, Nia et al. (2014) proposed a multi-item economic order quantity model that considers shortages under a vendor-managed inventory policy, with the goal of minimizing total supply chain costs in a single-vendor, single-buyer scenario. Efforts were made to achieve an optimal outcome regarding the total cost using the ant colony optimization algorithm.

At the same time, some authors have considered and explored challenging problems in this field in a more complicated scenario which is the single-vendor multi-retailer model. In this case, the vendor distributes its products across several retail channels. Zhang et al. (2007) presented an integrated vendor-managed inventory (VMI) model designed for a single vendor and multiple buyers where backlogging is allowed. The assumptions include varying cycle times among buyers and the vendor's production cycle structured to be a multiple of each buyer's replenishment cycle. The numerical examples illustrate that both the vendor and all buyers can achieve significant cost savings through reductions in ordering costs. In the following year, Qinglong et al. (2008) developed a dual-echelon model consisting of one vendor and multiple retailers. Immediate replenishment is considered without the allowance for backlogs. This study encompasses costs such as replenishment and inventory holding costs for both vendors and retailers. In comparison to the standard (s, S) policy, the modified inventory policy has shown potential cost savings ranging from 5% to 20%. After that, Sue-Ann et al. (2012) addressed the operational challenges in a two-echelon supply chain with a single vendor and multiple buyers, utilizing the VMI approach. The PSO algorithm is employed to investigate the model's performance across different parameter settings. In conclusion, the study reveals that adopting the VMI operation mode leads to higher channel profitability compared to the current independent operational mode. In 2014, Diabat (2014) examined the application of VMI within a network involving a two-echelon supply chain with a single vendor and multiple buyers. The researcher developed a hybrid genetic and annealing algorithm to solve the optimization problem associated with VMI models. The results indicate that the proposed hybrid algorithm performs better than existing methodologies and produces more robust solutions. Meanwhile, Rad et al. (2014) examined a two-echelon supply chain model involving a single vendor and two buyers in which the vendor supplies identical items to both buyers at a finite production rate where a continuous review policy is applied. Through their modeling approach, they investigated the total inventory costs in both VMI and traditional RMI systems, aiming to determine the optimal order quantity that minimizes overall costs. The findings suggest that the implementation of VMI provides greater benefits compared to the traditional RMI system. Moreover, Govindan (2015) proposed an optimal replenishment policy for handling stochastic demand variations in a two-echelon supply chain. The system included one vendor and several retailers under a vendor managed inventory framework. The researcher attempted to minimize system cost by comparing the performance between traditional and VMI systems. It is obvious that the application of VMI for time-varying stochastic products has proven effective, resulting in lower overall costs throughout the supply chain. Additionally, a greater number of buyers purchasing the same product from the vendor improves the vendor's capacity to aggregate demand and facilitates coordinated replenishment scheduling. Implementing this approach enables the vendor to explore reductions in operational costs. In the same year, Choudhary and Shankar (2015) figured out the advantages of VMI compared to IS for a supplier, multiple retailers, and the entire system, applying an (R, S) inventory policy in the context of varying stochastic demand. Results reveal that under conditions of low supplier setup costs and high order processing efficiency, VMI enhances performance metrics considerably. In 2017, Kaasgari et al. (2017) implemented VMI strategy to deal with perishable product inventory in a dual-echelon supply chain involving one vendor and multiple retailers. The model is formulated to minimize costs across the supply chain, containing fixed ordering, holding, discount, and deterioration costs. Finally, it was reported that the PSO algorithm outperformed in addressing the proposed model in this research. Han et al. (2017) considered a decentralized VMI planning challenge within a three-echelon supply chain network, involving multiple distributors and third-party logistics providers, aimed at balancing inventory between a vendor (manufacturer) and multiple buyers (manufacturers) under deterministic buyers' demand. The findings showed the improvement of individual decision-making performance and the balance of cost sharing within a decentralized VMI hierarchy. Additionally, the study explored how a vendor-buyer VMI system involving third-party logistics can minimize or even eliminate the need for holding inventory. Recently, Zhang et al. (2020) considered a supply chain model with one supplier and two competitive retailers, highlighting their research on simultaneous inventory competition and cross-retailer transshipment. The optimal ordering quantities for retailers were determined under both centralized and decentralized approaches, and comparative analyses were conducted. After that, they proved that in scenarios with low competitive intensity among retailers, achieving lateral coordination can be promoted through the implementation of an optimal transshipment pricing strategy. Without appropriate conditions, transshipment alone cannot enable lateral coordination in highly competitive environments. During the same period, Tarhini et al. (2020) investigated the collaboration between a single vendor and multiple buyers under VMI, applying a consignment stock policy. The collaboration among buyers is enabled through the allowance of transshipment goods between them. The model also took into account the constrained storage capacity of each buyer and the limitations on the size of shipments between locations. The study's results illustrated that enabling transshipments among buyers effectively lowers the optimal total cost per unit time since transshipment serves as a strategy to mitigate occurrences of stock-outs. Concurrently, Taleizadeh et al. (2020) devised a vendor-managed inventory system consisting of one vendor and two buyers in a two-echelon supply chain. Periodic replenishment (R, T) and continuous replenishment (r, Q) models with partial backordering, lost sales and backordering are considered in this study. From their analysis, it becomes evident that each replenishment system offers some advantages and drawbacks across various inventory systems or for different objectives.

In addition, fewer researchers have explored a model involving multiple vendors and retailers. Sadeghi et al. (2013) developed a multi-vendor, multi-retailer supply chain model with a single central warehouse, constrained by both storage capacity and the annual order count. This research seeks to determine the optimal order quantities and the number of shipments obtained by retailers and vendors to minimize the total inventory cost of the supply chain. According to the results, the hybrid PSO algorithm was more effective in addressing the considered problems. After that, Sadeghian et al.

(2015) proposed a three-stage supply chain model featuring three vendors, four retailers, and a warehouse under a VMI system. This model incorporates constraints on both the annual order quantity and warehouse capacity. The researchers attempted to reduce the overall supply chain expenses, encompassing the costs associated with ordering and holding for retailers, vendors, and warehouses. The final results showed that the proposed method performed well in 95% of situations. Similarly, Hong et al. (2016) developed a two-echelon distribution network composed of multiple vendors and retailers for a single item in traditional and vendor-managed inventory systems where unsatisfied demands is completely lost and considered the demand from retailers as a stochastic variable that follows a uniform distribution. The findings indicated that implementing vendor-managed inventory results in a lower total system inventory cost compared to a traditional system where shortages are permitted.

Among the majority of prior studies, it is found that consideration of a multi-vendor, single-retailer supply chain still lacks attention from researchers. There are few authors attempted to deal with this topic. Phong and Yenradee (2020) developed a VMI model for a supply chain with multiple vendors and a single manufacturer, incorporating traders in the first stage and a manufacturer in the second stage. Given that transportation costs constitute the largest component of the supply chain expenses in this study, the researchers mainly focused on optimizing the shipment sizes and truck numbers for transporting goods from the first to the second stage, aiming to minimize transportation cost. Based on the experimental results, the proposed VMI model has shown capability in identifying optimal solutions and providing insights into managing the VMI system efficiently.

2.3 Review of Previous Studies on Consumer Response to Stockout Situation

With the growing intensity of commerce nowadays, the lifespans of numerous products are progressively shortening, thereby amplifying the risk of excessive inventory holding. In addition, responses to increased competitiveness also affect overall inventory cost structures. For instance, when a new mobile phone model is launched to the market, there is a significant decrease in the value of earlier models (Zhang et al., 2020). Therefore, it is noticeable that the increase in competition presents greater challenges for both manufacturers and retailers in accurately forecasting product demand and effectively coordinating production and orders. To avoid the potential problem of overstocking, many retailers decide to hold lower inventory levels. However, this strategy can lead to a higher risk of stock-outs, which are a prevalent issue within the retail sector. Following such occurrences, retailers frequently engage in transshipment agreements with competitors to promptly meet customer demands and mitigate potential revenue losses. However, in the realm of business, even though retailers offer transshipment services during stock-outs, some customers may still look for alternative products from competitors (Lee and Lu, 2015; Olsen and Parker, 2014). According to Verbeke et al. (1998), approximately 34% of Coca-Cola customers opt to purchase competing products after experiencing stock-outs, despite the retailer offering a transshipment service. Tierney (2004) recorded that when a retailer runs out of P&G products, approximately 50% of its customers tend to shift to competitors. Moreover, around 40% of customers reject transshipment service offers and decide to purchase from alternative sources. This customer behavior can be attributed to two primary reasons. Firstly, some customers prefer to promptly satisfy their desires and are unwilling to wait for product replenishment. Secondly, others prefer to experience the products before making their purchase decision. Mishra and Raghunathan (2004) demonstrated that VMI amplifies competition among manufacturers of similar products, influenced by brand substitution. Because some customers may decide to purchase an alternative product if their desired item is unavailable, manufacturers experiencing stock-outs risk losing sales to competitors. Silver et al. (1998) mentioned the shortage cost as the cost of unsatisfied demand and categorized two distinct scenarios when items become unavailable for customers' orders. One involves complete backordering, where out-of-stock items are reserved and fulfilled upon the next replenishment, with customers accepting delayed delivery. In the other scenario, out-of-stock items are considered as complete lost sales, where customers reject delayed delivery. Although both stockouts and dead inventory may seem to be opposite problems, the core issue is the same. In the long term, they converge into the same results, namely making the supply chain less efficient, reflection of poor customer service and shrinking market share. Therefore, it is obvious that lost sales which are regarded as hidden costs of supply chain are another interesting topic to be further investigated.

2.4 Identifying Research Gaps

Based on past research, most researchers have focused on vendor-managed inventory system applied in supply chains with a single vendor and a single retailer. Some authors have considered single-vendor and multiple-retailer model and even the multiple-vendor and multiple-retailer model for VMI system. The summary of past research conducted on the VMI system is presented in Table 2.1. However, consideration of the case consisting of a retailer who sells several brands of a product manufactured by several manufacturers is still questionable. How many appropriate order quantities of each brand the retailer stocks in the inventory need to be further investigated to achieve the goal of maximizing the retailer's expected total profit. Furthermore, most studies examine the shortage when products are out of stock for a customer’s order and not able to fulfill on time as full backlogging, partial backlogging, or the transshipments between buyers are allowed in some cases. Nonetheless, the worst situation where customers do not want to wait and search for the same product of another brand with some probability still be a research gap and has not been addressed by any of the existing studies in this field. Therefore, this study aims to fill this research gap.

Table 2.1

The Summary of Past Research Conducted on the VMI System

Authors	Research Topic	Model Structure
Dong and Xu (2002)	A Supply Chain Model of Vendor Managed Inventory	Single vendor and single retailer
Ouyang et al. (2004)	Integrated Vendor-Buyer Cooperative Models with Stochastic Demand in Controllable Lead Time	Single vendor and single retailer
Song and Dinwoodie (2008)	Quantifying the Effectiveness of VMI and Integrated Inventory Management in a Supply Chain with Uncertain Lead-Times and Uncertain Demands	Single vendor and single retailer
Yao and Dresner (2008)	The Inventory Value of Information Sharing, Continuous Replenishment, and Vendor-Managed Inventory	Single vendor and single retailer

Authors	Research Topic	Model Structure
Razmi et al. (2010)	Developing a Two-Echelon Mathematical Model for a Vendor-Managed Inventory (VMI) System	Single vendor and single retailer
Nia et al. (2014)	A Fuzzy Vendor Managed Inventory of Multi-Item Economic Order Quantity Model under Shortage: An Ant Colony Optimization Algorithm	Single vendor and single retailer
Zhang et al. (2007)	An Integrated Vendor-Managed Inventory Model for a Two-Echelon System with Order Cost Reduction	Single vendor and multiple retailers
Qinglong et al. (2008)	A Modified Joint Inventory Policy for VMI Systems	Single vendor and multiple retailers
Sue-Ann et al. (2012)	Evolutionary Algorithms for Optimal Operating Parameters of Vendor Managed Inventory Systems in a Two-Echelon Supply Chain	Single vendor and multiple retailers
Diabat (2014)	Hybrid Algorithm for a Vendor Managed Inventory System in a Two-Echelon Supply Chain	Single vendor and multiple retailers
Rad et al. (2014)	Optimizing an Integrated Vendor-Managed Inventory System for a Single-Vendor Two-Buyer Supply Chain with Determining Weighting Factor for Vendor's Ordering Cost	Single vendor and multiple retailers
Govindan (2015)	The Optimal Replenishment Policy for Time-Varying Stochastic Demand under Vendor Managed Inventory	Single vendor and multiple retailers
Choudhary and Shankar (2015)	The Value of VMI beyond Information Sharing in a Single Supplier Multiple Retailers Supply Chain under a Non-Stationary (Rn, Sn) Policy	Single vendor and multiple retailers

Authors	Research Topic	Model Structure
Kaasgari et al. (2017)	Optimizing a Vendor Managed Inventory (VMI) Supply Chain for Perishable Products by Considering Discount: Two Calibrated Metaheuristic Algorithms	Single vendor and multiple retailers
Han et al. (2017)	Tri-Level Decision-Making for Decentralized Vendor-Managed Inventory	Single vendor and multiple retailers
Zhang et al. (2020)	Simultaneous Inventory Competition and Transshipment between Retailers	Single vendor and multiple retailers
Tarhini et al. (2020)	An Integrated Single-Vendor Multi-Buyer Production Inventory Model with Transshipments between Buyers	Single vendor and multiple retailers
Taleizadeh et al. (2020)	Stock Replenishment Policies for a Vendor Managed Inventory in a Retailing System	Single vendor and multiple retailers
Sadeghi et al. (2013)	Optimizing a Multi-Vendor Multi-Retailer Vendor Managed Inventory Problem: Two Tuned Metaheuristic Algorithms	Multiple vendors and multiple retailers
Sadeghian et al. (2015)	An Inventory Model for Tri-Stage Supply Chain with a Warehouse, Stochastic Demand and Multi Retailers and Vendors	Multiple vendors and multiple retailers
Hong et al. (2016)	Multiple-Vendor, Multiple-Retailer Based Vendor-Managed Inventory	Multiple vendors and multiple retailers
Phong and Yenradee (2020)	Vendor Managed Inventory for Multi-Vendor Single-Manufacturer Supply Chain: A Case Study of Instant Noodle Industry	Multiple vendors and multiple retailers

CHAPTER 3
MATHMATICAL MODEL DEVELOPMENT

The development of Vendor Managed Inventory (VMI) system involving two vendors and a single retailer for two competing products is presented and an order-up-to level policy is implemented in this chapter. Furthermore, shortages arise when customers are unwilling to wait for the fulfillment of their demands are considered in this study as completely lost. Therefore, to determine the appropriate order quantities from both vendors for delivery to the retailer in an inventory replenishment cycle of length T such that the expected total profit for the retailer is maximized, a mathematical inventory model is formulated based on the following assumptions and notations.

3.1 Assumptions

The methodology of this study will be conducted based on the following assumptions.

- The demand from both vendors will follow Normal distribution.
- Costs for each vendor are different.
- The shortage costs will occur when the demand is unfulfilled and be consider as fully lost sales.
- The replenishment cycles of two vendors are identical.
- The products of two vendors are alternative products. They can replace each other when another one is out of stock with probability α.

3.2 Notations

A mathematical model in this study is formulated upon the following notations.

Indices:

i, j	Vendors index, i, j = 1, 2
s	Scenarios, s = 1, 2, 3, and 4

Parameters:

$D_i(T)$	Demand of the vendor i during period T
$D_j(T)$	Demand of the vendor j during period T
K_i	A constant value of the ordering cost for each product
w_i	Unit purchasing cost for each product
v_i	Unit shortage cost for each product
r_i	Unit retail price for each product

s_i Unit salvage value for each product

T The cycle length

$E[\pi_i^s]$ The expected total profit of vendor i per cycle length in scenario *s*

$E[\pi_j^s]$ The expected total profit of vendor j per cycle length in scenario *s*

$E[\pi^s]$ The expected total profit of both vendors per time unit

α_i The fraction of search demand that customer of vendor *i* search for product from vendor *j*

α_j The fraction of search demand that customer of vendor *j* search for product from vendor *i*

Decision variables:

Q_i The optimal order quantity of vendor *i*

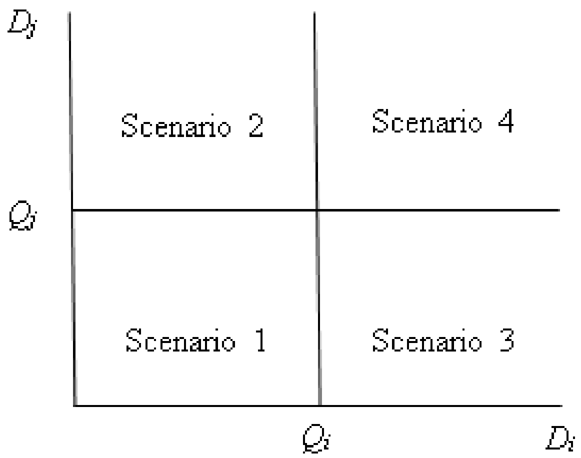
Q_j The optimal order quantity of vendor *j*

3.3 Development of Model Framework

In this study, each vendor will investigate their inventory level at the end of period *T* and decide the appropriate order quantities for delivery to the retailer in an inventory replenishment cycle of length *T*. When competing vendor undergoes a stock shortage of the item, the scenario that their customers will search for alternative products from the other vendor is possible if the other vendor still has remaining inventory stock. Therefore, the following four scenarios need to be examined, as outlined in Figure 3.1.

Figure 3.1

Graphical Overview of Four Scenarios Framework

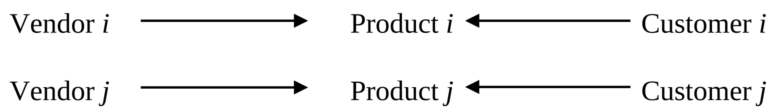


3.3.1 Scenario 1: When $D_i(T) \le Q_i$ and $D_j(T) \le Q_j$

The highest inventory levels at the beginning of period *T* of both vendors are greater than their customers' demands. As a result, this scenario considers when both vendors can fulfill all demands of their customers, as illustrated in Figure 3.2.

Figure 3.2

Graphical Representation of Scenario 1



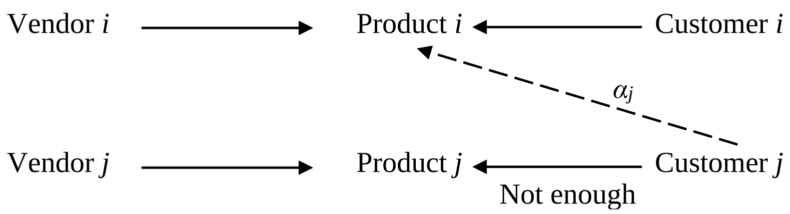
In this scenario, there is no switching occurring. Both vendors have sufficient product quantities to fulfill all customer demands. Therefore, customers of both vendors can normally come and get the product they are looking for. In summary, loyal customers do not search for an alternative product from the other vendor.

3.3.2 Scenario 2: When $D_i(T) \le Q_i$ and $D_j(T) > Q_j$

The maximum inventory level of vendor *i* at the end of the selling season is greater than vendor *i*'s demand during period *T*, while the highest inventory level of vendor *j* at the end of the selling season is less than vendor *j*'s demand during period *T*. As a result, this scenario considers when vendor *i* is able to fulfill all demands of its customers, but vendor *j* has insufficient inventory to meet the excessive demands of its customers, as illustrated in Figure 3.3.

Figure 3.3

Graphical Representation of Scenario 2



Since the product quantity of vendor i is sufficient to fulfill the demands of its customers in this scenario, customers of vendor i can normally come and get the product they are looking for. However, the product quantity from vendor j is not sufficient to fulfill some of its customers' demands. As a result, customers of vendor j will probably search for an alternative product from vendor i with probability α_j.

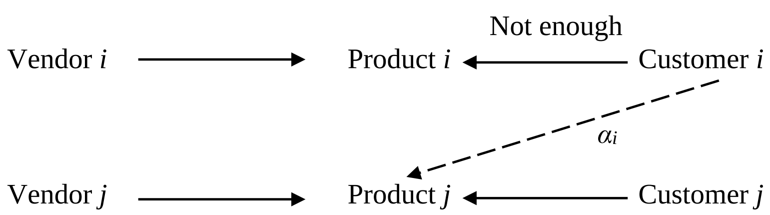
It can be concluded that there is a search demand quantity from customers of vendor j for vendor i, calculated as $\alpha_j(D_j(T) - Q_j)$. On the contrary, there is no search demand quantity from customers of vendor i for vendor j.

3.3.3 Scenario 3: When $D_i(T) > Q_i$ and $D_j(T) \leq Q_j$

The maximum inventory level of vendor i at the end of the selling season is less than vendor i's demand during period T, while the highest inventory echelon of vendor j at the end of the selling season is greater than vendor j's demand during period T. Consequently, this scenario considers when vendor i has insufficient inventory to fulfill the excessive demands of its customers, but vendor j is able to satisfy all demands of its customers, as illustrated in Figure 3.4.

Figure 3.4

Graphical Representation of Scenario 3



Since the product quantity of vendor i is not sufficient to meet some of its customers' demands, customers of vendor i will probably search for an alternative product from vendor j with probability α_i. On the other hand, the product quantity of vendor j is sufficient to fulfill the demands of its customers, customers of vendor j can normally come and get the product they are looking for.

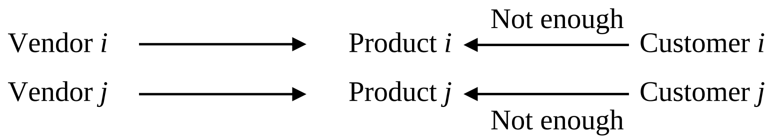
It is clear that there is a search demand quantity from customers of vendor *i* for vendor *j*, calculated as $\alpha_i(D_i(T) - Q_i)$. In contrast, there is no search demand quantity from customers of vendor *j* for vendor *i*.

3.3.4 Scenario 4: When $D_i(T) > Q_i$ and $D_j(T) > Q_j$

The highest inventory echelons at the beginning of period *T* of both vendors are greater than their demands. Consequently, this scenario considers when both vendors are not able to fulfill some of their customers' demands when a shortage occurs, as illustrated in Figure 3.5.

Figure 3.5

Graphical Representation of Scenario 4



There is no switching in this scenario. Customers of both vendors simply come and get the product they are looking for when it is available on the shelf during a certain period, given that they are loyal customers. On the other hand, as time passes during any period in this scenario, the product quantities of both vendors become insufficient to fulfill some of their customers' demands when a shortage occurs. Hence, there is no search demand quantity for both vendor *i* and vendor *j*.

3.4 Development of Profit Functions

The demand, cost, and profit of each vendor during time period T for each scenario have been defined in this section.

3.4.1 Scenario 1: When $D_i(T) \leq Q_i$ and $D_j(T) \leq Q_j$

The profit of vendor *i* and vendor *j* in scenario 1 can be derived step by step, as shown in Table 3.1 and Table 3.2, respectively.

Table 3.1

Profit Derivation for Vendor i in Scenario 1

Variable	Calculation
Search demand from customers of vendor *j*	0
Total demand of vendor i	$D_i(T)$
Sales amount of vendor *i*	$D_i(T)$
Salvage amount of vendor *i*	$Q_i – D_i(T)$
Shortage amount of vendor *i*	0
Ordering cost	K_i
Purchasing cost	w_iQ_i
Shortage cost	0
Revenue	$r_iD_i(T) + s_i(Q_i – D_i(T))$
Profit	$r_iD_i(T) + s_i(Q_i – D_i(T)) – K_i – w_iQ_i$

Table 3.2

Profit Derivation for Vendor j in Scenario 1

Variable	Calculation
Search demand from customers of vendor *i*	*0*
Total demand of vendor *j*	$D_j(T)$
Sales amount of vendor *j*	$D_j(T)$
Salvage amount of vendor *j*	$Q_j – D_j(T)$
Shortage amount of vendor *j*	*0*
Ordering cost	K_j
Purchasing cost	w_jQ_j
Shortage cost	*0*
Revenue	$r_jD_j(T) + s_j(Q_j – D_j(T))$
Profit	$r_jD_j(T) + s_j(Q_j – D_j(T)) – K_j – w_jQ_j$

3.4.2 Scenario 2: When $D_i(T) \le Q_i$ and $D_j(T) > Q_j$

The profit of vendor *i* and vendor *j* in scenario 2 can be derived step by step, as shown in Table 3.3 and Table 3.4, respectively.

Table 3.3

Profit Derivation for Vendor i in Scenario 2

Variable	Calculation
Search demand from customers of vendor *j*	$\alpha_j(D_j(T) – Q_j)$
Total demand of vendor *i*	$D_i(T) + \alpha_j(D_j(T) – Q_j)$
Sales amount of vendor *i*	$Min\ [D_i(T) + \alpha_j(D_j(T) – Q_j), Q_i]$
Salvage amount of vendor *i*	$Max\ [Q_i – D_i(T) – \alpha_j(D_j(T) – Q_j), 0]$
Shortage amount of vendor *i*	0
Ordering cost	K_i
Purchasing cost	w_iQ_i
Shortage cost	0
Revenue	$r_i\ Min\ [D_i(T) + \alpha_j(D_j(T) – Q_j), Q_i] +$ $s_i\ Max\ [Q_i – D_i(T) – \alpha_j(D_j(T) – Q_j), 0]$
Profit	$R_i\ Min\ [D_i(T) + \alpha_j(D_j(T) – Q_j), Q_i] +$ $s_i\ Max\ [Q_i – D_i(T) – \alpha_j(D_j(T) – Q_j), 0] –$ $K_i – w_iQ_i$

Table 3.4

Profit Derivation for Vendor j in Scenario 2

Variable	Calculation
Search demand from customers of vendor *i*	0
Total demand of vendor *j*	$D_j(T)$
Sales amount of vendor *j*	Q_j
Salvage amount of vendor *j*	0
Shortage amount of vendor *j*	$D_i(T) + \alpha_j(D_j(T) – Q_j) – Min\ [D_i(T) + \alpha_j(D_j(T) – Q_j), Q_i]$
Ordering cost	K_j
Purchasing cost	w_jQ_j
Shortage cost	$v_j\ \{D_i(T) + \alpha_j(D_j(T) – Q_j) – Min\ [D_i(T) + \alpha_j(D_j(T) – Q_j), Q_i]\}$
Revenue	r_jQ_j
Profit	$r_jQ_j – K_j – w_jQ_j – v_j\ \{D_i(T) + \alpha_j(D_j(T) – Q_j) – Min\ [D_i(T) + \alpha_j(D_j(T) – Q_j), Q_i]\}$

3.4.3 Scenario 3: When $D_i(T) > Q_i$ and $D_j(T) \leq Q_j$

The profit of vendor *i* and vendor *j* in scenario 3 can be derived step by step, as shown in Table 3.5 and Table 3.6, respectively.

Table 3.5

Profit Derivation for Vendor i in Scenario 3

Variable	Calculation
Search demand from customers of vendor *j*	0
Total demand of vendor *i*	$D_i(T)$
Sales amount of vendor *i*	Q_i
Salvage amount of vendor *i*	0
Shortage amount of vendor *i*	$D_j(T) + \alpha_i(D_i(T) - Q_i) - Min\ [D_j(T) + \alpha_i(D_i(T) - Q_i), Q_j]$
Ordering cost	K_i
Purchasing cost	w_iQ_i
Shortage cost	$v_i\ \{D_j(T) + \alpha_i(D_i(T) - Q_i) - Min\ [D_j(T) + \alpha_i(D_i(T) - Q_i), Q_j]\}$
Revenue	r_iQ_i
Profit	$r_iQ_i - K_i - w_i\,Q_i - v_i\ \{D_j(T) + \alpha_i(D_i(T) - Q_i) - Min\ [D_j(T) + \alpha_i(D_i(T) - Q_i), Q_j]\}$

Table 3.6

Profit Derivation for Vendor j in Scenario 3

Variable	Calculation
Search demand from customers of vendor *i*	$\alpha_i(D_i(T) - Q_i)$
Total demand of vendor *j*	$D_j(T) + \alpha_i(D_i(T) - Q_i)$
Sales amount of vendor *j*	$Min\ [D_j(T) + \alpha_i(D_i(T) - Q_i), Q_j]$
Salvage amount of vendor *j*	$Max\ [Q_j - D_j(T) - \alpha_i(D_i(T) - Q_i), 0]$
Shortage amount of vendor *j*	0
Ordering cost	K_j
Purchasing cost	w_jQ_j
Shortage cost	0
Revenue	$r_j\ Min\ [D_j(T) + \alpha_i(D_i(T) - Q_i), Q_j] + s_j\ Max\ [Q_j - D_j(T) - \alpha_i(D_i(T) - Q_i), 0]$

Variable	Calculation
Profit	r_j *Min* $[D_j(T) + \alpha_i(D_i(T) - Q_i), Q_j] +$ s_j *Max* $[Q_j - D_j(T) - \alpha_i(D_i(T) - Q_i), 0] -$ $K_j - w_jQ_j$

3.4.4 Scenario 4: When $D_i(T) > Q_i$ and $D_j(T) > Q_j$

The profit of vendor *i* and vendor *j* in scenario 4 can be derived step by step, as shown in Table 3.7 and Table 3.8, respectively.

Table 3.7

Profit Derivation for Vendor i in Scenario 4

Variable	Calculation
Search demand from customers of vendor *j*	0
Total demand of vendor *i*	$D_i(T)$
Sales amount of vendor *i*	Q_i
Salvage amount of vendor *i*	0
Shortage amount of vendor *i*	$D_i(T) - Q_i$
Ordering cost	K_i
Purchasing cost	w_iQ_i
Shortage cost	$v_i(D_i(T) - Q_i)$
Revenue	r_iQ_i
Profit	$r_iQ_i - K_i - w_iQ_i - v_i(D_i(T) - Q_i)$

Table 3.8

Profit Derivation for Vendor j in Scenario 4

Variable	Calculation
Search demand from customers of vendor *i*	0
Total demand of vendor *j*	$D_j(T)$
Sales amount of vendor *j*	Q_j
Salvage amount of vendor *j*	0
Shortage amount of vendor *j*	$D_j(T) - Q_j$
Ordering cost	K_j
Purchasing cost	w_jQ_j

Variable	Calculation
Shortage cost	$v_j(D_j(T) - Q_j)$
Revenue	r_jQ_j
Profit	$r_jQ_j - K_j - w_jQ_j - v_j(D_j(T) - Q_j)$

3.5 Development of Mathematical Model

After formulating the profit functions for each scenario, the expected profit functions for both vendors can be determined as follows.

For vendor *i*, the expected profit function can be formulated as follows:

$$E[\pi_i] = \int_0^{Q_i} \int_0^{Q_j} [r_i x + s_i(Q_i - x) - K_i - w_i Q_i]\, f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx$$

$$+ \int_0^{Q_i} \int_{Q_j}^{\infty} \{r_i\, Min[x + \alpha_j(y - Q_j), Q_i] + s_i\, Max[Q_i - x - \alpha_j(y - Q_j), 0] - K_i - w_i Q_i\}$$

$$f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx$$

$$+ \int_0^{Q_j} \int_{Q_i}^{\infty} \{r_i Q_i - K_i - w_i Q_i - v_i\{y + \alpha_i(x - Q_i) - Min[y + \alpha_i(x - Q_i), Q_j]\}\}$$

$$f_{D_i(T)}(x) f_{D_j(T)}(y)\, dx\, dy$$

$$+ \int_{Q_i}^{\infty} \int_{Q_j}^{\infty} [r_i Q_i - K_i - w_i Q_i - v_i(x - Q_i)]\, f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx \qquad (1)$$

For vendor *j*, the expected profit function can be formulated as follows:

$$E[\pi_j] = \int_0^{Q_i} \int_0^{Q_j} [r_j y + s_j(Q_j - y) - K_j - w_j Q_j]\, f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx$$

$$+ \int_0^{Q_i} \int_{Q_j}^{\infty} \{r_j Q_j - K_j - w_j Q_j - v_j\{x + \alpha_j(y - Q_j) - Min[x + \alpha_j(y - Q_j), Q_i]\}\}$$

$$f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx$$

$$+ \int_0^{Q_j} \int_{Q_i}^{\infty} \{r_j\, Min[y + \alpha_i(x - Q_i), Q_j] + s_j\, Max[Q_j - y - \alpha_i(x - Q_i), 0] - K_j - w_j Q_j\}$$

$$f_{D_i(T)}(x) f_{D_j(T)}(y)\, dx\, dy$$

$$+ \int_{Q_i}^{\infty} \int_{Q_j}^{\infty} [r_j Q_j - K_j - w_j Q_j - v_j(y - Q_j)]\, f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx \qquad (2)$$

This research assumes that the demand per unit time of vendor *i* will follow Normal distribution with mean μ_i and variance σ_i^2. Therefore, the demand of vendor *i* during time period *T* will follow Normal distribution with mean $\mu_i T$ and variance $\sigma_i^2 T$. Based on this assumption, the functions $f_{D_i(T)}(x)$ and $F_{D_i(T)}(x)$ can be derived. In the same way for vendor *j*, the demand of vendor *j* during time period *T* will follow Normal distribution with mean $\mu_j T$ and variance $\sigma_j^2 T$. Therefore, the functions $f_{D_j(T)}(y)$ and $F_{D_j(T)}(y)$ can be derived. The components of the profit functions will be further analyzed in the following sections.

3.5.1 Scenario 1: When $D_i(T) \leq Q_i$ and $D_j(T) \leq Q_j$

The calculation of the expected profit functions for vendor *i* within a cycle under scenario 1 is derived as detailed below:

$$E[\pi_i^1] = \int_0^{Q_i} \int_0^{Q_j} [r_i x + s_i(Q_i - x) - K_i - w_i Q_i]\, f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx$$

$$= \int_0^{Q_i} \int_0^{Q_j} (r_i - s_i) x\, f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx$$

$$+ \int_0^{Q_i} \int_0^{Q_j} [(s_i - w_i) Q_i - K_i]\, f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx$$

$$= (r_i - s_i) \int_0^{Q_i} x \left(\int_0^{Q_j} f_{D_j(T)}(y)\, dy \right) f_{D_i(T)}(x)\, dx$$

$$+ [(s_i - w_i)Q_i - K_i] \int_0^{Q_i} \left(\int_0^{Q_j} f_{D_j(T)}(y) dy \right) f_{D_i(T)}(x)\, dx$$

$$= (r_i - s_i) \int_0^{Q_i} x\, F_{D_j(T)}(Q_j)\, f_{D_i(T)}(x)\, dx$$

$$+ [(s_i - w_i)Q_i - K_i] \int_0^{Q_i} F_{D_j(T)}(Q_j)\, f_{D_i(T)}(x)\, dx$$

$$= (r_i - s_i)\, F_{D_j(T)}(Q_j) \int_0^{Q_i} x\, f_{D_i(T)}(x)\, dx$$

$$+[(s_i - w_i)Q_i - K_i]\, F_{D_j(T)}(Q_j)\, F_{D_i(T)}(Q_i) \tag{3}$$

The expected profit functions for vendor *j* in a cycle in the scenario 1 can be formulated as:

$$E[\pi_j^1] = \int_0^{Q_i} \int_0^{Q_j} [r_j y + s_j(Q_j - y) - K_j - w_j Q_j]\, f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx$$

$$= \int_0^{Q_i} \int_0^{Q_j} (r_j - s_j) y\, f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx$$

$$+ \int_0^{Q_i} \int_0^{Q_j} [(s_j - w_j)Q_j - K_j]\, f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx$$

$$= (r_j - s_j) \int_0^{Q_i} \left(\int_0^{Q_j} y\, f_{D_j(T)}(y)\, dy \right) f_{D_i(T)}(x)\, dx$$

$$+ [(s_j - w_j)Q_j - K_j] \int_0^{Q_i} \left(\int_0^{Q_j} f_{D_j(T)}(y)\, dy \right) f_{D_i(T)}(x)\, dx$$

$$= (r_j - s_j)\, F_{D_i(T)}(Q_i) \int_0^{Q_j} y\, f_{D_j(T)}(y)\, dy$$

$$+ \left[(s_j - w_j)Q_j - K_j\right] F_{D_j(T)}(Q_j)\, F_{D_i(T)}(Q_i) \tag{4}$$

3.5.2 Scenario 2: When $D_i(T) \le Q_i$ and $D_j(T) > Q_j$

The calculation of the expected profit functions for vendor *i* within a cycle under scenario 2 is derived as detailed below:

$$E[\pi_i^2] = \int_0^{Q_i} \int_{Q_j}^{\infty} \{r_i\, Min[x + \alpha_j(y - Q_j), Q_i] + s_i Max[Q_i - x - \alpha_j(y - Q_j), 0] - K_i - w_i Q_i\}$$

$$f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx$$

$$= \int_0^{Q_i} \int_{Q_j}^{\infty} r_i\, Min[x + \alpha_j(y - Q_j), Q_i]\, f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx$$

$$+ \int_0^{Q_i} \int_{Q_j}^{\infty} s_i\, Max[Q_i - x - \alpha_j(y - Q_j), 0]\, f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx$$

$$- \int_0^{Q_i} \int_{Q_j}^{\infty} (w_i Q_i + K_i)\, f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx$$

$$= r_i \int_0^{Q_i} \left[\int_{Q_j}^{\infty} Min[x + \alpha_j(y - Q_j), Q_i]\, f_{D_j(T)}(y)\, dy \right] f_{D_i(T)}(x)\, dx$$

$$+ s_i \int_0^{Q_i} \left[\int_{Q_j}^{\infty} Max[Q_i - x - \alpha_j(y - Q_j), 0]\, f_{D_j(T)}(y)\, dy \right] f_{D_i(T)}(x)\, dx$$

$$- (w_i Q_i + K_i) \int_0^{Q_i} \left(\int_{Q_j}^{\infty} f_{D_j(T)}(y)\, dy \right) f_{D_i(T)}(x)\, dx$$

$$= r_i \int_0^{Q_i} \left[\int_{Q_j}^{Q_j+\frac{Q_i-x}{\alpha_j}} \left[x + \alpha_j\left(y - Q_j\right)\right] f_{D_j(T)}(y)\, dy + \int_{Q_j+\frac{Q_i-x}{\alpha_j}}^{\infty} Q_i\, f_{D_j(T)}(y)\, dy \right] f_{D_i(T)}(x)\, dx$$

$$+ s_i \int_0^{Q_i} \left[\int_{Q_j}^{Q_j+\frac{Q_i-x}{\alpha_j}} \left[Q_i - x - \alpha_j\left(y - Q_j\right)\right] f_{D_j(T)}(y)\, dy \right] f_{D_i(T)}(x)\, dx$$

$$- (w_i Q_i + K_i) \int_0^{Q_i} \left[1 - F_{D_j(T)}\left(Q_j\right)\right] f_{D_i(T)}(x)\, dx$$

$$= r_i \int_0^{Q_i} \left[\int_{Q_j}^{Q_j+\frac{Q_i-x}{\alpha_j}} \left[x + \alpha_j\left(y - Q_j\right)\right] f_{D_j(T)}(y)\, dy + Q_i \left[1 - F_{D_j(T)}\left(Q_j + \frac{Q_i - x}{\alpha_j}\right)\right] \right] f_{D_i(T)}(x)\, dx$$

$$+ s_i \int_0^{Q_i} \left[\int_{Q_j}^{Q_j+\frac{Q_i-x}{\alpha_j}} \left[Q_i - x - \alpha_j\left(y - Q_j\right)\right] f_{D_j(T)}(y)\, dy \right] f_{D_i(T)}(x)\, dx$$

$$- (w_i Q_i + K_i) \left[1 - F_{D_j(T)}\left(Q_j\right)\right] \left[F_{D_i(T)}(Q_i)\right] \tag{5}$$

The expected profit functions for vendor j in a cycle in the scenario 2 can be formulated as:

$$E\left[\pi_j^2\right] = \int_0^{Q_i} \int_{Q_j}^{\infty} \{r_j Q_j - K_j - w_j Q_j - v_j\{x + \alpha_j\left(y - Q_j\right) - Min\left[x + \alpha_j\left(y - Q_j\right), Q_i\right]\}\}$$

$$f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx$$

$$= \int_0^{Q_i} \int_{Q_j}^{\infty} \left[\left(r_j - w_j\right) Q_j - K_j\right] f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx$$

$$- \int_0^{Q_i} \int_{Q_j}^{\infty} v_j\{x + \alpha_j\left(y - Q_j\right) - Min\left[x + \alpha_j\left(y - Q_j\right), Q_i\right]\}\, f_{D_i(T)}(x) f_{D_j(T)}(y)\, dy\, dx$$

$$= \left[(r_j - w_j)Q_j - K_j\right] \int_0^{Q_i} \left(\int_{Q_j}^{\infty} f_{D_j(T)}(y)\, dy \right) f_{D_i(T)}(x)\, dx$$

$$- v_j \int_0^{Q_i} \left\{ \int_{Q_j}^{\infty} \{x + \alpha_j(y - Q_j) - Min[x + \alpha_j(y - Q_j), Q_i]\} f_{D_j(T)}(y)\, dy \right\} f_{D_i(T)}(x)\, dx$$

$$= \left[(r_j - w_j)Q_j - K_j\right] \left[1 - F_{D_j(T)}(Q_j)\right] \left[F_{D_i(T)}(Q_i)\right]$$

$$- v_j \int_0^{Q_i} \left\{ \int_{Q_j + \frac{Q_i - x}{\alpha_j}}^{\infty} \left[x + \alpha_j(y - Q_j) - Q_i\right] f_{D_j(T)}(y)\, dy \right\} f_{D_i(T)}(x)\, dx$$

$$= \left[(r_j - w_j)Q_j - K_j\right] \left[1 - F_{D_j(T)}(Q_j)\right] \left[F_{D_i(T)}(Q_i)\right]$$

$$- v_j \int_0^{Q_i} \left\{ (x - Q_i)\left[1 - F_{D_j(T)}\left(Q_j + \frac{Q_i - x}{\alpha_j}\right)\right] + \int_{Q_j + \frac{Q_i - x}{\alpha_j}}^{\infty} \alpha_j(y - Q_j)\, f_{D_j(T)}(y)\, dy \right\}$$

$$f_{D_i(T)}(x)\, dx \qquad (6)$$

3.5.3 Scenario 3: When $D_i(T) > Q_i$ and $D_j(T) \le Q_j$

The calculation of the expected profit functions for vendor *i* within a cycle under scenario 3 is derived as detailed below:

$$E[\pi_i^3] = \int_0^{Q_j} \int_{Q_i}^{\infty} \left\{ r_i Q_i - K_i - w_i Q_i - v_i\{y + \alpha_i(x - Q_i) - Min[y + \alpha_i(x - Q_i), Q_j]\} \right\}$$

$$f_{D_i(T)}(x) f_{D_j(T)}(y)\, dx\, dy$$

$$= \int_0^{Q_j} \int_{Q_i}^{\infty} [(r_i - w_i)Q_i - K_i]\, f_{D_i(T)}(x) f_{D_j(T)}(y)\, dx\, dy$$

$$- \int_0^{Q_j} \int_{Q_i}^{\infty} v_i\{y + \alpha_i(x - Q_i) - Min[y + \alpha_i(x - Q_i), Q_j]\}\, f_{D_i(T)}(x) f_{D_j(T)}(y)\, dx\, dy$$

$$= [(r_i - w_i)Q_i - K_i] \int_0^{Q_j} \left[\int_{Q_i}^{\infty} f_{D_i(T)}(x)\, dx \right] f_{D_j(T)}(y)\, dy$$

$$- v_i \int_0^{Q_j} \left\{ \int_{Q_i}^{\infty} \{y + \alpha_i(x - Q_i) - Min[y + \alpha_i(x - Q_i), Q_j]\} f_{D_i(T)}(x)\, dx \right\} f_{D_j(T)}(y)\, dy$$

$$= [(r_i - w_i)Q_i - K_i][1 - F_{D_i(T)}(Q_i)] \left[F_{D_j(T)}(Q_j)\right]$$

$$- v_i \int_0^{Q_j} \left\{ \int_{Q_i + \frac{Q_j - y}{\alpha_i}}^{\infty} [y + \alpha_i(x - Q_i) - Q_j]\, f_{D_i(T)}(x)\, dx \right\} f_{D_j(T)}(y)\, dy$$

$$= [(r_i - w_i)Q_i - K_i][1 - F_{D_i(T)}(Q_i)] \left[F_{D_j(T)}(Q_j)\right]$$

$$- v_i \int_0^{Q_j} \left\{ (y - Q_j)\left[1 - F_{D_i(T)}\left(Q_i + \frac{Q_j - y}{\alpha_i}\right)\right] + \int_{Q_i + \frac{Q_j - y}{\alpha_i}}^{\infty} \alpha_i(x - Q_i)\, f_{D_i(T)}(x)\, dx \right\}$$

$$f_{D_j(T)}(y)\, dy \quad (7)$$

The expected profit functions for vendor *j* in a cycle in the scenario 3 can be formulated as:

$$E[\pi_j^3] = \int_0^{Q_j} \int_{Q_i}^{\infty} \{r_j\, Min[y + \alpha_i(x - Q_i), Q_j] + s_j Max[Q_j - y - \alpha_i(x - Q_i), 0] - K_j - w_j Q_j\}$$

$$f_{D_i(T)}(x) f_{D_j(T)}(y)\, dx\, dy$$

$$= \int_0^{Q_j} \int_{Q_i}^{\infty} r_j\, Min[y + \alpha_i(x - Q_i), Q_j]\, f_{D_i(T)}(x)\, f_{D_j(T)}(y)\, dx\, dy$$

$$+ \int_0^{Q_j} \int_{Q_i}^{\infty} s_j\, Max[Q_j - y - \alpha_i(x - Q_i), 0]\, f_{D_i(T)}(x)\, f_{D_j(T)}(y)\, dx\, dy$$

$$-\int_{0}^{Q_j}\int_{Q_i}^{\infty}\left(w_j Q_j + K_j\right) f_{D_i(T)}(x)\, f_{D_j(T)}(y)\, dx\, dy$$

$$= r_j \int_{0}^{Q_j}\left[\int_{Q_i}^{\infty} Min\left[y + \alpha_i(x - Q_i), Q_j\right] f_{D_i(T)}(x)\; dx\right] f_{D_j(T)}(y)\, dy$$

$$+ s_j \int_{0}^{Q_j}\left[\int_{Q_i}^{\infty} Max\left[Q_j - y - \alpha_i(x - Q_i), 0\right] f_{D_i(T)}(x)\, dx\right] f_{D_j(T)}(y)\, dy$$

$$-\left(w_j Q_j + K_j\right)\int_{0}^{Q_j}\left(\int_{Q_i}^{\infty} f_{D_i(T)}(x)\; dx\right) f_{D_j(T)}(y)\, dy$$

$$= r_j \int_{0}^{Q_j}\left[\int_{Q_i}^{Q_i+\frac{Q_j-y}{\alpha_i}} [y + \alpha_i(x - Q_i)]\, f_{D_i(T)}(x)\; dx + \int_{Q_i+\frac{Q_j-y}{\alpha_i}}^{\infty} Q_j\, f_{D_i(T)}(x)\; dx\right] f_{D_j(T)}(y)\, dy$$

$$+ s_j \int_{0}^{Q_j}\left[\int_{Q_i}^{Q_i+\frac{Q_j-y}{\alpha_i}} \left[Q_j - y - \alpha_i(x - Q_i)\right] f_{D_i(T)}(x)\, dx\right] f_{D_j(T)}(y)\, dy$$

$$-\left(w_j Q_j + K_j\right)\int_{0}^{Q_j}\left[1 - F_{D_i(T)}(Q_i)\right] f_{D_j(T)}(y)\, dy$$

$$= r_j \int_{0}^{Q_j}\left[\int_{Q_i}^{Q_i+\frac{Q_j-y}{\alpha_i}} [y + \alpha_i(x - Q_i)]\, f_{D_i(T)}(x)\; dx + Q_j\left[1 - F_{D_i(T)}\left(Q_i + \frac{Q_j - y}{\alpha_i}\right)\right]\right] f_{D_j(T)}(y)\, dy$$

$$+ s_j \int_{0}^{Q_j}\left[\int_{Q_i}^{Q_i+\frac{Q_j-y}{\alpha_i}} \left[Q_j - y - \alpha_i(x - Q_i)\right] f_{D_i(T)}(x)\, dx\right] f_{D_j(T)}(y)\, dy$$

$$-\left(w_j Q_j + K_j\right)\left[1 - F_{D_i(T)}(Q_i)\right]\left[F_{D_j(T)}\left(Q_j\right)\right] \qquad (8)$$

3.5.4 Scenario 4: When $D_i(T) > Q_i$ and $D_j(T) > Q_j$

The calculation of the expected profit functions for vendor *i* within a cycle under scenario 4 is derived as detailed below:

$$E\left[\pi_i^4\right] = \int_{Q_i}^{\infty}\int_{Q_j}^{\infty}[r_iQ_i - K_i - w_iQ_i - v_i(x - Q_i)]\, f_{D_i(T)}(x)f_{D_j(T)}(y)\, dy\, dx$$

$$= \int_{Q_i}^{\infty}\int_{Q_j}^{\infty}[(r_i - w_i + v_i)Q_i - K_i]\, f_{D_i(T)}(x)f_{D_j(T)}(y)\, dy\, dx$$

$$- \int_{Q_i}^{\infty}\int_{Q_j}^{\infty} v_i x\, f_{D_i(T)}(x)f_{D_j(T)}(y)\, dy\, dx$$

$$= [(r_i - w_i + v_i)Q_i - K_i]\int_{Q_i}^{\infty}\left(\int_{Q_j}^{\infty} f_{D_j(T)}(y)\, dy\right) f_{D_i(T)}(x)dx$$

$$- v_i\int_{Q_i}^{\infty} x\left(\int_{Q_j}^{\infty} f_{D_j(T)}(y)\, dy\right) f_{D_i(T)}(x)\, dx$$

$$= [(r_i - w_i + v_i)Q_i - K_i]\int_{Q_i}^{\infty}\left[1 - F_{D_j(T)}(Q_j)\right] f_{D_i(T)}(x)\, dx$$

$$- v_i\int_{Q_i}^{\infty} x\left[1 - F_{D_j(T)}(Q_j)\right] f_{D_i(T)}(x)\, dx$$

$$= [(r_i - w_i + v_i)Q_i - K_i]\left[1 - F_{D_j(T)}(Q_j)\right]\left[1 - F_{D_i(T)}(Q_i)\right]$$

$$- v_i\left[1 - F_{D_j(T)}(Q_j)\right]\int_{Q_i}^{\infty} x\, f_{D_i(T)}(x)\, dx \qquad (9)$$

The expected profit functions for vendor *j* in a cycle in the scenario 4 can be formulated as:

$$E\left[\pi_j^4\right] = \int_{Q_i}^{\infty}\int_{Q_j}^{\infty}\left[r_jQ_j - K_j - w_jQ_j - v_j\left(y - Q_j\right)\right] f_{D_i(T)}(x)f_{D_j(T)}(y)\, dy\, dx$$

$$= \int_{Q_i}^{\infty} \int_{Q_j}^{\infty} \left[(r_j - w_j + v_j) Q_j - K_j \right] f_{D_i(T)}(x) f_{D_j(T)}(y) \, dy \, dx$$

$$- \int_{Q_i}^{\infty} \int_{Q_j}^{\infty} v_j y \, f_{D_i(T)}(x) f_{D_j(T)}(y) \, dy \, dx$$

$$= \left[(r_j - w_j + v_j) Q_j - K_j \right] \int_{Q_i}^{\infty} \left(\int_{Q_j}^{\infty} f_{D_j(T)}(y) \, dy \right) f_{D_i(T)}(x) \, dx$$

$$- v_j \int_{Q_i}^{\infty} \left(\int_{Q_j}^{\infty} y f_{D_j(T)}(y) \, dy \right) f_{D_i(T)}(x) \, dx$$

$$= \left[(r_j - w_j + v_j) Q_j - K_j \right] \left[1 - F_{D_j(T)}(Q_j) \right] \left[1 - F_{D_i(T)}(Q_i) \right]$$

$$- v_j \left[1 - F_{D_i(T)}(Q_i) \right] \int_{Q_j}^{\infty} y \, f_{D_j(T)}(y) \, dy \qquad (10)$$

Following the analysis of the four scenarios, the expected total profit function for vendor i per a cycle time can be computed as

$$E[\pi_i^s] = E[\pi_i^1] + E[\pi_i^2] + E[\pi_i^3] + E[\pi_i^4] \qquad (11)$$

In a similar manner, the expected total profit function for vendor j per a cycle time can be computed as

$$E[\pi_j^s] = E[\pi_j^1] + E[\pi_j^2] + E[\pi_j^3] + E[\pi_j^4] \qquad (12)$$

Hence, the expected total profit function of both vendors per time unit can be computed as

$$E[\pi^s] = \frac{1}{T} \left(E[\pi_i^s] + E[\pi_j^s] \right) \qquad (13)$$

CHAPTER 4
NUMERICAL EXPERIMENTS AND SENSITIVITY ANALYSES

In this chapter, numerical experiments are conducted to illustrate the applicability of the mathematical models developed in chapter 3. Furthermore, sensitivity analyses are also presented. The dataset required for computational experiments is predefined. Thereafter, the impact of significant input parameters on decision variables and optimal solutions can be analyzed through sensitivity analyses using Genetic Algorithm (GA) solver in MATLAB.

4.1 Assumptions

A variety of numerical experiments of a VMI system involving two vendors and one retailer are performed to evaluate the effectiveness of the proposed inventory model. The optimal order quantities for both vendors delivered to the retailer (Q_i, Q_j) in an inventory replenishment cycle of length T will be examined with the aim to maximize the expected total profit per time unit for the retailer. Genetic Algorithm (GA) solver in MATLAB is conducted to find the optimal values of two decision variables.

In order to find the optimal solution, the following input parameters were provided and implemented for GA solver,

- The number of decision variables is 2.
- Lower bound = [1, 1]
- Upper bound = [2000, 2000]

Input parameter values for the base scenario are given in Table 4.1.

Table 4.1

Input Parameter Values for VMI System with Two Vendors and One Retailer

Input Parameter	Value	
	Vendor *i*	Vendor *j*
Mean demand (μ_i, μ_j) (unit per day)	150	100
Standard deviation of demand (σ_i, σ_j) (unit per day)	15	10
The fraction of search demand (α)	0.8	0.8
A constant value of the ordering cost (K)	50	50
Unit purchasing cost (w) ($ per unit)	4	4
Unit shortage cost (v) ($ per unit)	8	8
Unit retail price (r) ($ per unit)	10	10
Unit salvage value (s) ($ per unit)	3	3
The cycle length (T) (day)	6	6

Based on the input parameters provided in Table 4.1, genetic solver (GA) implemented in MATLAB identifies the optimal values of two decision variables: the order quantity of vendor i (Q_i) is 935 units, and the order quantity of vendor j (Q_j) is 631 units. The expected total profit for the retailer per day is $1468.85.

4.2 Sensitivity Analyses

Sensitivity analyses are conducted to investigate the impacts of input parameters, namely the mean demand, the standard deviation of demand, the fraction of search demand, the ordering cost, the purchasing cost, the shortage cost, the retail price, the salvage value, and the cycle length in the base case on decision variables and optimal solutions. However, it should be highlighted that the cycle length (T) in this study for both vendors are identical.

4.2.1 The Impact of Mean Demand

This section analyzes the impact of mean demand (μ_i, μ_j), while keeping other input parameters fixed in the base scenario. Vendor *i*'s mean demand (μ_i) ranges from 110 to 190, and Vendor *j*'s mean demand (μ_j) ranges from 8 to 16. The outcomes, including the order quantities of both vendors and the expected total profit per unit time, are presented and discussed in Table 4.2 and Figure 4.1.

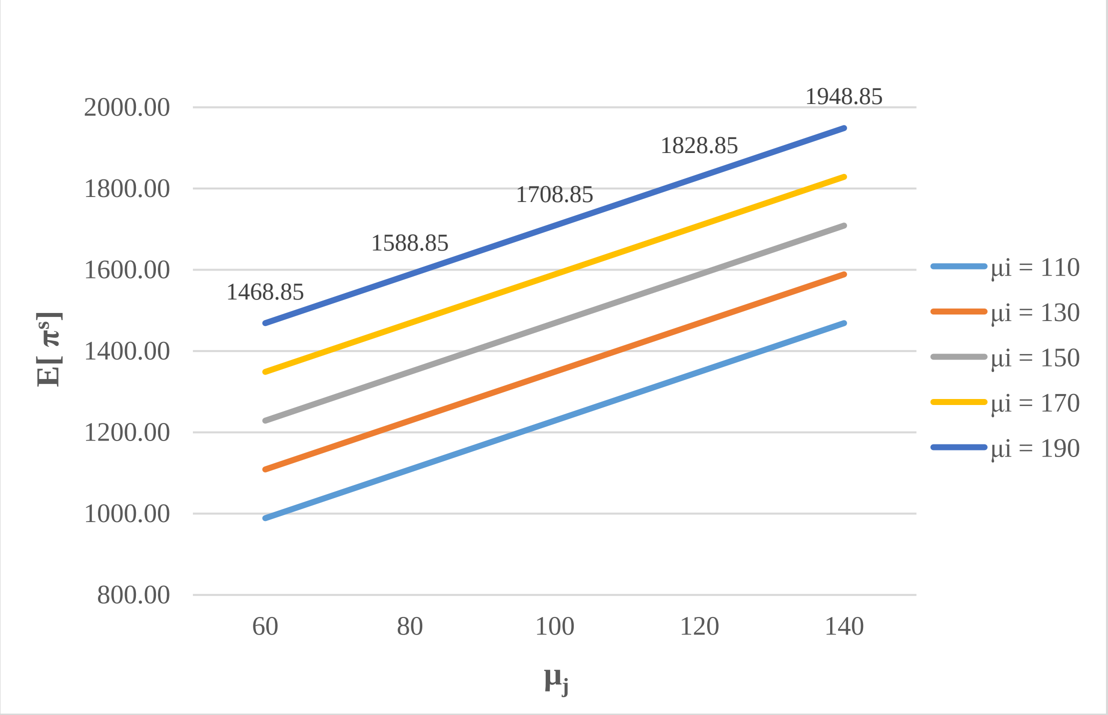
Table 4.2

Impact of Mean Demand on Order Quantities and Expected Total Profit

μ_i	μ_j	Q_i	Q_j	$E[\pi^s]$
	60	695	391	988.85
	80	695	511	1108.85
110	100	695	631	1228.85
	120	695	751	1348.85
	140	695	871	1468.85
	60	815	391	1108.85
	80	815	511	1228.85
130	100	815	631	1348.85
	120	815	751	1468.85
	140	815	871	1588.85
	60	935	391	1228.85
	80	935	511	1348.85
150	100	935	631	1468.85
	120	935	751	1588.85
	140	935	871	1708.85
	60	1055	391	1348.85
	80	1055	511	1468.85
170	100	1055	631	1588.85
	120	1055	751	1708.85
	140	1055	871	1828.85
	60	1175	391	1468.85
	80	1175	511	1588.85
190	100	1175	631	1708.85
	120	1175	751	1828.85
	140	1175	871	1948.85

Figure 4.1

Correlation between Mean Demand and Expected Total Profit per Time Unit



Based on the findings presented in Table 4.2 and Figure 4.1, it is obvious that when the mean demand of vendor *i* (μ_i) increases while the mean demand of vendor *j* (μ_j) maintains fixed or vice versa, the order quantity of vendor *i* (Q_i) will correspondingly rise, but the order quantity of vendor *j* (Q_j) stays unchanged. This trend is logical because the increased order quantity helps to meet the higher demand. Additionally, it should be highlighted that the expected total profit per time unit ($E[\pi^s]$) also goes up directly with the increased mean demand. Increased demand leads to higher sales revenue because more units are sold at the same fixed cost. Therefore, the additional revenue generated from higher demand contributes entirely to profit, without any additional costs incurred to meet the increased demand.

4.2.2 The Impact of the Standard Deviation of Demand

This section investigates the influence of the standard deviation of demand (σ_i, σ_j), while maintaining constant values for other parameters in the base scenario. The standard deviation of vendor *i*'s demand (σ_i) ranges from 11 to 19, whereas the standard deviation of vendor *j*'s demand (σ_j) varies between 6 and 14. The outcomes, including order quantities for both vendors and the expected total profit per time unit, are presented and analyzed in Table 4.3 and Figure 4.2.

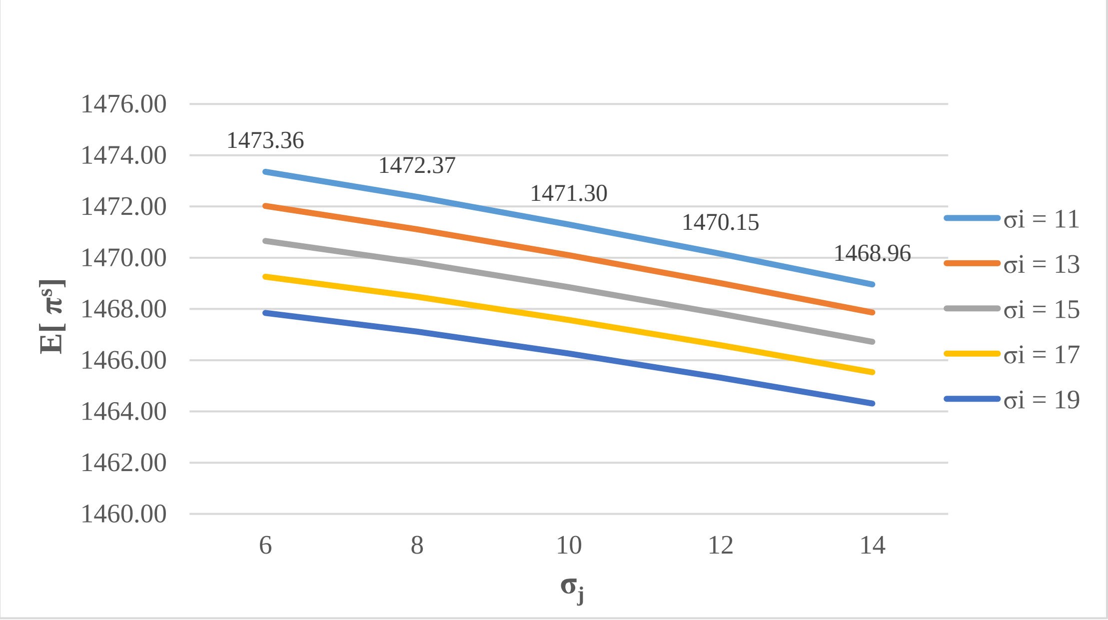
Table 4.3

Impact of Standard Deviation of Demand on Order Quantities and Expected Total Profit

σ_i	σ_j	Q_i	Q_j	$E[\pi^s]$
	6	925	621	1473.36
	8	926	624	1472.37
11	10	928	627	1471.30
	12	930	631	1470.15
	14	932	634	1468.96
	6	930	623	1472.02
	8	930	626	1471.11
13	10	931	629	1470.10
	12	933	633	1469.01
	14	935	636	1467.86
	6	934	624	1470.65
	8	934	628	1469.81
15	10	935	631	1468.85
	12	937	635	1467.82
	14	938	638	1466.72
	6	939	626	1469.26
	8	938	630	1468.48
17	10	939	633	1467.57
	12	940	637	1466.58
	14	942	640	1465.53
	6	944	627	1467.84
	8	943	631	1467.12
19	10	943	635	1466.26
	12	944	639	1465.32
	14	945	642	1464.31

Figure 4.2

Correlation between Standard Deviation of Demand and Expected Total Profit per Time Unit



As indicated in Table 4.3 and Figure 4.2, variations in the standard deviation of vendor *i*'s demand (σ_i), while the standard deviation of vendor *j*'s demand (σ_j) keeps constant or vice versa, result in marginal increases in both order quantities. An increase in the standard deviation of demand of vendor *i* (σ_i) indicates greater variability or uncertainty in the demand for vendor *i*'s products. In response to accommodate fluctuating demand patterns, the retailer adjusts order quantities slightly to manage inventory risks, aiming to mitigate potential stockouts or excess inventory situations caused by demand variability. Nonetheless, the expected total profit per time unit ($E[\pi^s]$) slowly declines, which is reasonable due to the associated costs increasing when the standard deviation of demand rises.

4.2.3 The Impact of the Fraction of Search Demand

This section explores the impact of the fraction of search demand (α). The fraction of search demand of vendor *i* (α_i) is varied from 0.2 to 1, while keeping other input parameters in the base scenario constant. The outcomes regarding the order quantities of both vendors and the expected total profit per time unit are presented and analyzed in Table 4.4 and Figure 4.3.

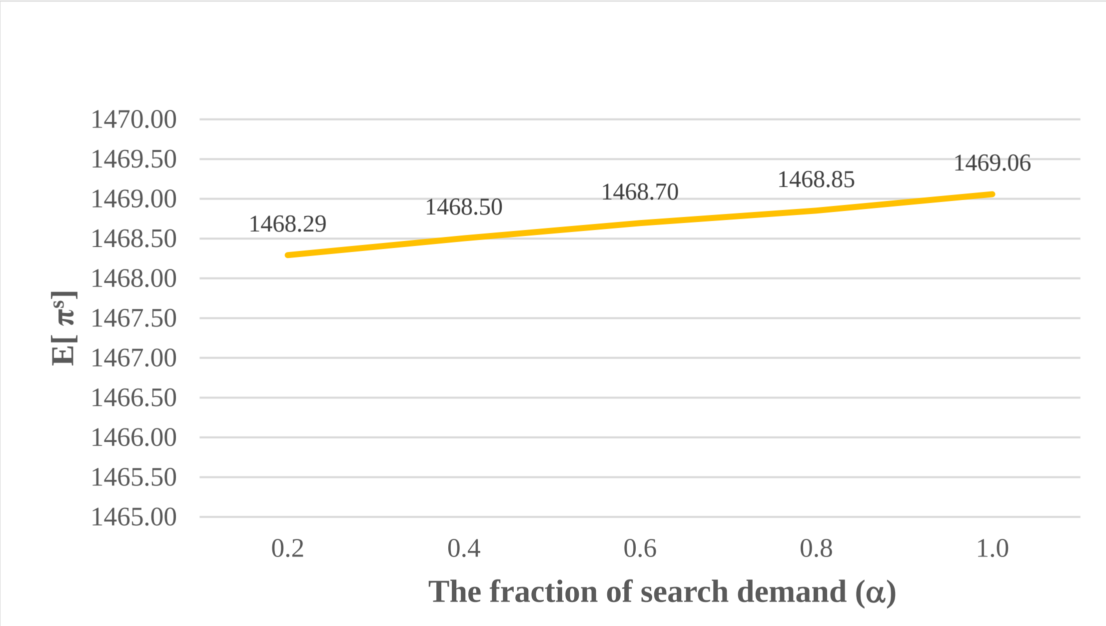
Table 4.4

Impact of Fraction of Search Demand on Order Quantities and Expected Total Profit

α_i	α_j	Q_i	Q_j	$E[\pi^s]$
0.2	0.8	949	619	1468.29
0.4	0.8	945	622	1468.50
0.6	0.8	941	626	1468.70
0.8	0.8	935	631	1468.85
1.0	0.8	784	784	1469.06

Figure 4.3

Correlation between Fraction of Search Demand and Expected Total Profit per Time Unit



As demonstrated in Table 4.4 and Figure 4.3, it can be concluded that as the fraction of search demand of vendor *i* (α_i) increases while the fraction of search demand of vendor *j* (α_j) continues unchanged, the order quantity of vendor *i* (Q_i) will slightly decrease, whereas the order quantity of vendor *j* (Q_j) will slowly increase. This pattern is generally understandable because when the fraction of search demand of vendor *i* (α_i) increases, it implies that more customers of vendor *i* become aware of vendor *j*'s product during shortages of vendor *i*'s product, which typically leads to increased demand and sales for vendor *j*. Hence, the retailer may adjust by potentially increasing the order quantity of vendor *j* (Q_j) to meet this higher demand. Accordingly, the expected total profit per unit time ($E[\pi^s]$) also slowly increases. Since a higher demand resulting from the increase in the faction of search demand of vendor *i* allows the retailer to sell more units with the increased order quantities, leading to higher revenue and, consequently, higher expected total profit per unit time.

4.2.4 The Impact of Ordering Cost

This section examines the impact of the ordering cost (*K*). The ordering cost for vendor *i* (K_i) is varied from 10 to 90, while all other input parameters in the base scenario are kept constant. The findings, which include the order quantities for both vendors and the expected total profit per time unit, are displayed and discussed in Table 4.5 and Figure 4.4.

Table 4.5

Impact of Ordering Cost on Order Quantities and Expected Total Profit

K_i	K_j	Q_i	Q_j	$E[\pi^s]$
10	50	935	631	1475.52
30	50	935	631	1472.19
50	50	935	631	1468.85
70	50	935	631	1465.52
90	50	935	631	1462.19

Figure 4.4

Correlation between Ordering Cost and Expected Total Profit per Time Unit

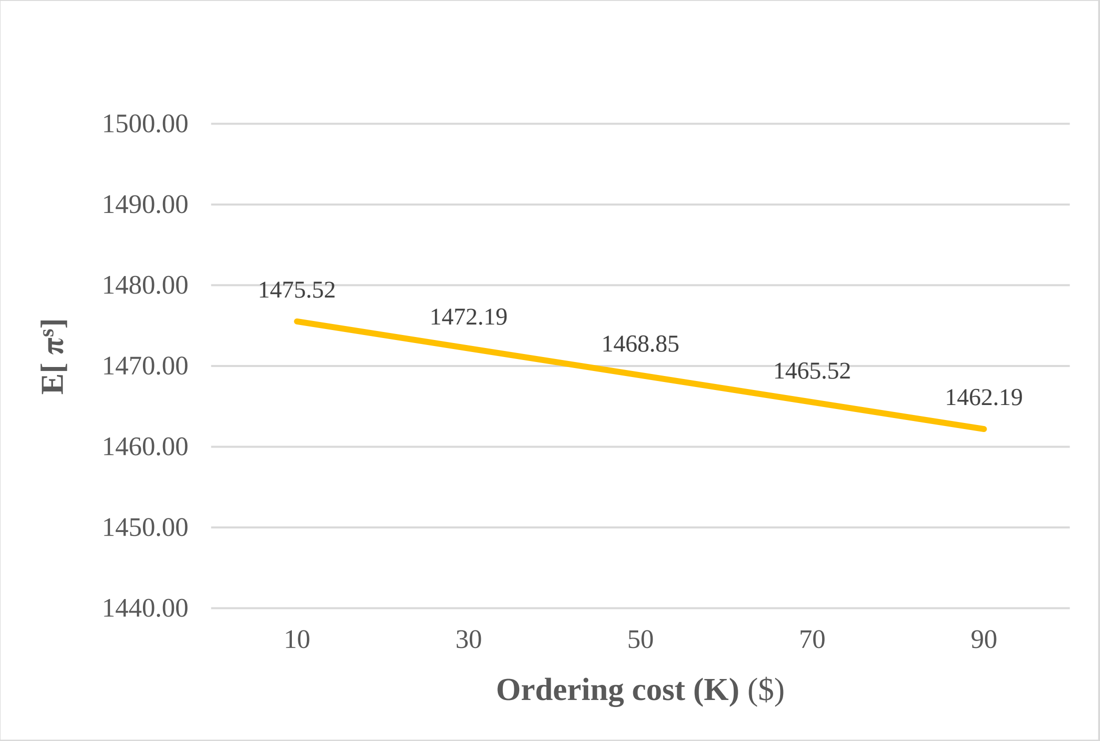


Table 4.5 and Figure 4.4 illustrate that when the ordering cost for vendor i (K_i) increases, while the ordering cost for vendor j (K_j) stays the same, the order quantities for vendor i (Q_i) and vendor j (Q_j) remain unchanged. It is indicated that the ordering cost does not affect order quantities. Both vendors may see no immediate need to adjust their order quantities in response to changes in ordering costs. On the contrary, the expected total profit per unit time ($E[\pi^s]$) steadily falls. This pattern indicates that as the ordering cost for vendor i rises, each order from vendor i becomes more expensive, thereby reducing the overall profitability of each unit sold.

4.2.5 The Impact of Purchasing Cost

This section investigates the impact of the unit purchasing cost (w). The purchasing cost per unit for vendor i (w_i) is varied from 3 to 5, while all other input parameters in the base scenario are kept stable. The outcomes, which include the order quantities for both vendors and the expected total profit per time unit, are presented and analyzed in Table 4.6 and Figure 4.5.

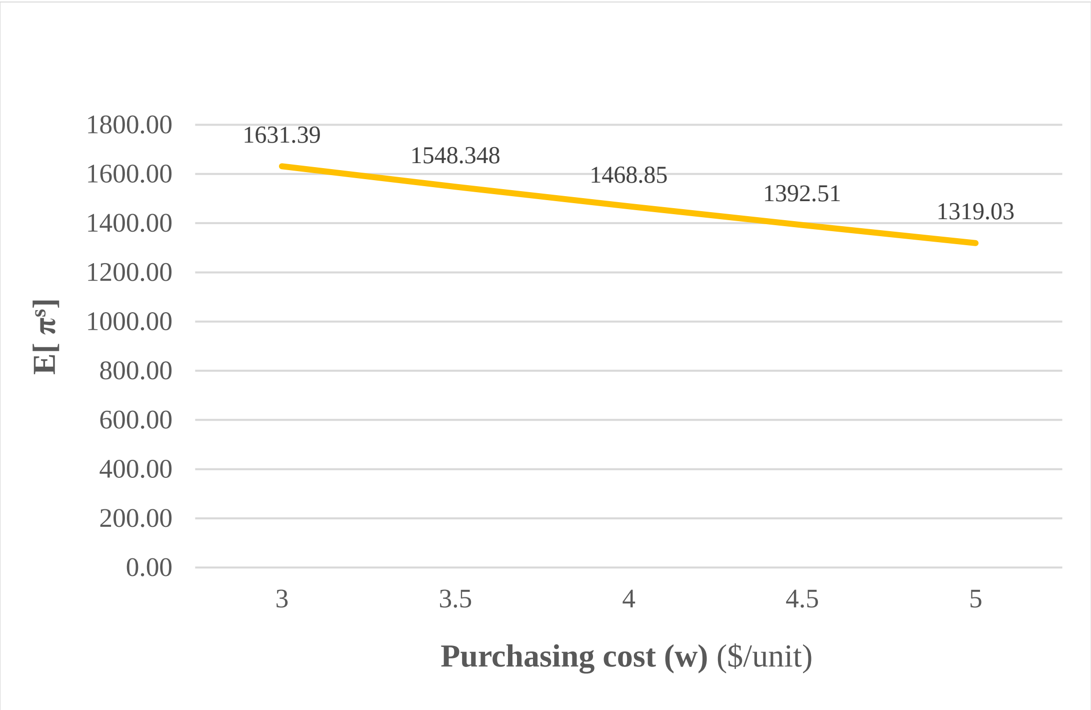
Table 4.6

Impact of Purchasing Cost on Order Quantities and Expected Total Profit

w_i	w_j	Q_i	Q_j	$E[\pi^s]$
3	4	1118	587	1631.39
3.5	4	973	606	1548.35
4	4	935	631	1468.85
4.5	4	900	659	1392.51
5	4	861	689	1319.03

Figure 4.5

Correlation between Purchasing Cost and Expected Total Profit per Time Unit



As illustrated in Table 4.6 and Figure 4.5, when the purchasing cost for vendor *i* (w_i) rises while the purchasing cost for vendor *j* (w_j) keeps stable, the order quantity for vendor *i* (Q_i) will gradually drop, but the order quantity for vendor *j* (Q_j) will steadily rise. This can be explained by the fact that higher unit purchasing cost typically means that it becomes more expensive to hold inventory. Therefore, for the retailer to minimize total inventory costs, fewer units will be ordered from vendor *i* (Q_i).

Furthermore, it can be clearly observed that when the unit purchasing cost approaches the salvage value, the retailer can potentially maximize profit by ordering larger quantities, with any unsold inventory being sold off at no loss. Conversely, as unit purchasing cost rises, the expected total profit per unit time ($E[\pi^s]$) moderately declines. This pattern is justifiable because a higher unit purchasing cost reduces the overall profitability of each unit sold, leading to a lower the expected total profit per unit time.

4.2.6 The Impact of Shortage Cost

This section examines the impact of the shortage cost per unit (v). The shortage cost per unit for vendor i (v_i) is varied from 6 to 10, while all other input parameters in the base scenario remain stable. The findings regarding the order quantities for both vendors and the expected total profit per time unit are presented and analyzed in Table 4.7 and Figure 4.6.

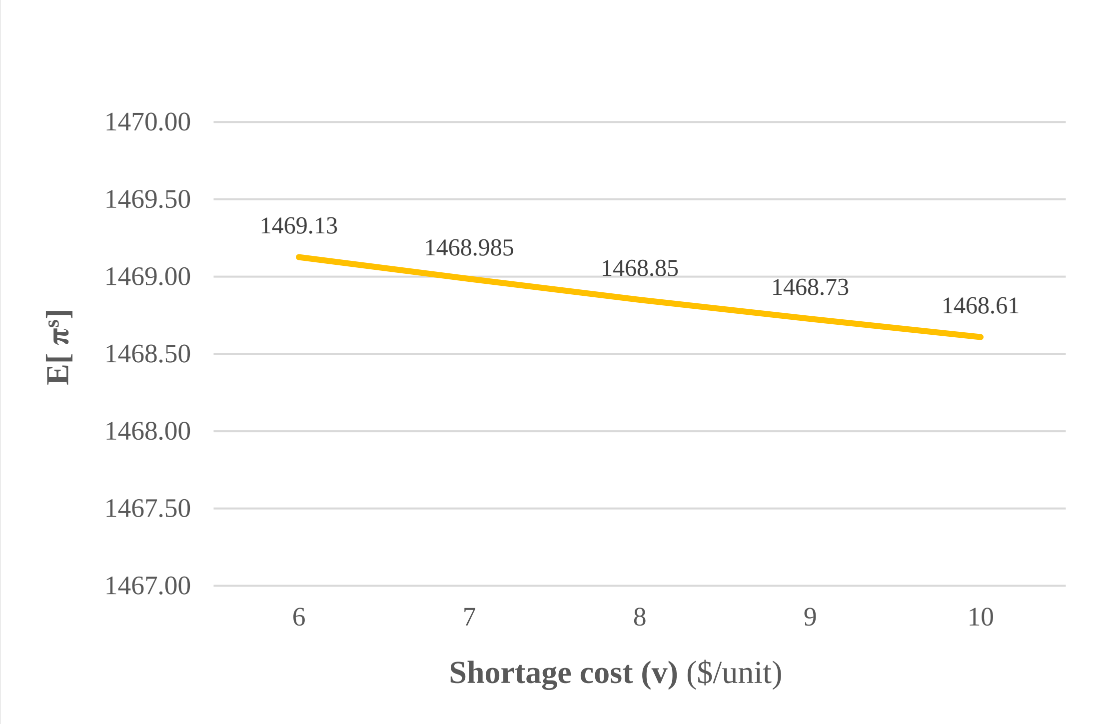
Table 4.7

Impact of Shortage Cost on Order Quantities and Expected Total Profit

v_i	v_j	Q_i	Q_j	$E[\pi^s]$
6	8	933	632	1469.13
7	8	934	632	1468.99
8	8	935	631	1468.85
9	8	936	631	1468.73
10	8	937	631	1468.61

Figure 4.6

Correlation between Shortage Cost and Expected Total Profit per Time Unit



According to Table 4.7 and Figure 4.6, it is observed that an increase in the shortage cost for vendor i (v_i), while the shortage cost for vendor j (v_j) stays stable, results in a slight increase in the order quantity for vendor i (Q_i) and a gradual decrease in the order quantity for vendor j (Q_j). In other words, both order quantities maintain relatively unchanged. It can be explained by the fact that the change in shortage cost does not significantly affect order quantities. In such cases, the retailer may not need to adjust the order quantities significantly, even when facing increased shortage costs. In addition, the expected total profit per unit time ($E[\pi^s]$) shows a gradual decline, which is a logical trend. A higher shortage cost implies a greater penalty for not meeting demand. As the shortage cost increases, the cost associated with lost sales or dissatisfied customers also increases. Consequently, the expected total profit per unit time reduces, albeit slowly.

4.2.7 The Impact of Retail Price

This section investigates the influence of the retail unit price (r). The retail price per unit of vendor i (r_i) varies between 8 and 12 , while keeping other input parameters constant in the base scenario. The findings, including order quantities from both vendors and the expected total profit per time unit, are presented and discussed in Table 4.8 and Figure 4.7.

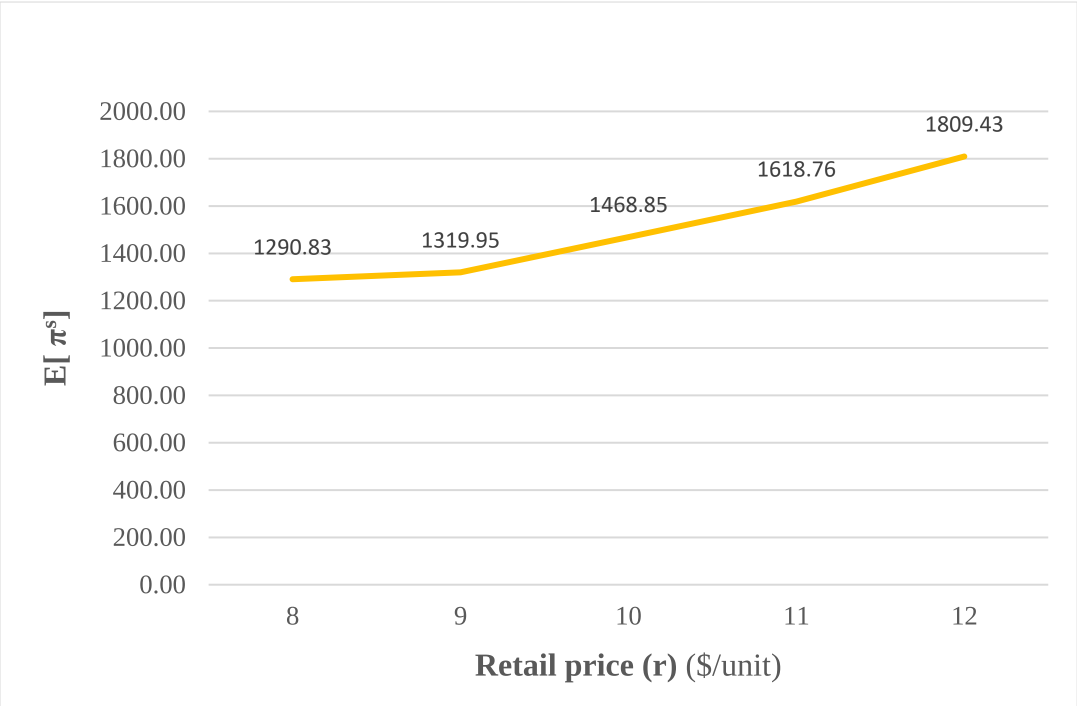
Table 4.8

Impact of Retail Price on Order Quantities and Expected Total Profit

r_i	r_j	Q_i	Q_j	$E[\pi^s]$
8	10	1	1377	1290.83
9	10	897	661	1319.95
10	10	935	631	1468.85
11	10	954	613	1618.76
12	10	1445	1	1809.43

Figure 4.7

Correlation between Retail Price and Expected Total Profit per Time Unit



According to the findings presented in Table 4.8 and Figure 4.7, when the retail price of vendor i (r_i) increases while retail price of vendor j (r_j) remains unchanged, the order quantity of vendor i (Q_i) will increase, but the order quantity of vendor j (Q_j) will decrease. This pattern is clear because a higher retail price typically enhances the profit margin per unit sold. This can incentivize the retailer to order more from vendor i, thereby increasing Q_i, because each unit sold generates more profit. Simultaneously, the expected total profit per unit time ($E[\pi^s]$) also rises, as more units sold at the higher price contribute to higher expected total profit per unit time.

4.2.8 The Impact of Salvage Value

This section examines the influence of the salvage value per unit (s). When the salvage value per unit of vendor i (s_i) varies between 2 and 4, while maintaining other input parameters constant in the base scenario. The findings regarding order quantities from both vendors and the expected total profit per time unit are presented and discussed in Table 4.9 and Figure 4.8.

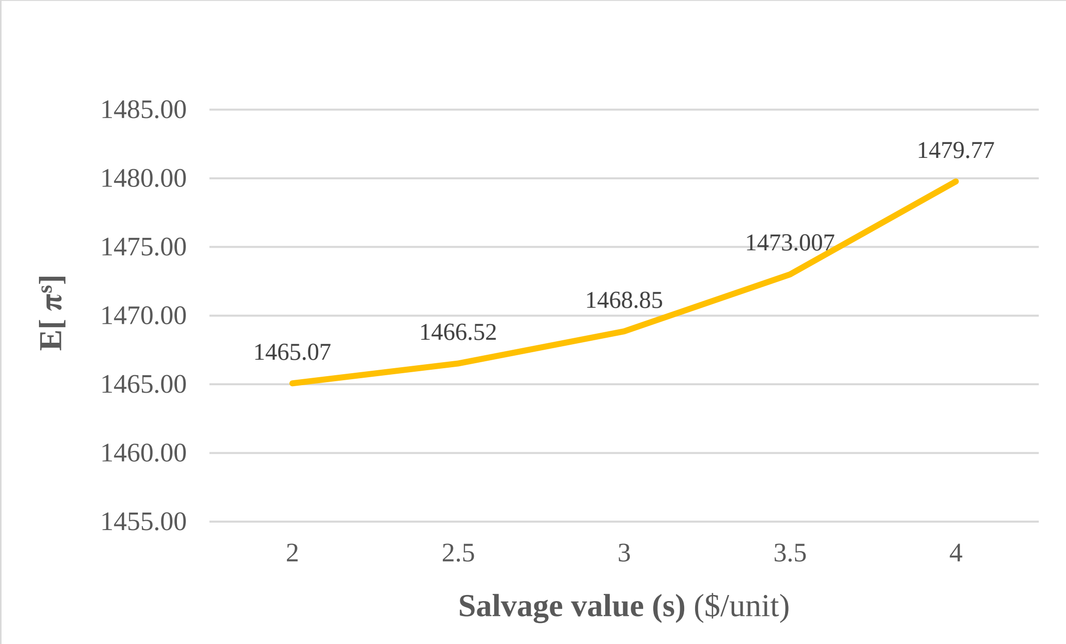
Table 4.9

Impact of Salvage Value on Order Quantities and Expected Total Profit

s_i	s_j	Q_i	Q_j	$E[\pi^s]$
2	3	900	659	1465.07
2.5	3	912	649	1466.52
3	3	935	631	1468.85
3.5	3	966	612	1473.01
4	3	1353	603	1479.77

Figure 4.8

Correlation between Salvage Value and Expected Total Profit per Time Unit



The findings from Table 4.9 and Figure 4.8 illustrate that when the salvage value of vendor *i* (s_i) increases while the salvage value of vendor *j* (s_j) remains steady, the order quantity of vendor *i* (Q_i) will increase, whereas the order quantity of vendor *j* (Q_j) will decline. This phenomenon occurs because an increase in salvage value means that the retailer receives a higher value for each unit of unsold product at the end of the selling period. Consequently, the retailer may feel more comfortable ordering larger quantity of vendor *i*'s product. Additionally, it is noted that when the salvage value goes up to reach the unit purchasing cost, the retailer can recover the entire cost of each unsold unit. This leads to a rapid and dramatic increase in the order quantity of vendor *i* as the retailer seeks to maximize potential sales without worrying about financial losses from unsold goods. Concurrently, the expected total profit per unit time ($E[\pi^s]$) tends to increase slowly. Since the salvage value contributes to revenue, an increase in salvage value directly leads to increased revenue.

4.2.9 The Impact of Cycle Length

This section examines the influence of the cycle length per day (T). It is noticed that the replenishment cycles for both vendors are identical within scope and limitations in this study. Therefore, the cycle length per day of both vendor *i* and vendor *j* (T_i, T_j) are systematically adjusted from 4 to 8, while other input parameters remain constant in the base scenario. The findings regarding the order quantities from both vendors and the expected total profit per time unit are presented and analyzed in Table 4.10 and Figure 4.9.

Table 4.10

Impact of Cycle Length on Order Quantities and Expected Total Profit

T	Q_i	Q_j	$E[\pi^s]$
4	629	426	1457.27
5	782	529	1464.14
6	935	631	1468.85
7	1088	734	1472.31
8	1240	836	1474.96

Figure 4.9

Correlation between Cycle Length and Expected Total Profit per Time Unit

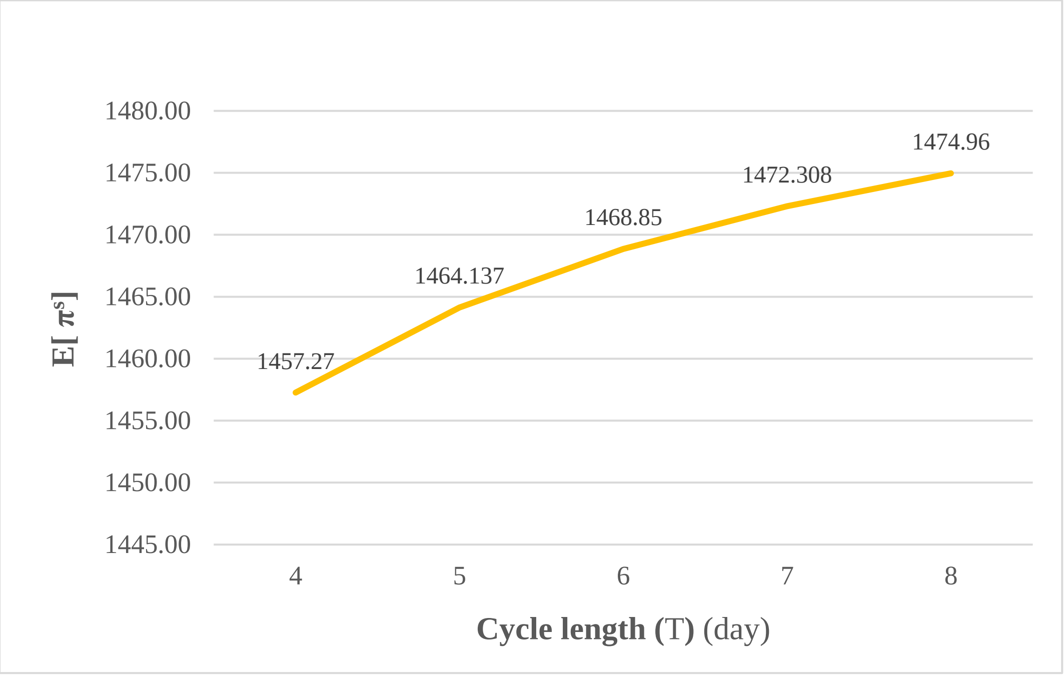


Table 4.10 and Figure 4.9 illustrate that an increase in the cycle length (T) results in a significant increase in the order quantities of vendor i (Q_i) and vendor j (Q_j). Likewise, the expected total profit per unit time ($E[\pi^s]$) rises correspondingly, albeit slightly. The trend of increasing order quantities and expected total profit per unit time with longer cycle lengths is reasonable. When the cycle length increases, it typically means that the retailer places orders less frequently. With a longer cycle length, both vendors are able to afford to order larger quantities of goods at once. This approach minimizes total inventory costs, including ordering and purchasing expenses, ultimately contributing to higher overall profitability.

CHAPTER 5
CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

This study investigates Vendor Managed Inventory (VMI) system involving two vendors supplying their products to a single retailer. The products from both vendors are assumed as competing products that can substitute one another. When shortages occur and customers' demands cannot be fulfilled, these demands are considered as completely lost. However, there is a probability that customers will search for an alternative product from the other vendor. To manage the inventory, an order-up-to level policy is implemented. This research developed a mathematical model aimed at maximizing the retailer's expected total profit by determining optimal order quantities both vendors for delivery to the retailer during each inventory replenishment cycle of length T. Following the development of the mathematical model, numerical experiments and sensitivity analyses are performed to evaluate its effectiveness and examine the effects of several input parameters, including mean demand (μ_i, μ_j), standard deviation of demand (σ_i, σ_j), search demand fraction (α), ordering cost (K), purchasing cost (w), shortage cost (v), retail price (r), salvage value (s), and cycle length (T). According to sensitivity analysis findings, the influence of all input parameters considered in this study can be noticed as follows:

- The increase in mean demand (μ_i, μ_j), search demand fraction (α), retail price (r), salvage value (s), and cycle length (T) will contribute to the increase in the total expected profit per time unit ($E[\pi^s]$).
- The increase in the standard deviation of demand (σ_i, σ_j), ordering cost (K), purchasing cost (w), and shortage cost (v) will result in the decrease in the total expected profit per time unit ($E[\pi^s]$).
- As the mean demand of vendor i (μ_i) increases, only the order quantity of vendor i (Q_i) will correspondingly increase while not significantly affecting the order quantities of vendor j (Q_j) as the increased order quantity helps to meet the higher demand. The same applies vice versa for vendor j.
- Changes in the order quantities of both vendors (Q_i ,Q_j) are observed when there are variations in the standard deviation of demand (σ_i, σ_j), search demand

fraction (α), purchasing cost (w), retail price (r), salvage value (s), or cycle length (T)

- Changes in the ordering cost (K) or the shortage cost (v) do not significantly affect the order quantities of both vendors.

5.2 Recommendations

Future studies could investigate this issue through diverse methodologies, including the following.

- This study considers a VMI system involving only two vendors and one retailer. Future research should enhance realism by investigating VMI systems with multiple vendors and one retailer.
- In this research, the replenishment cycles for both vendors are identical. To increase the realism of future studies, the cycle lengths for both vendors can be different.
- With the increasing intensity of competition between vendors nowadays, factors related to customer behavior should be further analyzed.

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APPENDIX

COMPUTER PROGRAM (MATLAB)

```
clc
clear all;
format long g

x0 = [0,0];

%Fminunc
[P,favl] = fmincon(@fun,x0,[],[],[],[],[1,1],[2000,2000],[])

function Profit = fun(P)

%Decision variables:
%Q1 = P(1);
%Q2 = P(2);

% Declare parameters
m1 = 150; %Mean i
m2 = 100; %Mean j
s1 = 15; %Standard deviation i
s2 = 10; %Standard deviation j

alpha = 0.8; %The fraction of search demand
K = 50; %A constant value of the ordering cost
w = 4;  %Purchasing cost per unit
v = 8;  %Shortage cost per unit
r = 10; %Retail price per unit
s = 3;  %Salvage value per unit
T = 6;  %The cycle length in day
```

```
mu1 = m1*T;
mu2 = m2*T;
sd1 = s1*sqrt(T);
sd2 = s2*sqrt(T);

%Total Profit of retailer from selling product i for each case

%Case1
TPi1 = (r-s)*integral2(@(x,y)x.*normpdf(y,mu2,sd2) ...
    .*normpdf(x,mu1,sd1),0,P(1),0,P(2)) ...
    + (((s-w)*P(1))-K).*integral2(@(x,y) normpdf(y,mu2,sd2) ...
    .*normpdf(x,mu1,sd1),0,P(1),0,P(2));

%Case2
TPi2 = (r*integral2(@(x,y) (x+(alpha*(y-P(2)))).*normpdf(x,mu1,sd1) ...
    .*normpdf(y,mu2,sd2),0,P(1),P(2),@(x) P(2)+((P(1)-x)/alpha))) ...
    + (r*integral2(@(x,y) P(1).*normpdf(x,mu1,sd1) ...
    .*normpdf(y,mu2,sd2),0,P(1),@(x) P(2)+((P(1)-x)/alpha),2000)) ...
    + (s*integral2(@(x,y) (P(1)-x-(alpha*(y-P(2)))).*normpdf(x,mu1,sd1) ...
    .*normpdf(y,mu2,sd2),0,P(1),P(2),@(x) P(2)+((P(1)-x)/alpha))) ...
    - ((w*P(1))+K)*integral2(@(x,y) normpdf(x,mu1,sd1) ...
    .*normpdf(y,mu2,sd2),0,P(1),P(2),2000);

%Case3
TPi3 = (((r-w)*P(1))-K)*integral2(@(y,x) normpdf(x,mu1,sd1) ...
    .*normpdf(y,mu2,sd2),0,P(2),P(1),2000) ...
    - (v*integral2(@(y,x) (y+(alpha*(x-P(1)))-P(2)).*normpdf(x,mu1,sd1) ...
    .*normpdf(y,mu2,sd2),0,P(2),@(y) P(1)+((P(2)-y)/alpha),2000));

%Case4
TPi4 = (((r-w+v)*P(1))-K)*integral2(@(x,y) normpdf(x,mu1,sd1) ...
    .*normpdf(y,mu2,sd2),P(1),2000,P(2),2000) ...
    - v*(integral2(@(x,y) x.*normpdf(x,mu1,sd1) ...
    .*normpdf(y,mu2,sd2),P(1),2000,P(2),2000));
```

```
TPi = TPi1+TPi2+TPi3+TPi4;

%Total Profit of retailer from selling product j for each case

%Case 1
TPj1 = (r-s)*integral2(@(x,y) y.*normpdf(x,mu1,sd1) ...
        .*normpdf(y,mu2,sd2),0,P(1),0,P(2)) ...
        + (((s-w)*P(2))-K).*integral2(@(x,y) normpdf(x,mu1,sd1) ...
        .*normpdf(y,mu2,sd2),0,P(1),0,P(2));

%Case 2
TPj2 = (((r-w)*P(2))-K)*integral2(@(x,y) normpdf(x,mu1,sd1) ...
        .*normpdf(y,mu2,sd2),0,P(1),P(2),2000) ...
        - (v*integral2(@(x,y) (x+(alpha*(y-P(2)))-P(1)).*normpdf(x,mu1,sd1) ...
        .*normpdf(y,mu2,sd2),0,P(1),@(x) P(2)+((P(1)-x)/alpha),2000));

%Case 3
TPj3 = (r*integral2(@(y,x) (y+(alpha*(x-P(1)))).*normpdf(x,mu1,sd1) ...
        .*normpdf(y,mu2,sd2),0,P(2),P(1),@(y) P(1)+((P(2)-y)/alpha)) ...
        + (r*integral2(@(y,x) P(2).*normpdf(x,mu1,sd1) ...
        .*normpdf(y,mu2,sd2),0,P(2),@(y) P(1)+((P(2)-y)/alpha),2000)) ...
        + (s*integral2(@(y,x) (P(2)-y-(alpha*(x-P(1)))).*normpdf(x,mu1,sd1) ...
        .*normpdf(y,mu2,sd2),0,P(2),P(1),@(y) P(1)+((P(2)-y)/alpha)) ...
        - ((w*P(2))+K)*integral2(@(y,x) normpdf(x,mu1,sd1) ...
        .*normpdf(y,mu2,sd2),0,P(2),P(1),2000);

%Case 4
TPj4 = (((r-w+v)*P(2))-K)*integral2(@(x,y) normpdf(x,mu1,sd1) ...
        .*normpdf(y,mu2,sd2),P(1),2000,P(2),2000) ...
        - v*(integral2(@(x,y) y.*normpdf(x,mu1,sd1) ...
        .*normpdf(y,mu2,sd2),P(1),2000,P(2),2000));

TPj = TPj1+TPj2+TPj3+TPj4;
```

```
TPij = (TPi+TPj)/T;

Profit = -TPij;

end
```